Quantum Mechanics Week 3: Class Activities

We'll go through these activities in Wednesday's Week 3 Quantum Mechanics tutorial.

Warm-up discussion

- What is a "bound state" versus an "unbound state"?
- How do we describe a free particle in quantum mechanics?

Bound states of a finite potential well

Consider a particle with energy E moving in a finite square well potential,

$$V(x) = \begin{cases} 0, & |x| < L \\ V_0, & |x| > L \end{cases}$$

where $E < V_0$ (i.e. the particle is in a "bound state").

1) Write down the Schrödinger equation in the region $|x| < L$, and show that $\psi = A\cos kx$ is a solution, finding an expression for k in terms of the other variables.

2) Write down the Schrödinger equation in the region $|x| > L$, and show that $\psi = Be^{\pm lx}$ is a solution, finding an expression for l in terms of the other variables.

3) By applying the boundary conditions on $\psi(x)$ at $x = \pm L$, show that $\tan\sqrt{\frac{2mEL}{\hbar^2}} = \sqrt{\frac{V_0}{E} - 1}$.

4) Sketch the wavefunction corresponding to the lowest energy state.

Quantum tunnelling in action!

Consider a beam of particles with energy $E = V_0/2$ incident on a potential barrier,

$$V(x) = \begin{cases} V_0, & 0 < x < L \\ 0, & x < 0, x > L \end{cases}$$

We suppose the wavefunctions of the beam to have the following forms in the different regions:

Incident wave in the $x < 0$ region: $\Psi_I(x,t) = e^{i(kx-\omega t)}$
Reflected wave in the $x < 0$ region: $\Psi_R(x,t) = Re^{i(-kx-\omega t)}$
Transmitted wave in the $x > L$ region: $\Psi_T(x,t) = Te^{i(kx-\omega t)}$
Wavefunction in the $0 < x < L$ barrier region: $\Psi_B(x,t) = (Ae^{lx} + Be^{-lx})\, e^{-i\omega t}$

where $k^2 = 2mE/\hbar^2$ and $l^2 = 2m(V_0 - E)/\hbar^2$ and $k = l$ because $E = V_0/2$.

5) By applying boundary conditions at $x = 0$ and $x = L$, show that:

$$1 + R = A + B$$
$$Te^{ikL} = Ae^{kL} + Be^{-kL}$$
$$(1 - R)i = A - B$$
$$i\,Te^{ikL} = Ae^{kL} - Be^{-kL}$$

6) Re-arrange these equations to show that:

$$\frac{B}{A} = \left(\frac{1-i}{1+i}\right) e^{2kL} \qquad \frac{T}{A} = \frac{2\,e^{kL}\,e^{-ikL}}{1+i}$$

Hence show that the amplitude of the transmitted wavefunction is.

$$|T|^2 = \frac{4}{(e^{kL} + e^{-kL})^2}$$

The rest of this week's activities

Please complete the Week 3 Online Quiz (10 multiple choice questions) by the end of Sunday. Each Online Quiz represents 1% of the unit grade.

Assignment 1: You can now solve Q3 in Assignment 1 (which is due at the end of Friday Week 3).