Quantum Mechanics Week 2: Class Prep solutions

Q1)

$$\hat{p} = -i\hbar\frac{d}{dx} \qquad \hat{x} = x \qquad \hat{E} = \hat{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)$$

Q2) Consider:

$$\hat{p}\phi(x) = \hat{p}\left(e^{ipx/\hbar}\right) = -i\hbar\frac{d}{dx}\left(e^{ipx/\hbar}\right) = -i\hbar\cdot\frac{ip}{\hbar}e^{ipx/\hbar} = p\phi(x)$$

This show that $e^{ipx/\hbar}$ is an eigenfunction of momentum with eigenvalue p.

Q3) The expectation value of position is given by:

$$\langle x\rangle = \int_{-\infty}^{\infty}\psi^*\hat{x}\psi\,dx = \int_{-\infty}^{\infty}x\,|\psi(x)|^2\,dx = 105\int_0^1 x\,(x^2 - x^3)^2\,dx$$

$$= 105\int_0^1 (x^5 - 2x^6 + x^7)\,dx = 105\left[\frac{x^6}{6} - \frac{2x^7}{7} + \frac{x^8}{8}\right]_0^1 = 105\left(\frac{1}{6} - \frac{2}{7} + \frac{1}{8}\right)$$

$$= \frac{5}{8}$$

Q4)

(i) For two commuting operators, the result of applying the operators in turn does not depend on the order.

(ii) The values of two compatible observables may be simultaneously known with 100% certainty.

(iii) Simultaneous eigenfunctions are eigenfunctions of more than one operator.

Q5) Substituting in the two operators and differentiating the second term as a product:

$$\hat{x}(\hat{p}f) - \hat{p}(\hat{x}f) = x\left(-i\hbar\frac{df}{dx}\right) + i\hbar\frac{d}{dx}(xf) = -i\hbar x\frac{df}{dx} + i\hbar f + i\hbar x\frac{df}{dx} = i\hbar f$$

Q6) Substituting in the two operators, using the form of the Hamiltonian with $V(x) = 0$:

$$\left[\hat{H},\hat{p}\right]f = \left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}, -i\hbar\frac{d}{dx}\right]f = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\left(-i\hbar\frac{df}{dx}\right) + i\hbar\frac{d}{dx}\left(\frac{\hbar^2}{2m}\frac{d^2f}{dx^2}\right)$$

$$= -i\frac{\hbar^3}{2m}\frac{d^3f}{dx^3} + i\frac{\hbar^3}{2m}\frac{d^3f}{dx^3} = 0$$