Quantum Mechanics Week 4: Class Prep

Please study this worksheet before Wednesday's Week 4 tutorial, and submit your solutions to the Class Prep exercises by the end of Tuesday. The estimated total time requirement is 2 hours.

Overview

This week we'll explore how angular momentum is used in Quantum Mechanics! This is essential for studying quantum systems in three dimensions, such as atoms. We'll learn about:

- The quantum operators representing angular momentum, and their properties
- The observable values of angular momentum, and their representation as quantum numbers
- The eigenfunctions of angular momentum, also known as the spherical harmonic functions

This worksheet will help you get the most out of this week's tutorial, by covering some introductory aspects in advance of the class.

Resources for Learning

Please study the Week 4 resources. The estimated time requirement is 1 hour.

Video: The Week 4 video summarises the topics we'll study this week. The running time is 28:17.

Textbook: You can find more information on these topics in Griffiths: Chapter 4.3.

You are free to search for and use other resources in addition to (or instead of) the above, as long as you can answer the following Class Prep exercises.

Class Prep Exercises

Please answer these short exercises on paper and submit your scanned answers by the end of Tuesday. After studying the above resources, the estimated time requirement for these exercises is 1 hour.

1) Let's get to know the operators representing the angular momentum of a particle! Write down the operators for $\hat{L}_x$, $\hat{L}_y$ and $\hat{L}_z$ in terms of (x, y, z) co-ordinates.

2) For any function $f(x, y, z)$, show that $[\hat{L}_x, \hat{L}_y]f = i\hbar\hat{L}_z f$, which is written as the commutation relation $[\hat{L}_x, \hat{L}_y] = i\hbar\hat{L}_z$.

3) Which subset of the observables (L_x, L_y, L_z, L^2) are simultaneous observables?

4) What is the operator for $\hat{L}_z$ in spherical co-ordinates? Show that $e^{im\phi}$ is an eigenfunction of $\hat{L}_z$ with eigenvalue $m\hbar$.

5) If Y is an angular momentum eigenfunction (i.e., an eigenfunction of $\hat{L}^2$ and $\hat{L}_z$), what are the corresponding eigenvalues of $\hat{L}^2$ and $\hat{L}_z$ and what values can they take?

6) The angular momentum eigenstates may be characterised by quantum numbers l and m. How many distinct eigenstates are possible with $l = 3$? How many distinct eigenstates are possible with $m = 3$?

Grading and getting help

How this is graded: Class Prep assignments are always graded on *completeness & effort*. If you submit a reasonable effort for all parts on time, you will receive full marks regardless of all details being correct. Otherwise, you will not receive a grade. Each Class Prep assignment represents 1% of the unit grade.

After you have studied this week's learning resources, these exercises are not supposed to take more than 1 hour to complete. If you've been working purposefully on these exercises for 30 minutes and you're struggling with the content, please stop and ask for help. You can work with a friend, e-mail the instructor (cblake@swin.edu.au) or ask a question on the Week 4 Discussion Board in Canvas.

The rest of this week's activities

In Wednesday's tutorial class we'll expand on the Class Prep exercises to:

- Verify the full eigenfunctions of angular momentum
- Demonstrate the orthogonality and normalisation of the spherical harmonic functions by integration over spherical co-ordinates
- Analyse a given wavefunction on the sphere to find the possible results of angular momentum measurements and their relative probabilities.

Finally, **please complete the Week 4 Online Quiz** (10 multiple choice questions) by the end of Sunday Week 4. Each Online Quiz represents 1% of the unit grade.