Quantum Mechanics Week 2: Class Activities

We'll go through these activities in Wednesday's Week 2 Quantum Mechanics tutorial.

Warm-up discussion

- What's this uncertainty principle – conceptually and mathematically?
- How can quantum mechanics be both deterministic and uncertain?

Making momentum measurements

1) A particle in an infinite potential well has the same wavefunction as in the Week 1 class,

$$\psi(x) = \frac{2}{\sqrt{5L}} \sin\left(\frac{\pi x}{L}\right)\left[1 + \cos\left(\frac{\pi x}{L}\right)\right]$$

defined in the range $|x| < L$. By expressing the wavefunction as a sum of momentum eigenfunctions, determine what momentum values can be measured, and with what probabilities? *[Hint: you can use the formulae* $\sin x = (e^{ix} - e^{-ix})/2i$ *and* $\cos x = (e^{ix} + e^{-ix})/2$ *and compare the result with the momentum eigenfunctions.]*

2) Using these results, determine the expectation value of a momentum measurement of this particle.

Time-evolution of the wavefunction

3) At time $t = 0$, a particle in an infinite potential well has the same wavefunction as in Q1,

$$\Psi(x, 0) = \frac{2}{\sqrt{5L}} \sin\left(\frac{\pi x}{L}\right)\left[1 + \cos\left(\frac{\pi x}{L}\right)\right]$$

By using the expression of this function as a combination of energy eigenfunctions from the Week 1 class, write down the wavefunction of the particle $\Psi(x, t)$ at time t.

4) Use the definition of an expectation value of energy, $\langle E \rangle = \int_{-\infty}^{\infty} \Psi^* \hat{H} \Psi \, dx$, to show that the expectation value of energy for this particle remains constant with time. *[Hint: you can substitute in the result of Q3 and use the fact that the energy eigenfunctions are orthogonal.]*

If you have time and want to try something ...

5) Consider a particle in the ground state of an infinite potential well, with wavefunction $\psi(x) = \frac{1}{\sqrt{L}} \cos\left(\frac{\pi x}{2L}\right)$ defined in the range $|x| < L$. Find the values of $\langle x^2 \rangle$ and $\langle p^2 \rangle$ (the expectation values of the position squared, and momentum squared). Use these results to determine $\sqrt{\langle x^2 \rangle \langle p^2 \rangle}$, and compare your result to the uncertainty principle. *[Hint: You may use the integral* $\int u^2 cos^2 u \; du = \frac{1}{6}u^3 + \frac{1}{4}u^2 \sin 2u + \frac{1}{4}u \cos 2u - \frac{1}{8}\sin 2u + C$*.]*

The rest of this week's activities

Please complete the Week 2 Online Quiz (10 multiple choice questions) by the end of Sunday. Each Online Quiz represents 1% of the unit grade.

Assignment 2: You can now solve Q1(c), Q1(d), Q2(c) and Q2(d) in Assignment 1 (which is due at the end of Friday Week 3).