Quantum Mechanics Week 2: Class Prep

Please study this worksheet before Wednesday's Week 2 tutorial, and submit your solutions to the Class Prep exercises by the end of Tuesday. The estimated total time requirement is 2 hours.

Overview

This week we'll complete our exploration of the fundamentals of Quantum Mechanics! We'll learn about:

- The operators for momentum, position and energy
- The time-independent Schrödinger equation for the energy eigenfunctions
- How to tell if two observables can be simultaneously known, and the connection with commuting operators
- The uncertainty principle of quantum mechanics, and its mathematical expression
- How to determine the evolution of a wavefunction with time

This worksheet will help you get the most out of this week's tutorial, by covering some introductory aspects in advance of the class.

Resources for Learning

Please study the Week 2 resources. The estimated time requirement is 1 hour.

Video: The Week 2 video summarises the topics we'll study this week. The running time is 29:25.

Textbook: You can find more information on these topics in Griffiths: Chapter 1.5-1.6, 2.1.

You are free to search for and use other resources in addition to (or instead of) the above, as long as you can answer the following Class Prep exercises.

Class Prep Exercises

Please answer these short exercises on paper and submit your scanned answers by the end of Tuesday. After you have studied the above resources, the estimated time requirement for these exercises is 1 hour.

1) Write down the operators for momentum, position and energy for a particle moving in one dimension in a potential $V(x)$.

2) Use the momentum operator to show that $\phi(x) = e^{ipx/\hbar}$ is an eigenfunction of momentum with eigenvalue p.

3) A particle in an infinite potential well in the region $0 < x < 1$ has a normalised wavefunction,

$$\psi(x) = \sqrt{105}(x^2 - x^3)$$

where $\psi = 0$ for $x < 0$ and $x > 1$. What is the expectation value of its position? *[Hint: you can use the definition of the expectation value,* $\langle a \rangle = \int_{-\infty}^{\infty} \psi^* \hat{A} \psi \, dx$*.]*

4) Write one sentence describing each of the following concepts: (i) commuting operators, (ii) compatible observables, (iii) simultaneous eigenfunctions.

5) If $\hat{x}$ is the position operator and $\hat{p}$ is the momentum operator, show that for any function $f(x)$

$$\hat{x}(\hat{p}f) - \hat{p}(\hat{x}f) = i\hbar f$$

which may also be written as the commutation relation, $[\hat{x}, \hat{p}] = i\hbar$. *[Hint: you can substitute in the expressions for the operators* $\hat{x}$ *and* $\hat{p}$*.]*

6) If $\widehat{H}$ is the energy operator of a particle moving in 1D in free space (potential $V(x) = 0$), show that $[\widehat{H}, \hat{p}] = 0$. What does this result imply about the energy and momentum of this particle?

Grading and getting help

How this is graded: Class Prep assignments are always graded on *completeness & effort*. If you submit a reasonable effort for all parts on time, you will receive full marks regardless of all details being correct. Otherwise, you will not receive a grade. Each Class Prep assignment represents 1% of the unit grade.

After you have studied this week's learning resources, these exercises are not supposed to take more than 1 hour to complete. If you've been working purposefully on these exercises for 30 minutes and you're struggling with the content, please stop and ask for help. You can work with a friend, e-mail the instructor (cblake@swin.edu.au) or ask a question on the Week 2 Discussion Board in Canvas.

The rest of this week's activities

In Wednesday's tutorial class we'll expand on the Class Prep exercises to:

- Analyse a given wavefunction to determine the momentum values that can be obtained in a particular measurement, and their probabilities
- Determine the time evolution of a wavefunction from its stationary states
- Analyse the time evolution of expectation values

Finally, **please complete the Week 2 Online Quiz** (10 multiple choice questions) by the end of Sunday Week 2. Each Online Quiz represents 1% of the unit grade.