

Quantum Mechanics Week 4: Class Activities

We'll go through these activities in Wednesday's Week 4 Quantum Mechanics tutorial.

Warm-up discussion

- Why should we care about angular momentum?
 - Quantum numbers
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The eigenfunctions of angular momentum

1) The simultaneous eigenfunctions of \hat{L}^2 and \hat{L}_z can be written as the spherical harmonic functions,

$$Y_{lm}(\theta, \phi) = N P_{lm}(\theta) e^{im\phi}$$

where $P_{lm}(\theta)$ satisfies the equation,

$$\frac{1}{\sin \theta} \frac{d}{d\theta} \left(\sin \theta \frac{dP_{lm}}{d\theta} \right) + \left[l(l+1) - \frac{m^2}{\sin^2 \theta} \right] P_{lm} = 0$$

Show that

- $P_{00} = 1$ satisfies this equation with $l = 0$ and $m = 0$
- $P_{10} = \cos \theta$ satisfies this equation with $l = 1$ and $m = 0$
- $P_{11} = -\sin \theta$ satisfies this equation with $l = 1$ and $m = 1$

Hence, write down the mathematical functions for Y_{00} , Y_{10} and Y_{11} – no need to evaluate the normalisation constants yet, this is coming next!

Integrations over a sphere!

2) The spherical harmonic functions are normalised by ensuring

$$\int_0^{2\pi} d\phi \int_0^\pi d\theta \sin \theta |Y_{lm}(\theta, \phi)|^2 = 1$$

Why does the “ $\sin \theta$ ” appear in this equation? Find the normalisation constants N for the spherical harmonics Y_{00} , Y_{10} and Y_{11} .

[Hint: you can use the triple-angle formula, $\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$.]

3) Show that Y_{10} and Y_{11} are orthogonal on the sphere such that

$$\int_0^{2\pi} d\phi \int_0^\pi d\theta \sin \theta Y_{10}(\theta, \phi) Y_{11}^*(\theta, \phi) = 0$$

Analysing a wavefunction on a sphere

4) A particle has a wavefunction on a sphere given by

$$\psi(\theta, \phi) = \frac{1}{\sqrt{4\pi}} \left[\frac{1}{\sqrt{2}} + \cos \theta + \frac{1}{2} \sin \theta e^{i\phi} \right]$$

Express this function as a sum of spherical harmonics Y_{00} , Y_{10} and Y_{11} and hence determine the possible results of measuring L^2 and L_z , and their probabilities.

The rest of this week's activities

Please complete the Week 4 Online Quiz (10 multiple choice questions) by the end of Sunday. Each Online Quiz represents 1% of the unit grade.

Assignment 2: You can now solve Q1 in Assignment 2 (which is due at the end of Friday Week 6).