PHYSICS 2B: SPECIAL RELATIVITY WORKBOOK

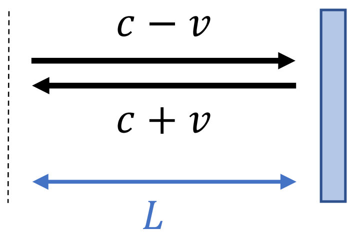
Class 1: Frames and Events

1) Write a brief statement explaining what is meant by an "inertial frame".

2) Write down the "principle of relativity".

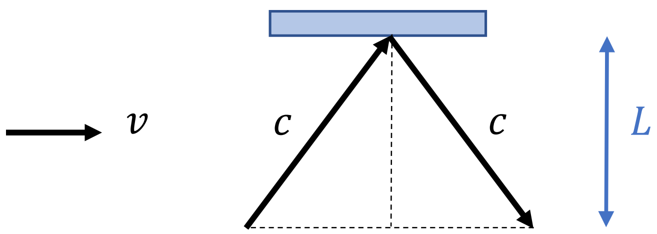
3) The Michelson-Morley experiment provided strong evidence against changes in light speed from relative motion. Let's compute the difference in light travel time of the two beams, predicted using the Galilean transformations.

First, consider the beam oriented parallel to the Earth's motion:



What is the total travel time?

Now, consider the beam oriented perpendicular to the Earth's motion:

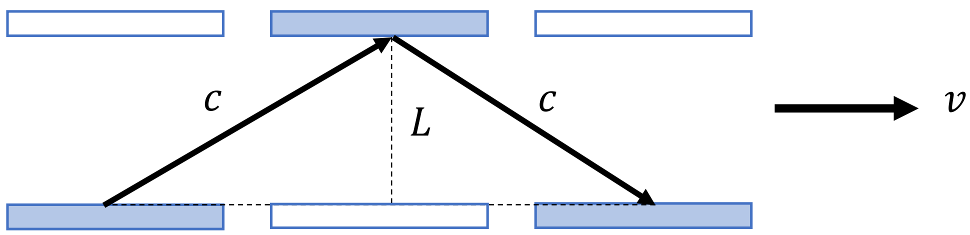


What is the total travel time?

Class 2: Einstein's Postulate

1) Write down Einstein's postulate regarding the speed of light in different frames.

2) Calculate the time period of the moving light clock from its geometry:



3) Write a concise statement describing the phenomenon of "time dilation".

4) Write a definition of the "proper time difference" between 2 events.

5) Write a concise statement describing the phenomenon of "length contraction".

6) How fast would you need to travel to reach a star located 4.2 light years away in only 3 months, as measured by your own wristwatch?

Class 3: Lorentz Transformations

1) Write down the Lorentz transformations, which relate the space-time co-ordinates of an event in frames S and S'.

2) A clock at rest at the origin of S' ticks at times $t' = 0$ and $t' = \tau$.

a. What are the full space-time co-ordinates (x', y', z', t') of these 2 events in S'?

b. Use the Lorentz transformations to determine the space-time co-ordinates (x, y, z, t) of these 2 events in S.

c. What is the time difference between the ticks as measured in S?

3) Consider a ruler at rest in S', with ends at $x' = 0$ and $x' = L$. Observers in S measure the length of the ruler by marking the positions of the two ends simultaneously in S, at time t. Use the Lorentz transformations to determine the length of the ruler as measured in S.

4) The space-time interval, Δs^2, is an important quantity in relativity.

a. Write down the definition of the space-time interval between two events.

b. Compute the space-time interval between the two events in Q2), in S and S'.

c. What is significant about the value of Δs^2?

5) Addition of velocities is more complicated in relativity.

a. The Amazon-space delivery service, a rocket moving at speed $v = 0.5c$, fires a package towards the Earth at speed $u' = 0.75c$ in the rocket frame. What is the package speed as measured by the Earth observer?

b. How does your answer change if the package is accidentally fired in the opposite direction, at the same speed in the rocket frame?

Class 4: Space-time Diagrams

1) The event E_1 is at the origin of a space-time diagram. Sketch the regions of the diagram containing (a) events which can be caused by E_1, (b) events which can cause E_1.

2) Consider an event E_2 which takes place at the same spatial location in S as E_1. What mathematical curve does the event E_2 trace out in a space-time diagram, when it is viewed from a series of inertial frames S' with varying speeds with respect to S?

3) Consider 4 events which take place in frame S at space-time co-ordinates $(x, ct) = (0,0), (1,0), (0,1), (1,1)$. Using the Lorentz transformations, plot these events in a space-time diagram for S' with axes (x', ct'), if $v = 0.6c$.

Class 5: Energy and Momentum

1) Write down an expression for the relativistic momentum of a particle of rest-mass m_0 travelling with speed u. Show that at low speeds, $u \ll c$, we recover the classical formula, $p = m_0 u$.

2) What is the expression for the relativistic kinetic energy of the particle? Show that at low speeds, we recover the classical formula, $T = \frac{1}{2} m_0 u^2$.

3) Estimate the rest-mass energy of a human!

4) A mass of 1 kg, moving at speed $u = 0.8c$, is absorbed by a stationary mass of 2 kg. At what speed U does the combined mass recoil? What is the combined mass M?

a. Write down the total relativistic momentum before, and after, the collision.

b. Write down the total relativistic energy before, and after, the collision.

c. Eliminate variables between these two equations to solve for the two unknowns, U and M.

Class 6: Relativistic Phenomena

1) The Sun generates energy by fusing hydrogen into helium. The mass of a helium nucleus is $4.001503\ au$, where $1\ au = 1.66\times10^{-27}\ kg$, and the combined mass of the products forming the nucleus is $4.03188\ au$. If the luminosity of the Sun is $3.8\times10^{26}\ W$, what mass of hydrogen is converted in the Sun every second?

2) In the process of pair production, a photon produces an electron and positron pair.

a. Using mass-energy equivalence, what photon threshold energy is required to produce the electron-positron pair?

b. What wavelength of radiation is required to achieve this?

3) In a Compton scattering event, a photon of wavelength λ_1 interacts with a stationary electron of mass m_e, and is scattered backward with new wavelength λ_2.

a. Write an expression for the momentum of the electron after the collision, p_e, in terms of λ_1 and λ_2.

b. Write an expression for the energy of the electron after the collision, E_e, in terms of λ_1, λ_2 and m_e.

c. Use the relation ${E_e}^2 - (p_e c)^2 = (m_e c^2)^2$ to find an expression for $\lambda_2 - \lambda_1$.

4) A relativity student is caught running a red light on the rocket freeway. In court they plead that they were driving so fast that the red light ($\lambda = 650\ nm$) looked green ($\lambda = 530\ nm$). How fast would their rocket car have been travelling?