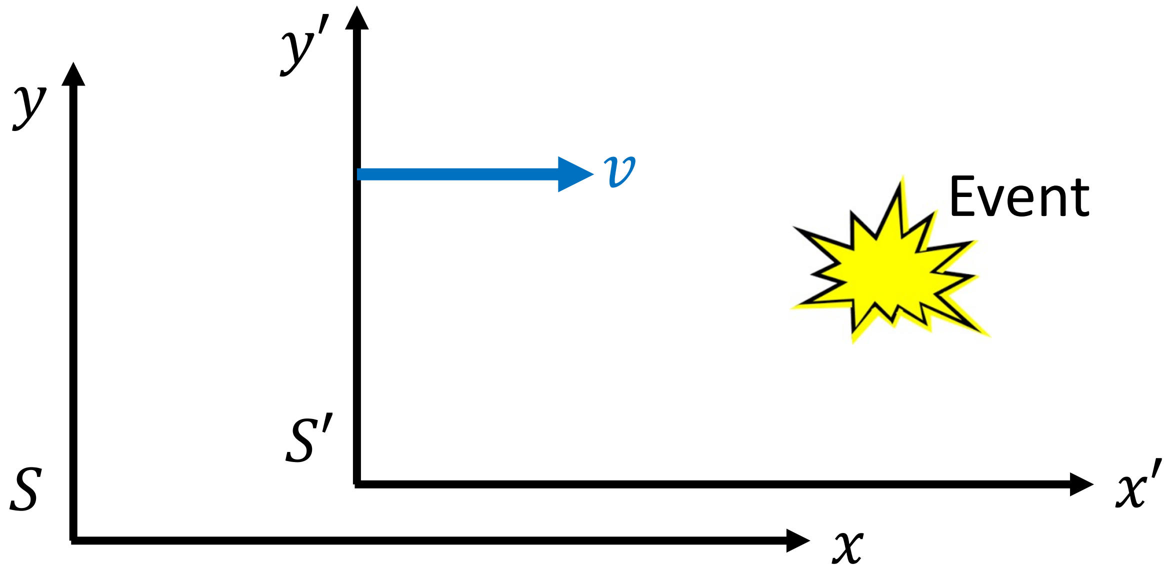
Week 7 Special Relativity Tutorial
Relativistic Kinematics

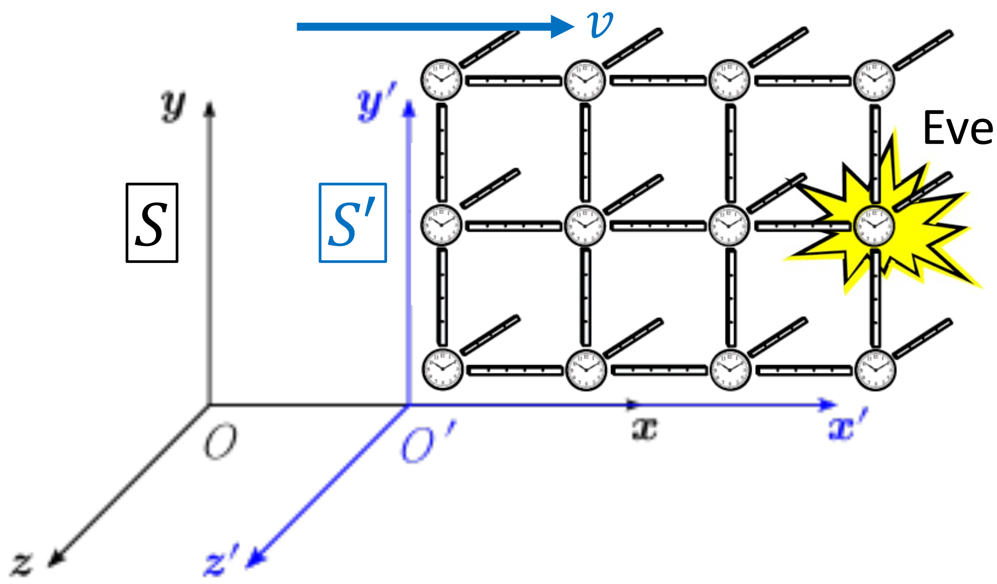
- Class Prep concept review
- Example calculations in relativistic kinematics



Class Prep concept review

Setting up a reference frame

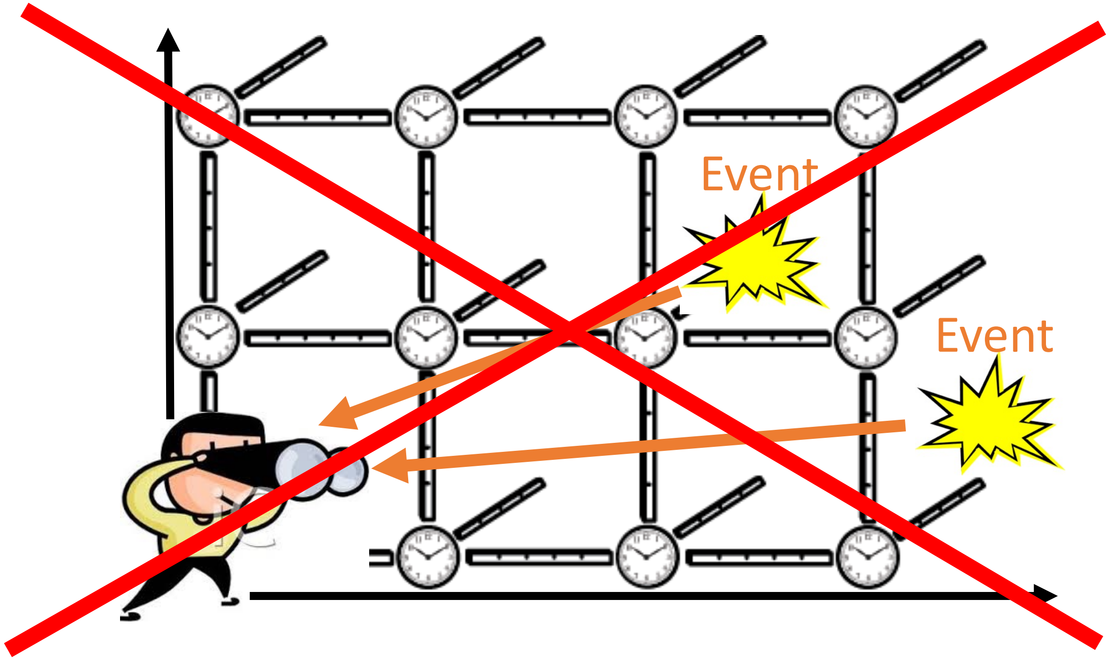
In special relativity, we assign co-ordinates to events using **reference frames**. How is a reference frame constructed?



A reference frame is constructed from many local observers with synchronized clocks on a co-ordinate grid

Setting up a reference frame

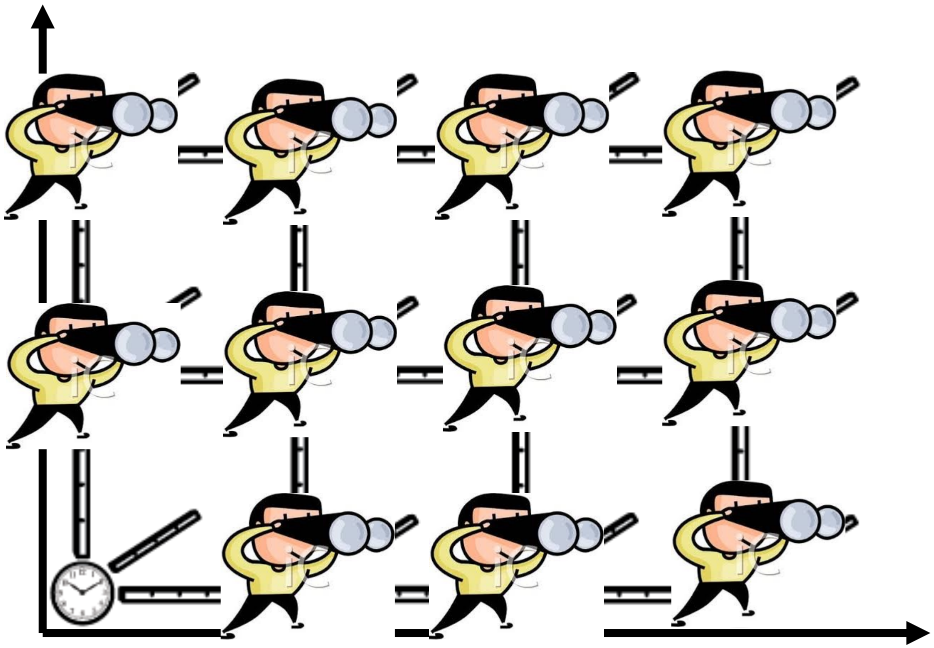
- A reference frame is constructed using **many local observers**, **not** a single observer sitting at the origin:



This is bad, because simultaneous events would not appear to be simultaneous, because of the light travel time!

Setting up a reference frame

- A reference frame is constructed using **many local observers**, **not** a single observer sitting at the origin:

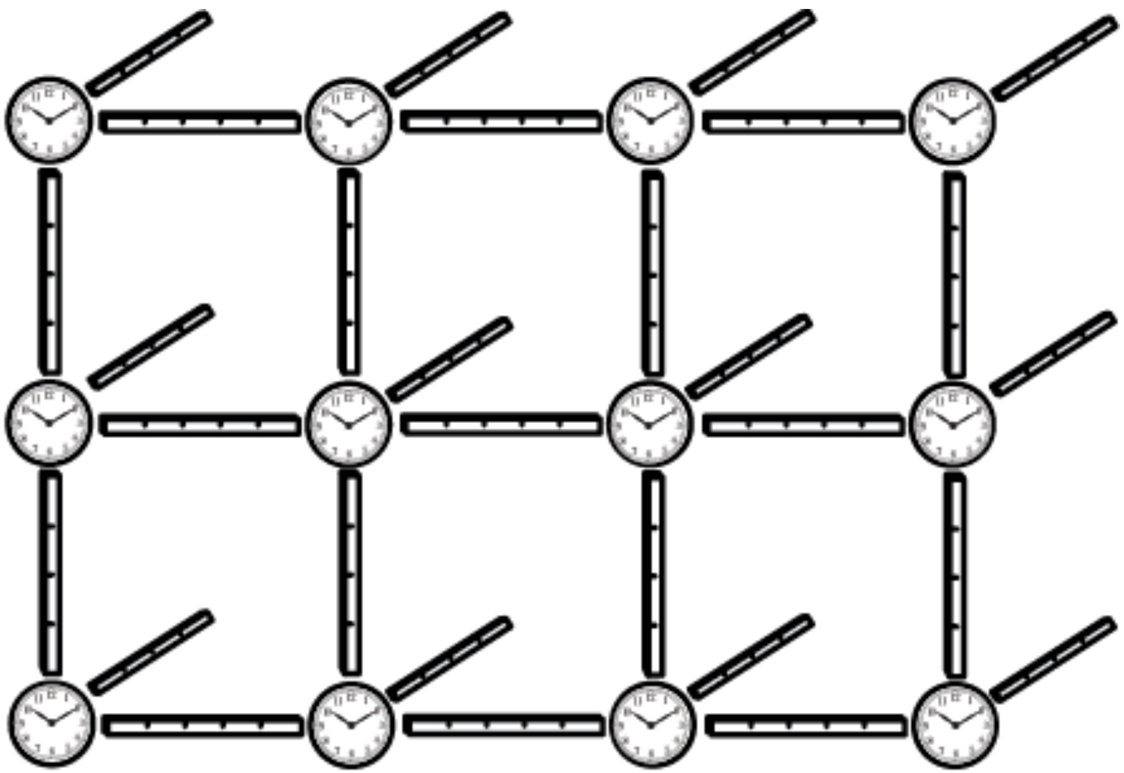


This is good, because simultaneous events are measured as simultaneous!

- We always record events **at their space-time location**, rather than worrying about the travel time of light

Setting up a reference frame

What methods could local observers at each vertex of a given reference frame use to **synchronize their clocks**?



Setting up a reference frame

- What is an **inertial** reference frame?
- How would you know if you were in a **non-inertial** frame?

In an inertial frame, a freely-moving object moves with constant velocity

In a non-inertial frame, observers will notice "weird pseudo-forces"

Postulates of relativity

- What is the **principle of relativity**? (not due to Einstein!)
- What is Einstein's **postulate of special relativity**?

Principle of relativity: *"The laws of nature are identical in all inertial frames"*

… also known as …

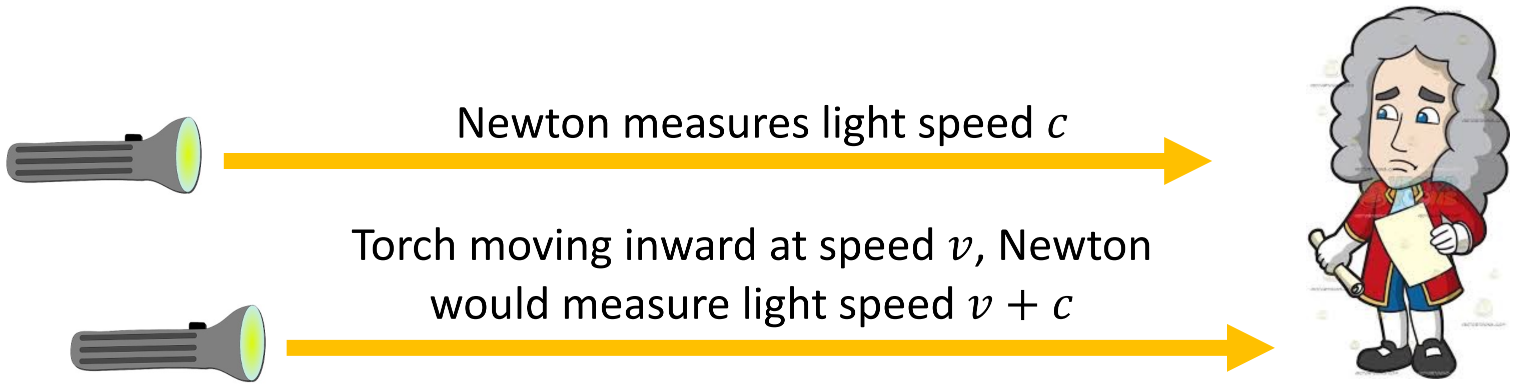
"Without looking out of the aeroplane window, we can't tell whether we're in smooth flight or on the ground"

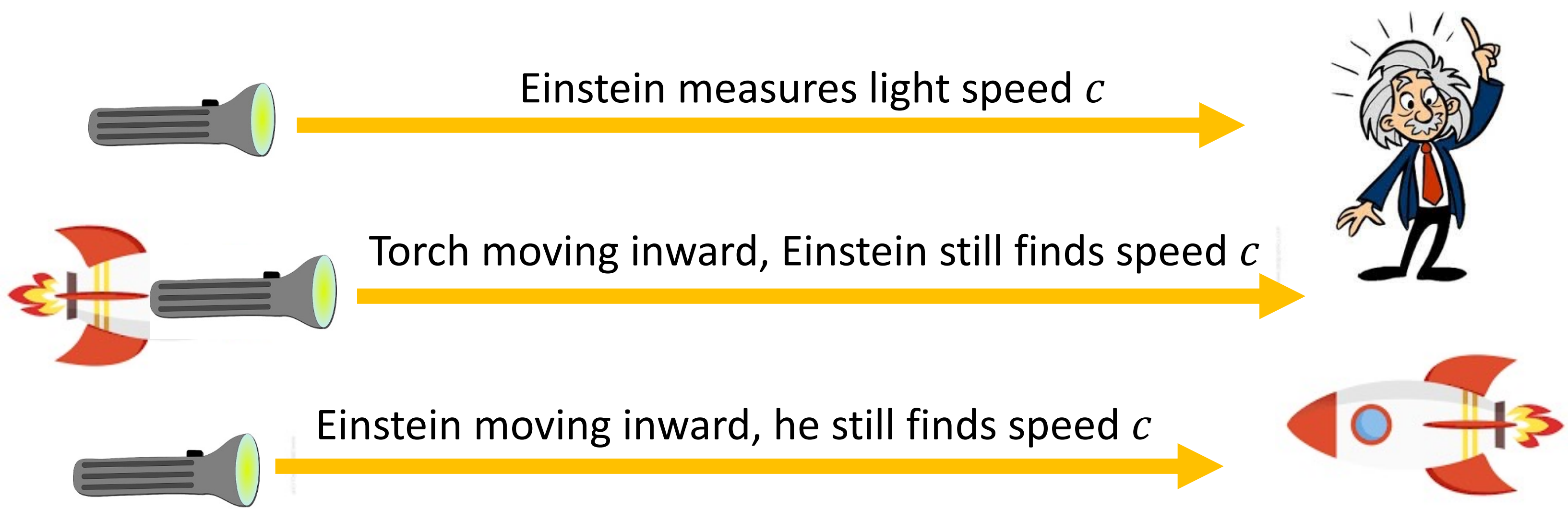
Einstein's postulate:

The speed of light is constant in all inertial reference frames, independent of the motion of the source and observer

Postulates of relativity

The speed of light is constant in all inertial reference frames, independent of the motion of the source and observer





Postulates of relativity

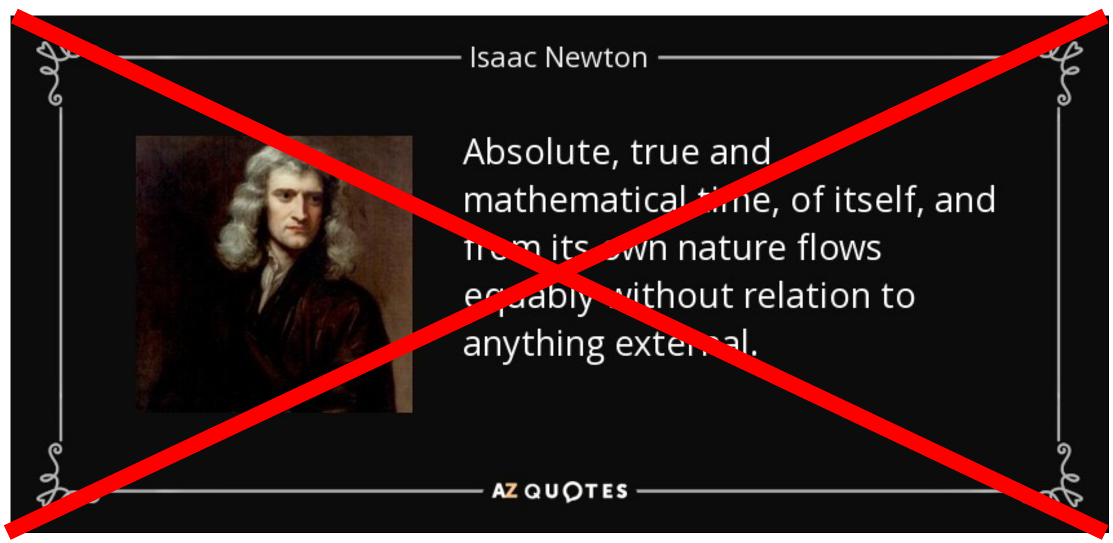
In what ways does Einstein's postulate of special relativity **contradict classical physics**?

Postulates of relativity

In what ways does Einstein’s postulate of special relativity **contradict classical physics**?

There is no such thing as absolute time!

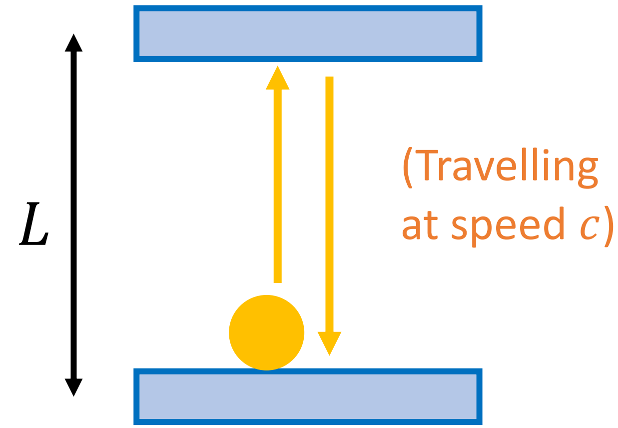
- Two events which are simultaneous in one reference frame are not simultaneous in another frame
- Moving clocks run slow
- Moving objects are shorter
- Information cannot travel faster than the speed of light
- Relative velocities do not add and subtract



Time dilation

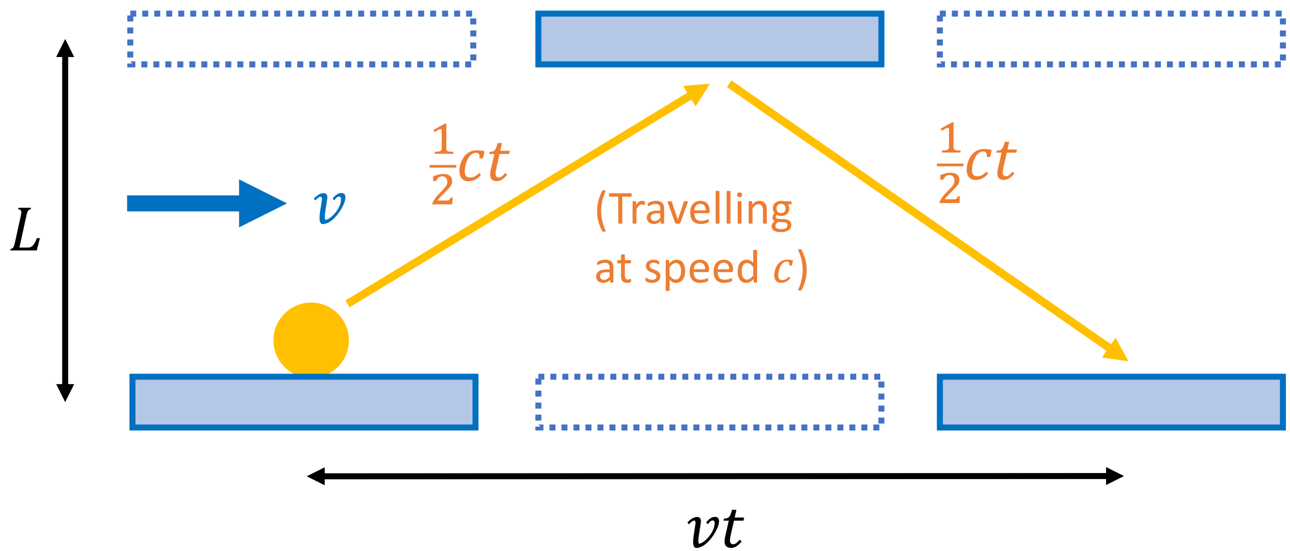
- Here's how Einstein's postulate leads to time dilation via a very simple clock!

Here's the clock in its rest frame:



Time period
$t = 2L/c$

Here's a moving clock:



Pythagoras' theorem:

$$\left(\frac{vt}{2}\right)^2 + L^2 = \left(\frac{ct}{2}\right)^2$$

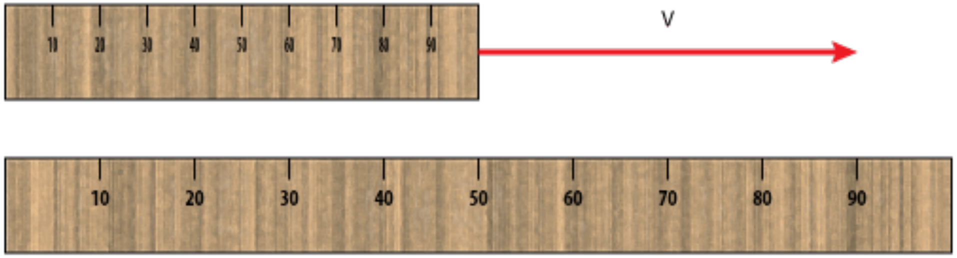
Re-arrange:

$$t = \frac{2L}{c}\frac{1}{\sqrt{1 - v^2/c^2}}$$

Predictions of relativity

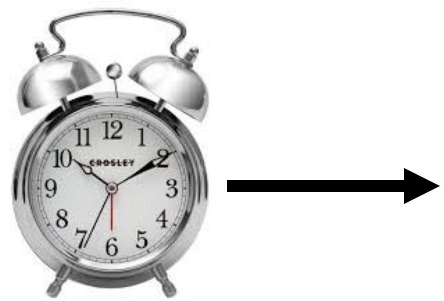
Time dilation:
Clock ticks are further apart, if recorded in a frame through which the clock is moving

Length contraction:
The length of an object is measured to be shorter in a frame in which it is moving



Predictions of relativity

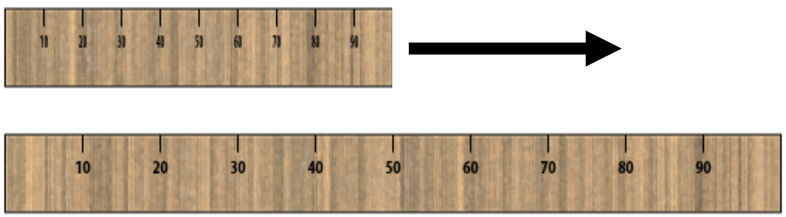
Time dilation



$$\Delta t = \gamma\,\Delta t_{\text{proper}}$$

Δt_{proper} is the time interval in the frame in which the events occur at the same place (e.g. the clock frame), Δt is measured in a moving frame

Length contraction



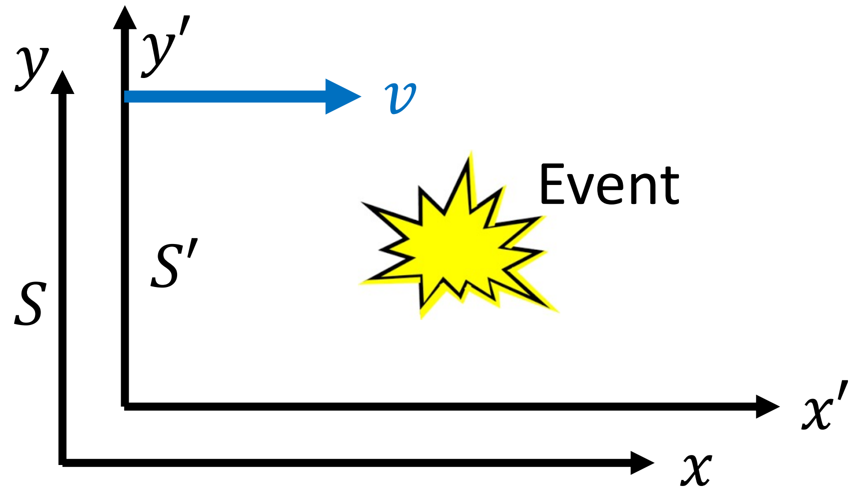
$$L = L_{\text{proper}}/\gamma$$

L_{proper} is the length in the frame in which the object is at rest (e.g. the ruler frame), L is measured in the frame in which the object is moving

$\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$ is from the relative speed v of the moving frame

Lorentz transformations

- The Lorentz transformations are the **algebraic relations for the co-ordinates of an event** in two different inertial reference frames



Lorentz transformations:

$$t' = \gamma(t - vx/c^2)$$
$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$

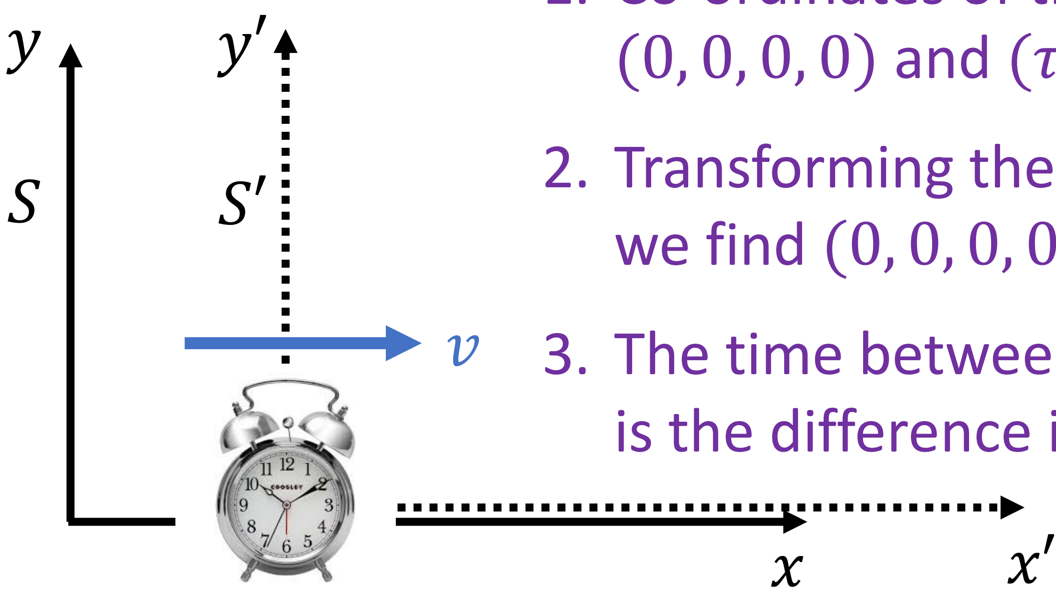
Inverse transformations:

$$t = \gamma(t' + vx'/c^2)$$
$$x = \gamma(x' + vt')$$
$$y = y'$$
$$z = z'$$

Time dilation

A clock at rest at the origin of S' ticks at times $t' = 0$ and $t' = \tau$.

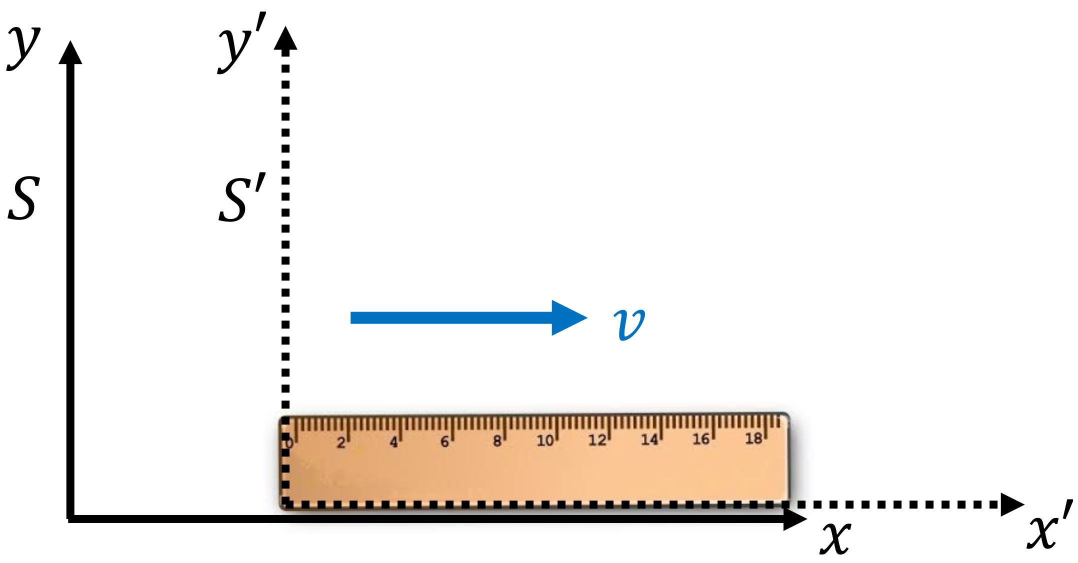
1. What are the co-ordinates (t', x', y', z') of these events in S'?
2. Use the Lorentz transformations to determine the space-time co-ordinates (t, x, y, z) of these events in S.
3. What is the time between the ticks as measured in S?



1. Co-ordinates of these ticks in S' are $(0, 0, 0, 0)$ and $(\tau, 0, 0, 0)$
2. Transforming these to S with the inverse LTs, we find $(0, 0, 0, 0)$ and $(\gamma\tau, \gamma v\tau, 0, 0)$
3. The time between the ticks as measured in S is the difference in t co-ordinates, $\Delta t = \gamma\tau$

Length contraction

Consider a ruler at rest in S', with ends at $x' = 0$ and $x' = L$. Observers in S measure the length of the ruler by **marking the positions of the 2 ends simultaneously** in S, at time t *(these are the 2 events)*. Use the Lorentz transformations to determine the length of the ruler as measured in S.



$$t' = \gamma(t - vx/c^2)$$

$$x' = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$

Length contraction

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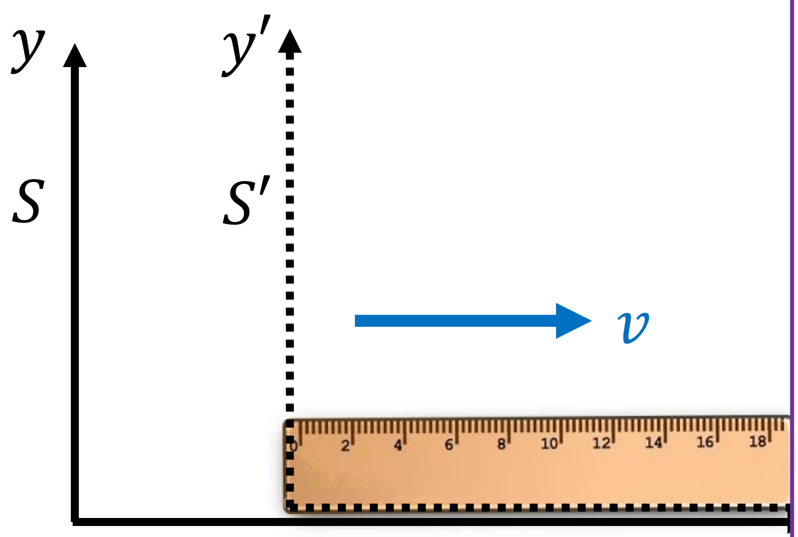


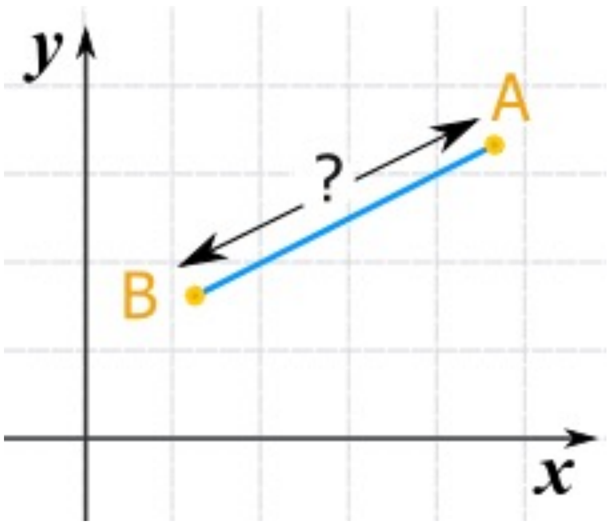
- The co-ordinates of the 2 events in S are $(t, x, y, z) = (T, x_1, 0,0)$ and $(t, x, y, z) = (T, x_2, 0,0)$ where length $= x_2 - x_1$
- The co-ordinates in S' are $(t', x', y', z') = (t_1', 0,0,0)$ and $(t_2', L, 0,0)$
- Using the "$x' = \gamma(x - vt)$" Lorentz transformation: $x_2 - x_1 = L/\gamma$

Space-time interval

- The space-time interval between two events, which are separated in space-time by co-ordinate differences $(\Delta x, \Delta y, \Delta z, \Delta t)$, is $\boldsymbol{\Delta s^2 = -c^2\Delta t^2 + \Delta x^2 + \Delta y^2 + \Delta z^2}$

- The space-time interval is an **invariant** – what does this mean?

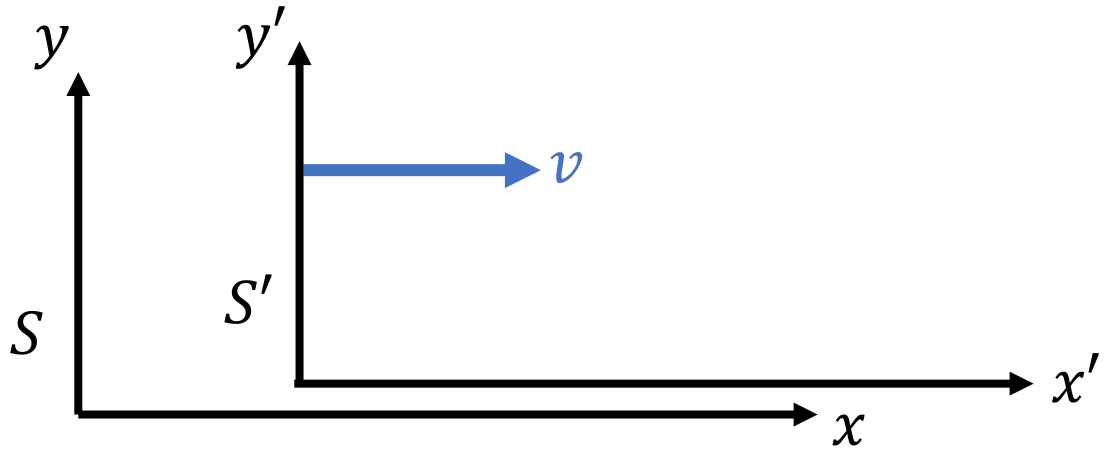
- We can show this is the case using the two ticks of the clock: space-time interval is $\Delta s^2 = -c^2\tau^2$ in both frames



The **space-time interval in relativity** (same in all inertial frames) is analogous to a **distance in Euclidean geometry** (same in all rotated frames)

Some other invariants …

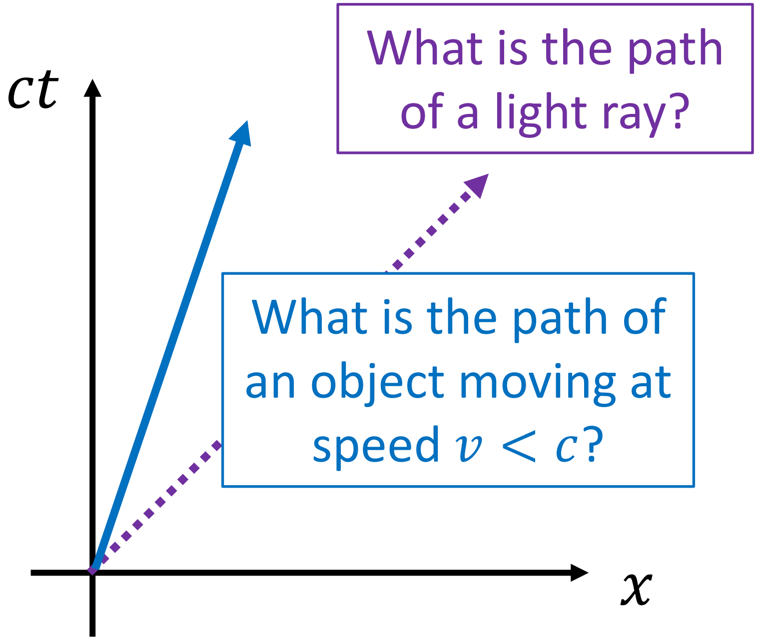
Which of the following quantities are **necessarily the same** when measured in two inertial frames, S and S'? Which are not?



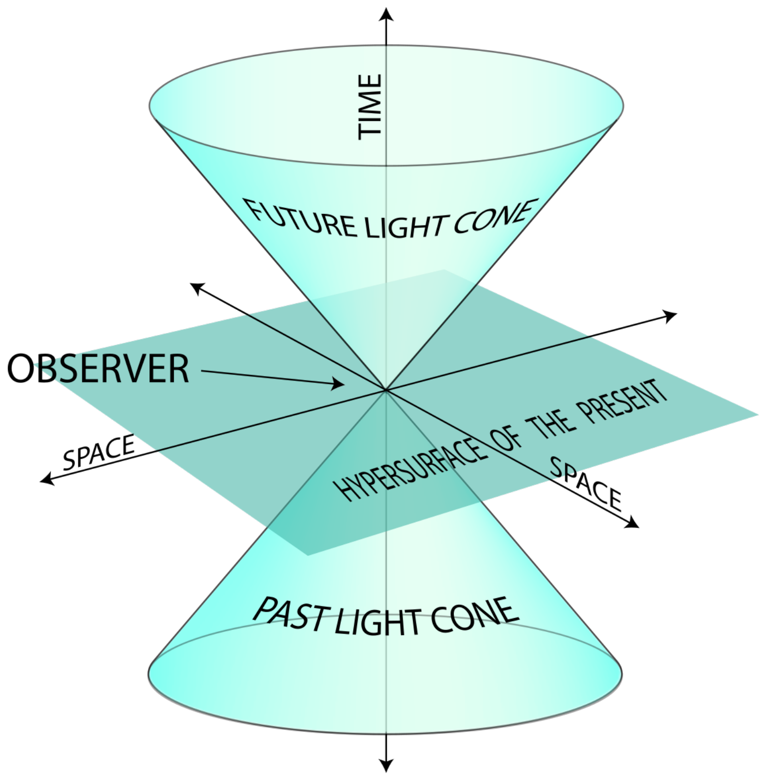
- Value of the speed of light in a vacuum
- Speed of an electron
- Value of the charge on an electron
- Kinetic energy of a proton
- Distances in the y-direction
- Value of the electric field at a point
- Time between two events
- Order of elements in the periodic table
- Newton's First Law of Motion
- Distances in the x-direction

Space-time diagrams

- A space-time diagram is a graph showing the **position of objects (events) in a reference frame, as a function of time**



What is represented by a light cone?



Causality

How does the space-time interval help us answer the question: **can one event cause another event**?

- To be causally connected, we require $|\Delta x| < c|\Delta t|$, or the **space-time interval** $\Delta s^2 = -c^2\Delta t^2 + \Delta x^2$ **is negative**
- The space-time interval is an invariant – so **if 2 events are causally connected in one frame, they are in all frames –** i.e. relativity preserves cause-and-effect

Example calculations in relativistic kinematics

Example 1

As part of Swinburne's Physics class in the year 2122, you suggest a rather ambitious 3rd year Grand Challenge project to visit the closest star Proxima Centauri, located 4.2 light years from the Earth.

Your supervisor (who may or may not know any relativity) complains that it will take you at least 4.2 years to get there, since you cannot travel faster than light, but you only have 3 months to finish the project! However, having recently studied time dilation, you know better ...

1. How fast would you need to travel in order to reach Proxima Centauri in 3 months, according to your own wristwatch?
2. How can your rocket ship travel 4.2 light years in only 3 months? Doesn't that involve faster-than-light travel?
3. Is it possible to hand in your project back at Swinburne on time?

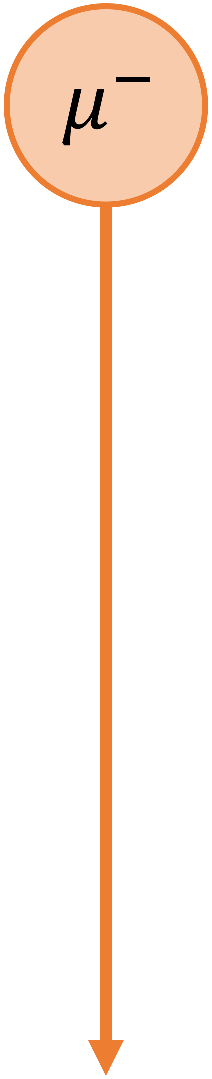
Example 2

Particles called muons are produced in the Earth's atmosphere by cosmic rays. Muons travel at speed $v = 0.999c$, and decay into an electron after average time $\tau = 2\ \mu s$, as measured in a frame of reference in which the muon is at rest.

1. Ignoring the effects of relativity, how far does an average muon travel in its lifetime?

2. Many muons reach the ground, $10\ km$ away. **Consider the situation from the ground frame**. Using time dilation, what is the muon lifetime measured by ground observers? How far can a muon travel, as measured by ground observers?

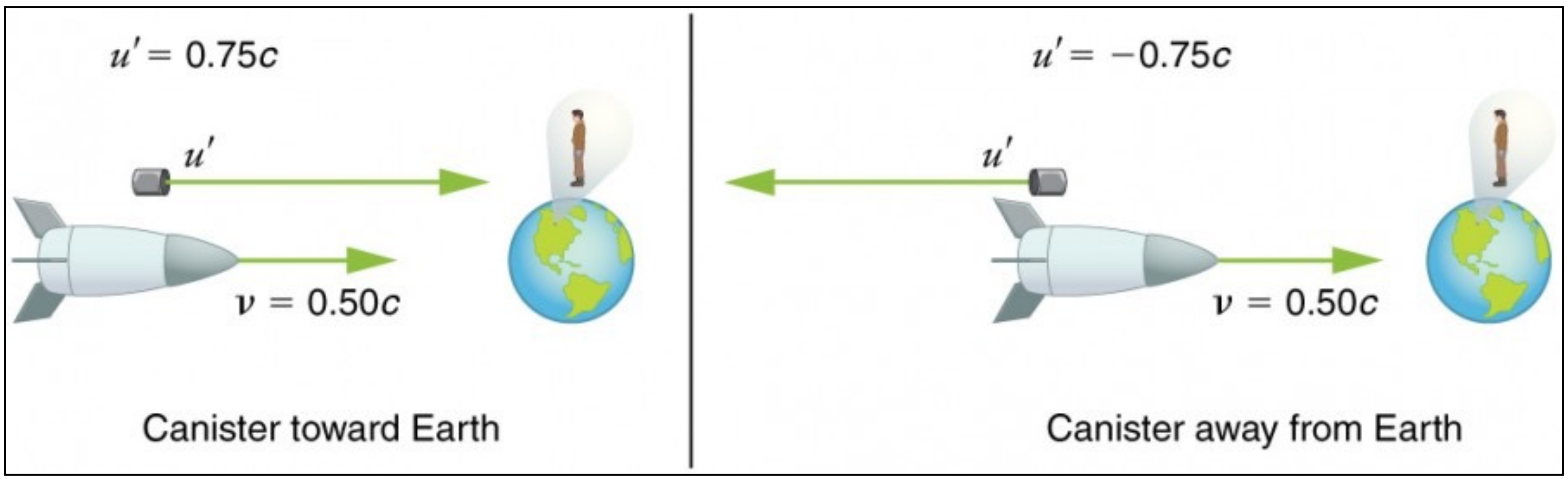
3. Now **consider the situation from the muon rest frame**. Using length contraction, by what factor is distance contracted when measured by muon observers? How far can a muon travel in the ground frame before decaying?



Example 3

The Amazon space delivery service, a rocket moving at speed $v = 0.5\ c$, fires a package towards the Earth at speed $u' = 0.75\ c$ in the rocket frame. What is the package speed as measured by the Earth observer?

How does your answer change if the package is accidentally fired in the opposite direction, at the same speed in the rocket frame?



Combination of velocities: $u = \dfrac{u' + v}{1 + u'v/c^2}$ $\quad u' = \dfrac{u - v}{1 - uv/c^2}$

Example 4

The Doppler effect is more complicated in relativity! A relativity student is caught running a red light on the rocket freeway. In court they plead that they were driving so fast that the red light ($\lambda = 650$ nm) looked green ($\lambda = 530$ nm). How fast would their rocket car have been travelling?

$$\lambda_o = \lambda_e \sqrt{\frac{1 + v/c}{1 - v/c}} \text{ (receding)} \qquad \lambda_o = \lambda_e \sqrt{\frac{1 - v/c}{1 + v/c}} \text{ (approaching)}$$

Example 5

A spaceship is approaching a planet-moon system with speed $v = 0.98\ c$ on a straight line that will take it first past the moon and then past the planet. The distance between the moon and the planet is measured to be $4{\times}10^8$ m in the spaceship's reference frame, S'.

The spaceship detects a burst of radiation on the moon (Event 1) at time $t' = 0$, and an explosion on the planet (Event 2) at time $t' = 1.1$ s, where these times are measured in the spaceship's reference frame, S'.

1. What is the distance between the planet and the moon, as measured in the planet-moon reference frame, S?
2. What is the time interval between the burst of radiation and the explosion, as measured in the planet-moon reference frame, S?
3. Could the burst of radiation cause the explosion?

That’s all for today!