Tutorial 5: Time-Varying Fields

1. **Inductance**

   Consider two solenoids, both of length L. One has radius a and N_a turns, the other has radius $b > a$ and N_b turns. One is placed inside the other.

   (a) Work out the mutual inductance M of the system by passing current I through the *smaller* solenoid and working out the magnetic flux Φ through the *larger* solenoid.

   (b) Work out the mutual inductance M of the system by passing current I through the *larger* solenoid and working out the magnetic flux Φ through the *smaller* solenoid.

   [Hint: Your answers should be the same.]

2. **Electromagnetic induction**

   (a) What is meant by *magnetic flux* and *electromotive force* (e.m.f)? State how Faraday's Law relates e.m.f. to changing magnetic flux. Explain how Lenz's Law specifies the direction of an induced e.m.f.

   (b) A rod of length l lies parallel to the y-axis. It starts to move in the x-direction at speed u through a region in which there is a uniform magnetic field in the z-direction of strength B. If the rod has resistance R, what is the current in the rod just after it starts moving?

   (c) A generator consists of a conducting disk of radius a which rotates with angular velocity ω about its axis in a uniform magnetic field B, which is parallel to the axis. Show that the potential difference between the axis and rim of the disk is $\omega a^2 B/2$.

3. **Electromagnetic Waves**

   (a) Consider a sinusoidal electromagnetic wave of angular frequency ω propagating along the z-axis. The wave is plane polarized along the x-axis. Explain why we can write the electric field variation as

   $$\vec{E} = \begin{pmatrix} E_0 \cos{(kz - \omega t)} \\ 0 \\ 0 \end{pmatrix} \tag{1}$$

   where $k = \omega/c$, in terms of the wave speed c.

   (b) Use Maxwell's equations to find $\vec{B}$ in this case. Show that the directions of the electric field, magnetic field and direction of propagation are all perpendicular, and that the amplitudes of the field variations are related by $B_0 = E_0/c$. Sketch the resulting pattern of electric and magnetic fields at $t = 0$.