Quantum Mechanics Week 4: Class Prep solutions

Q1)

$$\begin{pmatrix} \hat{L}_x \\ \hat{L}_y \\ \hat{L}_z \end{pmatrix} = -i\hbar \begin{pmatrix} y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y} \\ z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z} \\ x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x} \end{pmatrix}$$

Q2) Consider:

$$\begin{aligned}
&[\hat{L}_x, \hat{L}_y]\, f = \hat{L}_x\hat{L}_y f - \hat{L}_y\hat{L}_x f \\
&= -i\hbar\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right)\cdot -i\hbar\left(z\frac{\partial f}{\partial x} - x\frac{\partial f}{\partial z}\right) - -i\hbar\left(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\right)\cdot -i\hbar\left(y\frac{\partial f}{\partial z} - z\frac{\partial f}{\partial y}\right) \\
&= -\hbar^2\left[y\frac{\partial f}{\partial x} + yz\frac{\partial^2 f}{\partial z\,\partial x} - xy\frac{\partial^2 f}{\partial z^2} - z^2\frac{\partial^2 f}{\partial y\,\partial x} + zx\frac{\partial^2 f}{\partial y\,\partial z}\right] \\
&\qquad + \hbar^2\left[zy\frac{\partial^2 f}{\partial x\,\partial z} - z^2\frac{\partial^2 f}{\partial x\,\partial y} - xy\frac{\partial^2 f}{\partial z^2} + x\frac{\partial f}{\partial y} + xz\frac{\partial^2 f}{\partial z\,\partial y}\right] \\
&\qquad = -\hbar^2\left(y\frac{\partial f}{\partial x} - x\frac{\partial f}{\partial y}\right) \\
&\qquad = i\hbar\hat{L}_z f
\end{aligned}$$

Q3) L^2, and any single component of (L_x, L_y, L_z), are simultaneous observables.

Q4) The operator for $\hat{L}_z$ in spherical co-ordinates is $-i\hbar\frac{\partial}{\partial\phi}$. Applying this operator to the proposed eigenfunction,

$$\hat{L}_z(e^{im\phi}) = -i\hbar\frac{\partial}{\partial\phi}(e^{im\phi}) = -i\hbar\cdot im(e^{im\phi}) = m\hbar\,(e^{im\phi})$$

Hence $e^{im\phi}$ is an eigenfunction of $\hat{L}_z$ with eigenvalue $m\hbar$.

Q5)

- The eigenvalues of $\hat{L}^2$ are $l(l+1)\hbar^2$ where l is an integer
- The eigenvalues of $\hat{L}_z$ are $m\hbar$ where m is an integer

Q6) The eigenstates can be characterised by quantum numbers l and m.

- There are 7 states with $l = 3$, where $m = -3, -2, -1, 0, 1, 2, 3$
- There are infinite states with $m = 3$, since $l = 3, 4, 5, 6, \ldots$