Quantum Mechanics Week 1: Class Activities

We'll go through these activities in Wednesday's Week 1 Quantum Mechanics tutorial.

Warm-up discussion

- What's all this about the wavefunction?
- What makes quantum mechanics different: the measurement process

What is "orthogonality"?

In "Physics 2A" you'll have met the solution of the Schrödinger equation for the infinite potential well, which has potential $V(x) = 0$ for $|x| < L$ and $V(x) = \infty$ for $|x| > L$. We will use these solutions to illustrate some of the properties of eigenfunctions. The energy eigenfunctions of the infinite potential well, defined over the range $|x| < L$, are:

$$\psi_n(x) = \begin{cases} \frac{1}{\sqrt{L}} \cos\left(\frac{n\pi x}{2L}\right), & n = 1,3,5, \dots \\ \frac{1}{\sqrt{L}} \sin\left(\frac{n\pi x}{2L}\right), & n = 2,4,6, \dots \end{cases}$$

and $\psi_n(x) = 0$ for $|x| > L$.

1) Make a sketch of $\psi_n(x)$ against x for $n = 1,2,3,4$.

2) Show that $\int_{-\infty}^{\infty} |\psi_1(x)|^2 \, dx = 1$ and $\int_{-\infty}^{\infty} \psi_1^*(x)\, \psi_2(x)\, dx = 0$. *[Hint: $\psi = 0$ for $|x| > L$, so you can restrict the integration range to $-L < x < L$.]*

Give me a wavefunction, what measurements can I make?

A particle in the same infinite potential well as above has the following normalised wavefunction defined over the range $|x| < L$:

$$\psi(x) = \frac{2}{\sqrt{5L}} \sin\left(\frac{\pi x}{L}\right) \left[1 + \cos\left(\frac{\pi x}{L}\right)\right]$$

3) Express $\psi(x)$ as a linear combination of energy eigenfunctions of the infinite potential well. *[Hint: you could break the equation into 2 terms and use $2 \sin\theta \cos\theta = \sin 2\theta$.]*

4) Given that the energy levels of the infinite potential well are $E_n = \frac{n^2\pi^2\hbar^2}{8mL^2}$, what energy values can be measured for the particle, and with what probabilities?

What's meant by an expectation value?

5) The expectation value of an operator $\hat{A}$ applied to a wavefunction $\psi(x)$ is,

$$\langle a \rangle = \int_{-\infty}^{\infty} \psi^*(x)\, \hat{A}\psi(x)\, dx$$

The energy operator is $\hat{E} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$. What is the expectation value of energy, $\langle E \rangle$, for the particle with the same wavefunction as in Q3?

If you have time and want to try something ...

6) A general wavefunction $\psi(x)$ may be expressed as a linear combination of eigenfunctions $\phi_n(x)$,

$$\psi(x) = \sum_n c_n\, \phi_n(x)$$

Use the normalisation condition $\int_{-\infty}^{\infty} |\psi(x)|^2\, dx = 1$, and the relation for the orthogonality of eigenfunctions, to show that $\sum_n |c_n|^2 = 1$, such that the $|c_n|^2$ can be interpreted as probabilities.

7) By substituting in the expression for $\psi(x)$ as a linear combination of eigenfunctions, $\psi(x) = \sum_n c_n\, \phi_n(x)$ to the equation for the expectation value given in Q5, show that

$$\langle a \rangle = \sum_n |c_n|^2\, a_n$$

where a_n are the eigenvalues corresponding to ϕ_n.

The rest of this week's activities

Please complete the Week 1 Online Quiz (10 multiple choice questions) by the end of Sunday. Each Online Quiz represents 1% of the unit grade.

Assignment 1: You can now solve Q1(a), Q1(b), Q2(a) and Q2(b) in Assignment 1 (which is due at the end of Friday Week 3).