Quantum Mechanics Week 3: Class Prep

Please study this worksheet before Wednesday's Week 3 tutorial, and submit your solutions to the Class Prep exercises by the end of Tuesday. The estimated total time requirement is 2 hours.

Overview

This week we'll study how to solve the Schrödinger equation in 1 dimension, for particles bound in a potential well, and for beams of particles. We'll learn about:

- How to use boundary and continuity conditions to determine the energy eigenfunctions and eigenvalues of particles bound in a finite or infinite potential well
- The energy eigenstates and other properties of the quantum harmonic oscillator
- How beams of particles are represented in quantum mechanics, and how to determine their reflection and transmission probabilities at potential steps or barriers

This worksheet will help you get the most out of this week's tutorial, by covering some introductory aspects in advance of the class.

Resources for Learning

Please study the Week 3 resources. The estimated time requirement is 1 hour.

Video: The Week 3 video summarises the topics we'll study this week. The running time is 29:45.

Textbook: You can find more information on these topics in Griffiths: Chapter 2.2-2.4, 2.6.

You are free to search for and use other resources in addition to (or instead of) the above, as long as you can answer the following Class Prep exercises.

Class Prep Exercises

Please answer these short exercises on paper and submit your scanned answers by the end of Tuesday. After studying the above resources, the estimated time requirement for these exercises is 1 hour.

1) State the boundary and continuity conditions that a wavefunction $\psi(x)$ that is a solution to the time-independent Schrödinger equation must satisfy.

2) Write down the time-independent Schrödinger equation for the quantum harmonic oscillator, which has potential $V(x) = \frac{1}{2}m\omega^2 x^2$.

3) If $a = m\omega/2\hbar$, show that $\psi_1(x) = Ne^{-ax^2}$ is a solution of this equation, where N is a normalisation constant you don't need to determine, and find the corresponding value of the energy eigenvalue E. (This is the lowest energy eigenfunction of the oscillator.)

4) In the class slides we met the "ladder operator" for the harmonic oscillator,

$$\hat{A}_+ = \sqrt{\frac{m\omega}{2\hbar}}\,\hat{x} - \frac{i}{\sqrt{2m\omega\hbar}}\,\hat{p} = \sqrt{a}\,x - \frac{1}{2\sqrt{a}}\frac{d}{dx}$$

where $\hat{x}$ and $\hat{p}$ are the position and momentum operators. Show that the result of applying the ladder operator $\hat{A}_+$ to $\psi_1(x)$ produces the function $\psi_2(x) \propto x\,e^{-ax^2}$.

5) Show that $\psi_2(x)$ is also an energy eigenfunction of the Schrödinger equation for a harmonic oscillator, and find the corresponding energy eigenvalue.

6) Hence, write a sentence describing what the ladder operator $\hat{A}_+$ does.

7) The wavefunction of a beam of free particles can be represented by, $\Psi(x,t) = Ne^{i(kx-\omega t)}$. What is the physical interpretation of N, given that $\int_{-\infty}^{\infty} |\Psi(x,t)|^2\,dx = \infty$?

Grading and getting help

How this is graded: Class Prep assignments are always graded on *completeness & effort*. If you submit a reasonable effort for all parts on time, you will receive full marks regardless of all details being correct. Otherwise, you will not receive a grade. Each Class Prep assignment represents 1% of the unit grade.

After you have studied this week's learning resources, these exercises are not supposed to take more than 1 hour to complete. If you've been working purposefully on these exercises for 30 minutes and you're struggling with the content, please stop and ask for help. You can work with a friend, e-mail the instructor (cblake@swin.edu.au) or ask a question on the Week 3 Discussion Board in Canvas.

The rest of this week's activities

In Wednesday's tutorial class we'll expand on the Class Prep exercises to:

- Apply boundary/continuity conditions to solve the Schrödinger equation for bound states of a finite potential well
- Evaluate the reflection/transmission fractions for particles incident on a potential barrier

Assignment 1 is due to be submitted by the end of Friday Week 3.

Finally, **please complete the Week 3 Online Quiz** (10 multiple choice questions) by the end of Sunday Week 3. Each Online Quiz represents 1% of the unit grade.