Module 3: Probability and Statistics

Week 10 Tutorial

Binomial and Poisson distributions

Key goals for the class

1. What situations are described by a **Binomial distribution**, and how do we predict the probabilities of outcomes in this case?

2. What situations are described by a **Poisson distribution**, and how do we predict the probabilities of outcomes in this case?

3. In what circumstances can a Binomial distribution be **approximated by** a Poisson distribution?

Binomial distribution

- The **Binomial distribution** applies in random processes with **two possible outcomes**

What examples are there?

Binomial distribution

- The **Binomial distribution** applies in random processes with **two possible outcomes** with probabilities p and $1 - p$

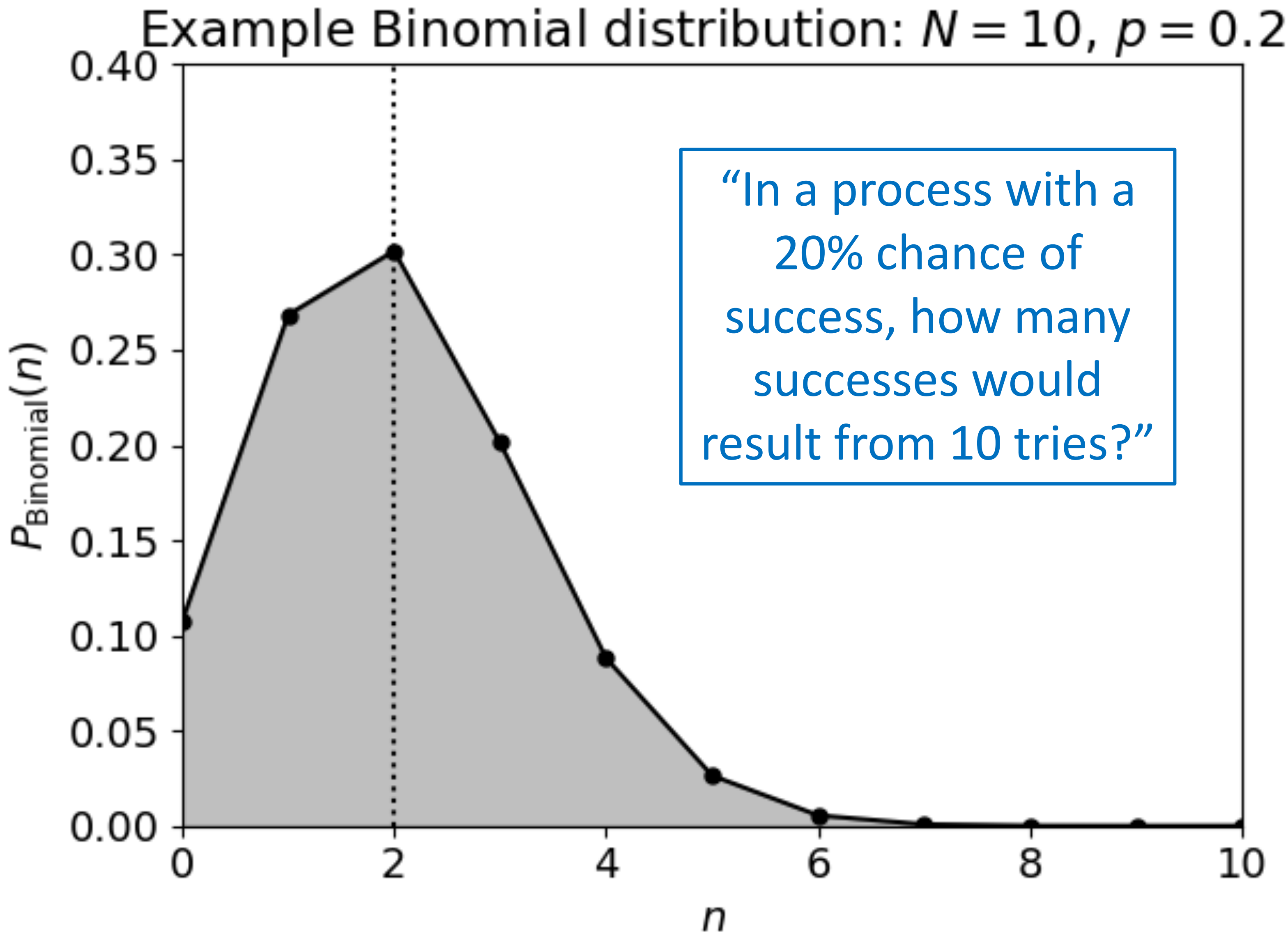
- *Example: tossing a coin*

- If we have N trials, and the probability of success in each is p, then the probability of obtaining n successes is:

$$P_{\text{Binomial}}(n) = \frac{N!}{n!\,(N-n)!}\,p^n\,(1-p)^{N-n}$$

- The **mean** and **variance** of this distribution are $\bar{n} = pN$, $\text{Var}(n) = Np(1-p)$

Binomial distribution



Tutorial question

Try Q1 on the tutorial sheet (**Binomial distribution**).

- 1. In a large factory 30% of employees is female. A ten-member union committee is chosen without regard to gender.
  - (a) What is the probability distribution function of the number of women in the committee?
  - (b) What is the probability distribution function of the number of men in the committee?
  - (c) What is the probability that 2 or fewer members of the committee are female?
  - (d) What is the probability that 2 or more members are male?
  - (e) Determine the expected value and the variance of the number of men in the committee.
  - (f) Determine the expected value and the variance of the number of women in the committee.

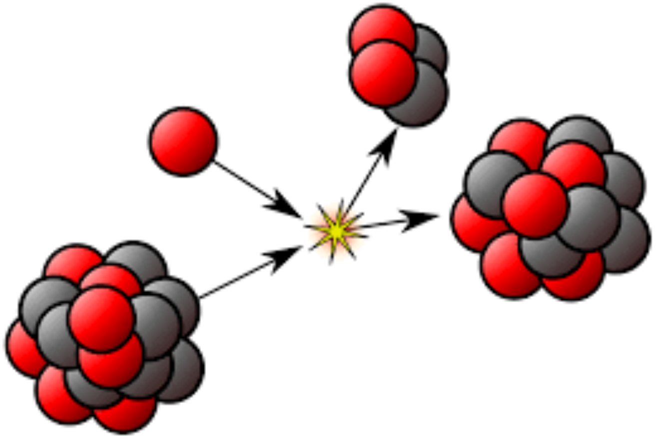
Poisson distribution

- The **Poisson distribution** applies in random processes where we are **counting something** in a fixed interval

What examples are there?

Poisson distribution

- The **Poisson distribution** applies in random processes where we are **counting something** in a fixed interval



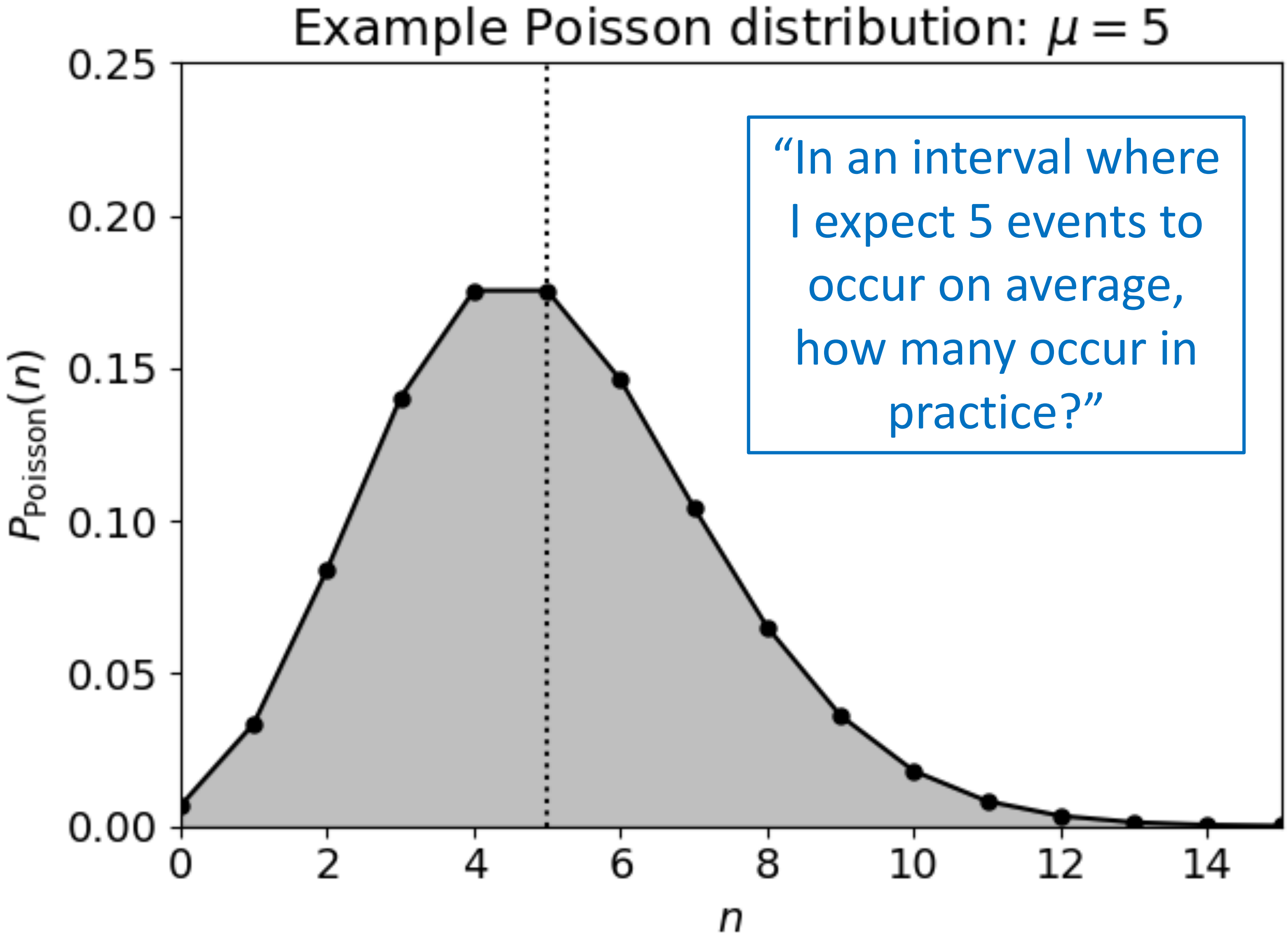
- *Example: radioactive decay, trams passing a tram stop!*

- If the mean number of events expected in some interval is μ, the probability of observing n events is

$$P_{\text{Poisson}}(n) = \frac{\mu^n \, e^{-\mu}}{n!}$$

- The **mean** and **variance** of this distribution are equal, $\bar{n} = \mathrm{Var}(n) = \mu$

Poisson distribution



"In an interval where I expect 5 events to occur on average, how many occur in practice?"

Tutorial question

Try Q3 on the tutorial sheet (**Poisson distribution**).

- 3. The average number of oil tankers arriving each day at a certain port city is known to be 4.

  (a) What is the probability distribution function of the number of oil tankers arriving each day at this port?

  (b) The facilities at the port can handle at most 6 tankers per day. What is the probability that on a given day tankers will be sent away?

  (c) Assume that for a different port, the average number of oil tankers arriving each day is also four. How many tankers per day should the facilities of this port be able to handle, if the accepted probability of failure is less than 3% ? (Note that failure corresponds to the event when some tankers are sent away).

  (d) What is the probability that port facilities will be busy the next day?

  (e) Determine the expected value and the variance of the number of tankers arriving each day at this port.

Tutorial question

Try Q4 on the tutorial sheet (**approximating the Binomial distribution with the Poisson distribution**).

- 4. Binomial distribution $X \sim \text{Binomial}(n, p)$ can be approximated by a Poisson distribution $\text{Poisson}(\lambda)$, when $n > 20$ and $p < 0.05$. If such approximation is applied, what should be the value of λ in terms of n and p? You are rolling a pair of dies 100 times. What is the probability of rolling two sixes exactly 5 times?

Takeaway: if $n > 20$ and $p < 0.05$, the Binomial distribution is very similar to a Poisson distribution with $\lambda = n \times p$

Tutorial question

Try Q2 on the tutorial sheet (**if time**).

- 2. You are tossing a symmetric coin 4 times. Determine the probability to toss three heads in a row, followed by tails. Compare with the probability of tossing three heads and one tail. Discuss the difference.

That’s all for today!