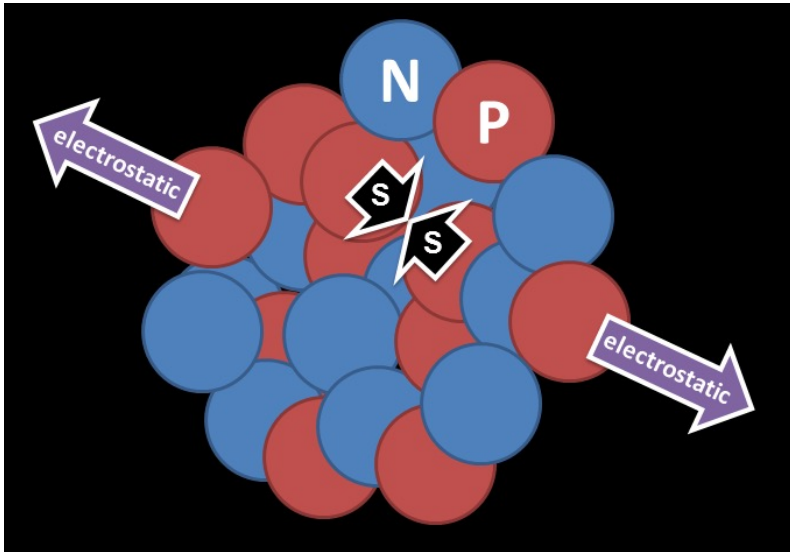
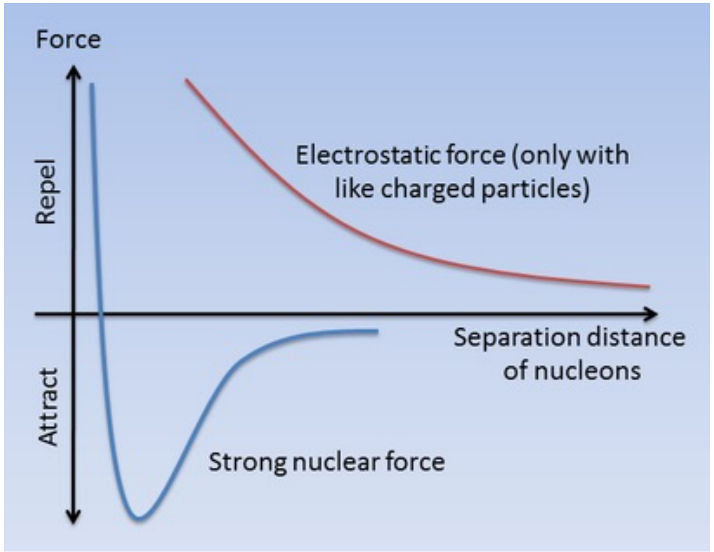
Week 9 Nuclear Physics Tutorial

Nuclear Models & Radioactivity

- This week's Class Prep
- Concept review of nuclear models & radioactivity
- Example calculations



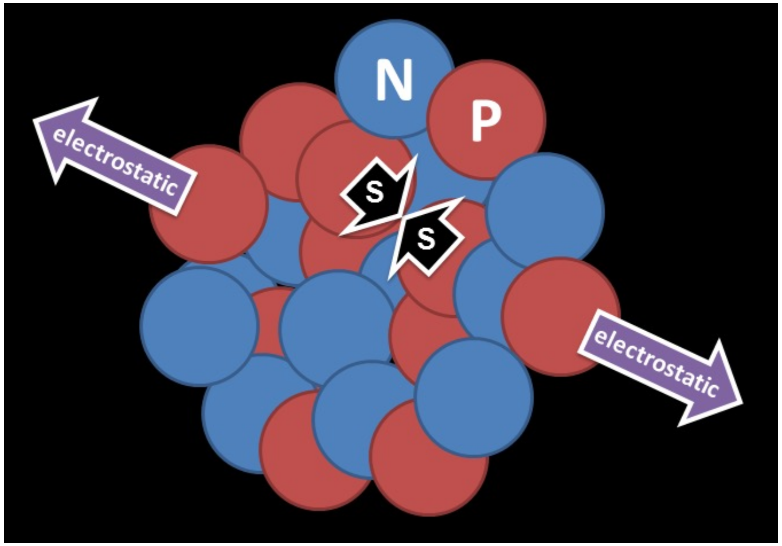


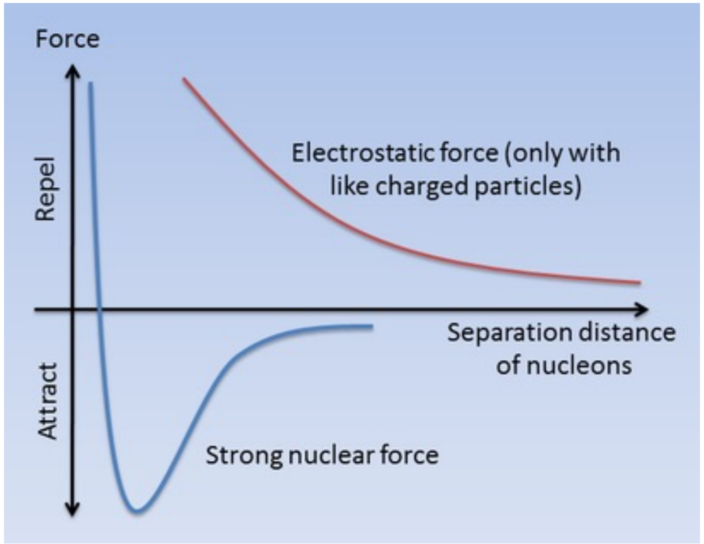
Introducing the Nucleus:

Concepts & Examples

Introducing the Nucleus

How can the nucleus exist, when protons should repel each other by Coulomb's Law?

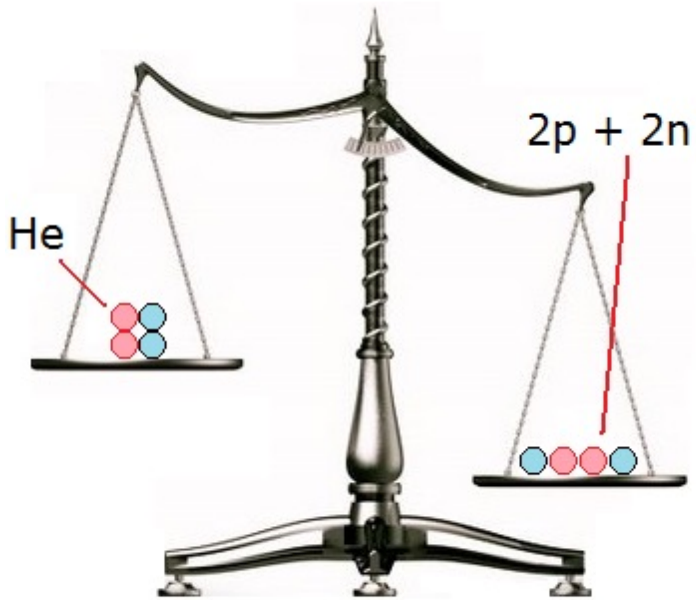




The electrostatic force is overcome by the **strong nuclear force**, which binds nucleons together

The strong nuclear force is a **short-range force** which falls to zero after a few femtometres ($1\ \mathrm{fm} = 10^{-15}\ \mathrm{m}$)

Introducing the Nucleus



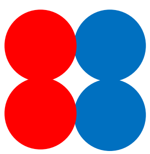
Key fact: the **total mass of a nucleus is less than the sum of the mass of its constituents** – this is known as the **mass defect** ΔM

$$M_{nucleus} = Zm_P + Nm_N - \Delta M$$

- This mass defect is equivalent to the **binding energy** of the nucleus, $B = \Delta M\, c^2$
- This is the energy needed to break the nucleus apart
- This energy can also be **released by nuclear reactions**

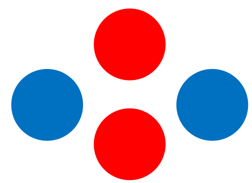
Introducing the Nucleus

- Binding energy is actually a **negative energy** – we have to supply energy to break up a nucleus



Helium nucleus

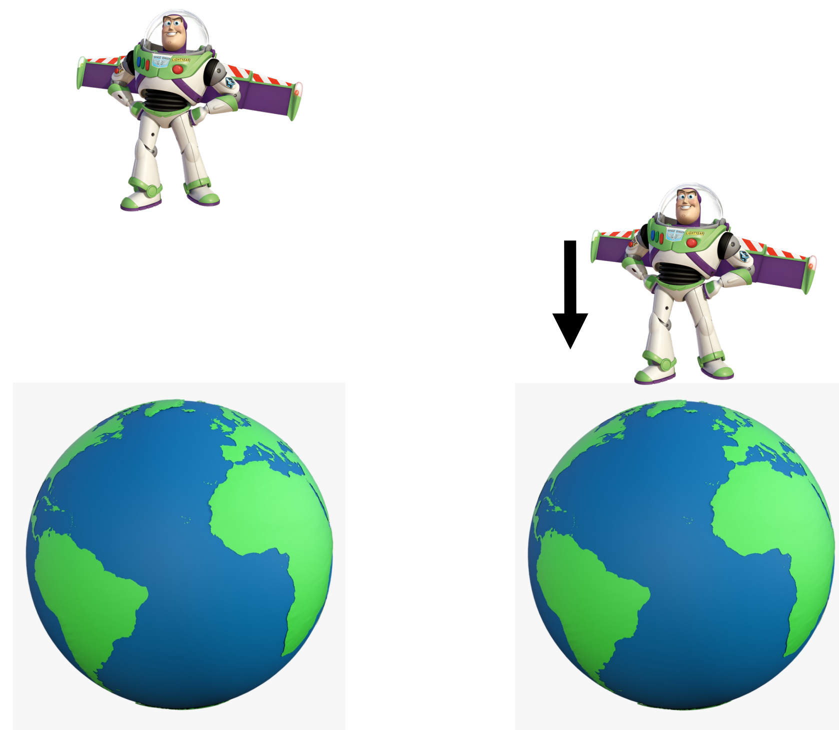
+ Energy =



2 protons & 2 neutrons

- If the binding energy of a system increases, it becomes **more negative**, such that energy is released

Sounds weird!! … we can try an analogy to gravity?



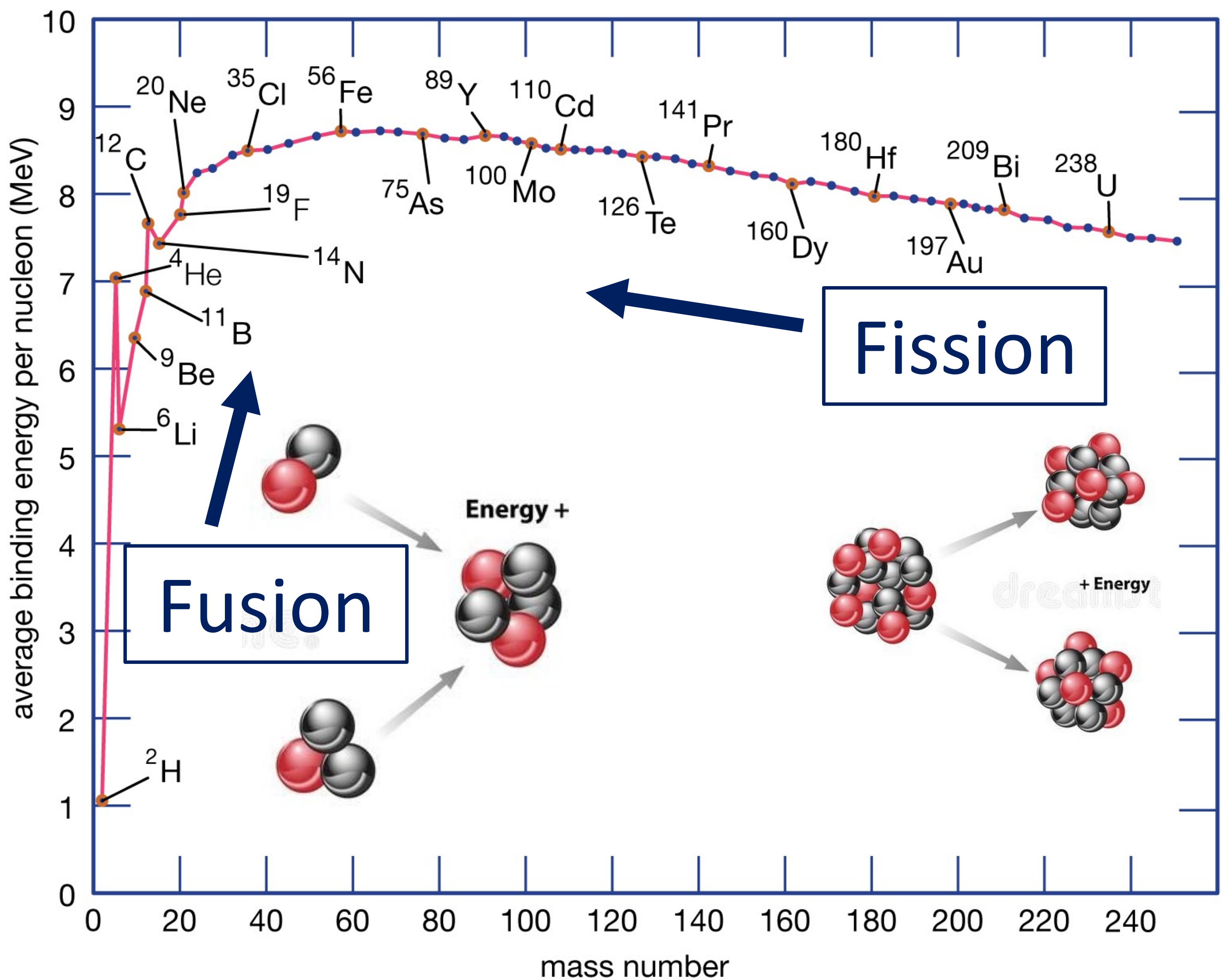
Approaching the Earth, the astronaut becomes more tightly bound (negative gravitational potential energy) and gains kinetic energy!

Introducing the Nucleus

More tightly bound

Energy is released

Less tightly bound



Introducing the Nucleus

- **Here's the most useful form of the mass defect equation …**

- Our previous formula for the binding energy:

$$M_{nucleus} = Zm_P + Nm_N - \Delta M$$

- However, usually **atomic masses** of nuclides are given – which include both the nucleus **and** the electrons

- Therefore, it's useful to write the above formula in terms of atomic masses and m_H, the mass of the hydrogen atom including the electron:

$$\boxed{M_{atom} = Zm_H + Nm_N - \Delta M}$$

Introducing the Nucleus Example 1

Group the following nuclides into **isotopes**, **isotones** and **isobars**:

$$^{40}\mathrm{Ca},\ ^{40}\mathrm{S},\ ^{38}\mathrm{Ar},\ ^{44}\mathrm{Ca},\ ^{40}\mathrm{Cl},\ ^{42}\mathrm{Ca},\ ^{39}\mathrm{K}$$

Isotopes (same Z, different A and N): $^{40}\mathrm{Ca}$, $^{42}\mathrm{Ca}$, $^{44}\mathrm{Ca}$

Isotones (same N, different Z and A): $^{38}\mathrm{Ar}$, $^{39}\mathrm{K}$, $^{40}\mathrm{Ca}$

Isobars (same A, different Z and N): $^{40}\mathrm{S}$, $^{40}\mathrm{Cl}$, $^{40}\mathrm{Ca}$

Introducing the Nucleus Example 2

The largest known stable nucleus is lead-208 ($^{208}_{82}\mathrm{Pb}$). Estimate the radius of a lead-208 nucleus in units of fm.

Using the nuclear radius formula $R = R_0\, A^{1/3}$, where $R_0 = 1.2\ \mathrm{fm}$, we have for $A = 208$, $R = 1.2 \times 208^{1/3} = 7.1\ \mathrm{fm}$.

Introducing the Nucleus Example 3

The atomic mass unit is $1\ \mathrm{u} = 1.66054 \times 10^{-27}\ \mathrm{kg}$.

Verify that this amount of mass, when converted to energy, yields $931.5\ \mathrm{MeV}$.

$$c = 2.99792 \times 10^{8}\ \mathrm{m/s}$$
$$1\ \mathrm{eV} = 1.60218 \times 10^{-19}\ \mathrm{J}$$

$$\text{Energy equivalent in MeV} = \frac{1.66054 \times 10^{-27} \times \left(2.99792 \times 10^{8}\right)^{2}}{1.60218 \times 10^{-13}} = 931.5$$

Introducing the Nucleus Example 4

Nickel-62 ($^{62}_{28}\mathrm{Ni}$) has the highest binding energy per nucleon, B/A, of any known nuclide. The atomic mass of nickel-62 is $61.92834\ \mathrm{u}$. Determine the values of the mass defect in atomic mass units, and B/A in MeV units, for this nuclide.

Mass of hydrogen atom $= m_H = 1.00783\ \mathrm{u}$
Mass of neutron $= m_n = 1.00866\ \mathrm{u}$
$\Delta M = Zm_H + Nm_n - M_{atom}$

Nickel-62 has $Z = 28$, $N = 34$ and $A = 62$

The mass defect is given by $\Delta M = Zm_H + Nm_n - M_{atom} = 28\times1.00783 + 34\times1.00866 - 61.92834 = 0.58534\ \mathrm{u}$

Converting the mass defect to a binding energy, $B = 0.58534\times931.5 = 545.2\ \mathrm{MeV}$, hence $B/A = 8.79\ \mathrm{MeV}$

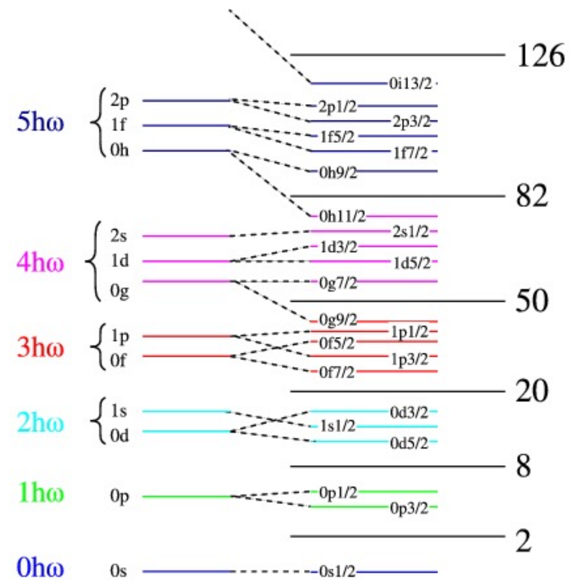
Nuclear Models:

Concepts & Examples

Nuclear Models

How would you describe the **"liquid drop"** description of the nucleus?

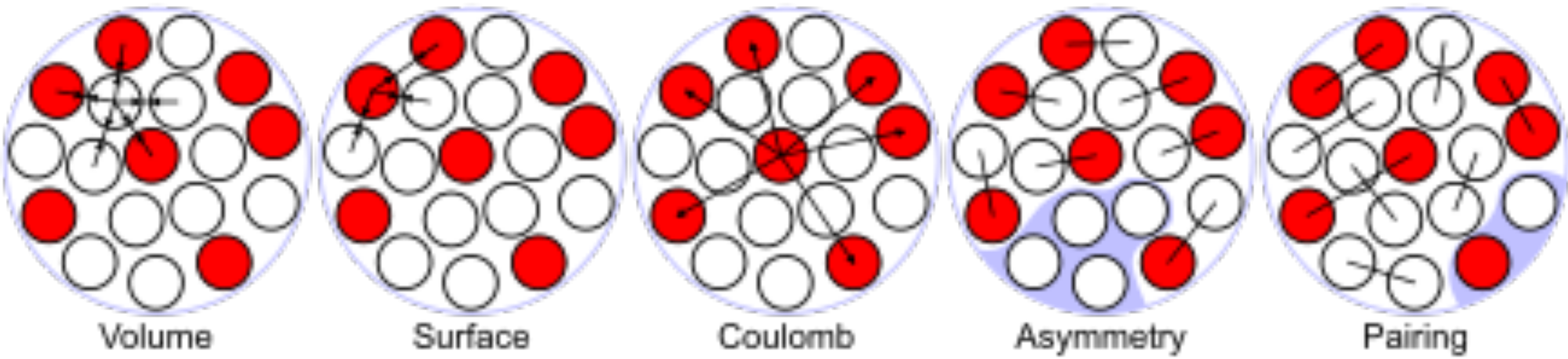
How would you describe the **"shell model"** description of the nucleus?



The shell model is more accurate, but also more difficult to apply since it uses complicated quantum mechanics calculations!

Nuclear Models

- The liquid drop model motivates a model for the mass (hence binding energy) of a nucleus, in terms of Z, N and A, called the **semi-empirical mass formula** (or SEMF for short)



Volume effect: the binding energy increases with the number of nucleons

Surface effect: nucleons on the surface have a reduced binding

Coulomb effect: there is electrostatic potential energy between protons

Asymmetry effect: nuclei prefer to have roughly balanced numbers of protons and neutrons

Nuclear Models

- The SEMF for the **nuclear binding energy** B in terms of the mass number A, atomic number Z and neutron number N:

$$B = a_V\, A - a_S\, A^{2/3} - a_C\, \frac{Z^2}{A^{1/3}} - a_A\, \frac{(N-Z)^2}{A}$$

(we'll neglect the "pairing term" here)

Volume term: the strong force increases B by a constant amount per nucleon

Surface term: nucleons on the surface are not as strongly bound, decreases B as the surface area $\propto R^2 \propto A^{2/3}$

Coulomb term: protons repel each other, decreases B by electrostatic potential energy $\propto \frac{Q^2}{R} \propto \frac{Z^2}{A^{1/3}}$

Asymmetry term: accounts for the tendency of nuclei to have $N \sim Z$

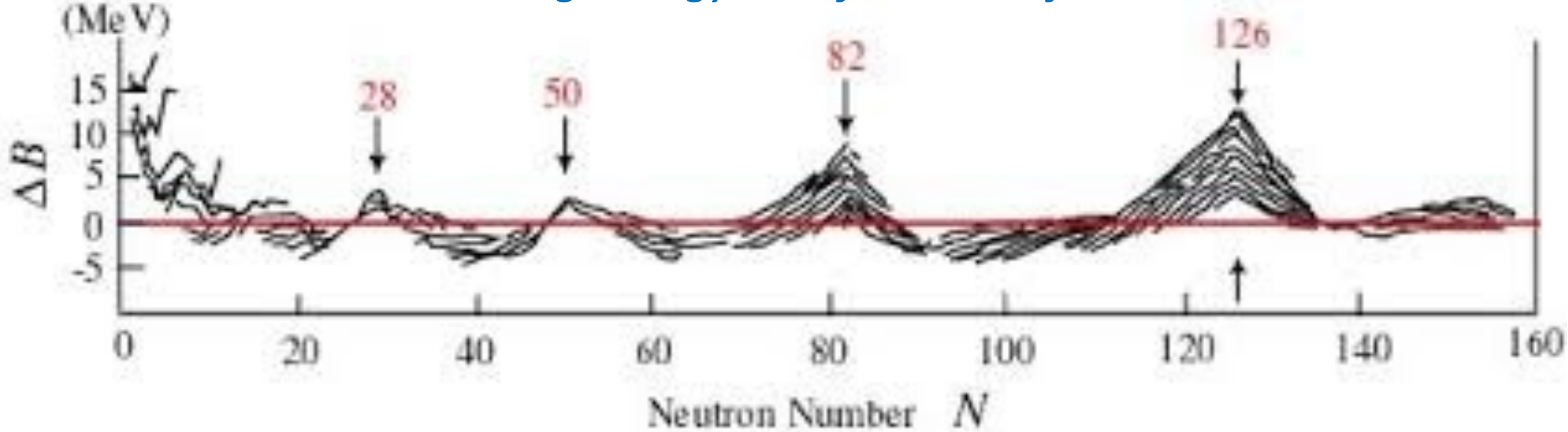
- The **coefficients** in the SEMF formula are found to be: $a_V = 15.8\ \mathrm{MeV}$, $a_S = 18.0\ \mathrm{MeV}$, $a_C = 0.72\ \mathrm{MeV}$, $a_A = 23.5\ \mathrm{MeV}$

Nuclear Models

- The SEMF predicts the *average* nuclear binding energy, but **some nuclei are significantly more stable** (i.e. have higher binding energy) than calculated by the model

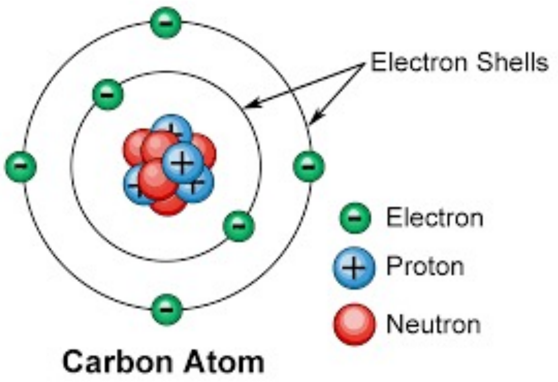
- These cases happen for nuclei with so-called **"magic numbers"** of protons and/or neutrons: Z and/or $N = 2, 8, 20, 28, 50, 82, 126$ (effects are more pronounced for "doubly magic")

Excess binding energy as a function of neutron number:



Nuclear Models

- The presence of magic numbers suggests a **shell model** – where the *magic numbers correspond to the closing of shells*



Atomic structure

- Electrons **move independently** in an effective Coulomb potential
- Electrons occupy “shells” (energy levels) because of **quantum mechanics** and the **Pauli exclusion principle**
- **Shell closure** gives the most inert/stable atoms (Nobel gases)

Nuclear structure

- Nucleons **move independently** in an effective strong-force potential
- Nucleons also obey the laws of **quantum mechanics** and the **Pauli exclusion principle**, and likewise occupy shells
- **Shell closure** gives the most stable nuclei

Nuclear Models Example

(a) Three different isotopes of calcium, $^{40}\mathrm{Ca}$, $^{44}\mathrm{Ca}$ and $^{48}\mathrm{Ca}$, have atomic masses $39.96259\ \mathrm{u}$, $43.95548\ \mathrm{u}$ and $47.95253\ \mathrm{u}$, respectively. What are the binding energies per nucleon of these isotopes?

Mass of hydrogen atom $= m_H = 1.00783\ \mathrm{u}$
Mass of neutron $= m_n = 1.00866\ \mathrm{u}$

(b) The SEMF predicts that these 3 nuclides should have $\frac{B}{A} = 8.43, 8.65, 8.54\ \mathrm{MeV}$, respectively. Explain these results in terms of the shell model of nuclear structure.

Nuclear Models Example

(a) For ^{40}Ca, $Z = 20$ and $N = 20$, hence the mass defect is $\Delta m = Zm_H + Nm_n - m(^{40}\text{Ca}) = 20\times 1.00783 + 20\times 1.00867 - 39.96259 = 0.367\text{ u}$, which corresponds to binding energy $0.367\times 931.5 = 342.24\text{ MeV}$ or $B/A = 8.56\text{ MeV}$.

For ^{44}Ca, $Z = 20$ and $N = 24$, hence the mass defect is $\Delta m = Zm_H + Nm_n - m(^{44}\text{Ca}) = 20\times 1.00783 + 24\times 1.00867 - 43.95548 = 0.409\text{ u}$, which corresponds to binding energy $0.409\times 931.5 = 381.17\text{ MeV}$ or $B/A = 8.66\text{ MeV}$.

For ^{48}Ca, $Z = 20$ and $N = 28$, hence the mass defect is $\Delta m = Zm_H + Nm_n - m(^{48}\text{Ca}) = 20\times 1.00783 + 28\times 1.00867 - 47.95253 = 0.447\text{ u}$, which corresponds to binding energy $0.447\times 931.5 = 416.22\text{ MeV}$ or $B/A = 8.67\text{ MeV}$.

(b) The nuclides ^{40}Ca and ^{48}Ca are more stable (have higher binding energies) than predicted by the SEMF, because both the proton and neutron numbers are "magic" numbers (20 or 28).

Break time!

Radioactivity:

Concepts & Examples

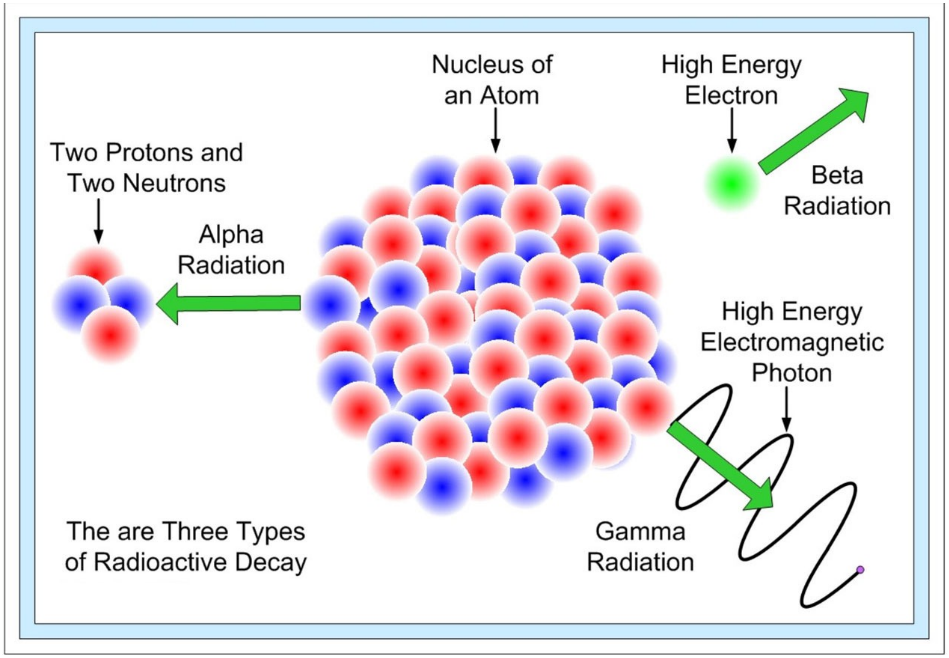
Radioactivity

- Radioactivity is the **spontaneous transformation of an unstable nucleus**, involving the emission of radiation
- Radioactive decay can occur if the nucleons are arrange-able in a **lower energy state** than their current configuration

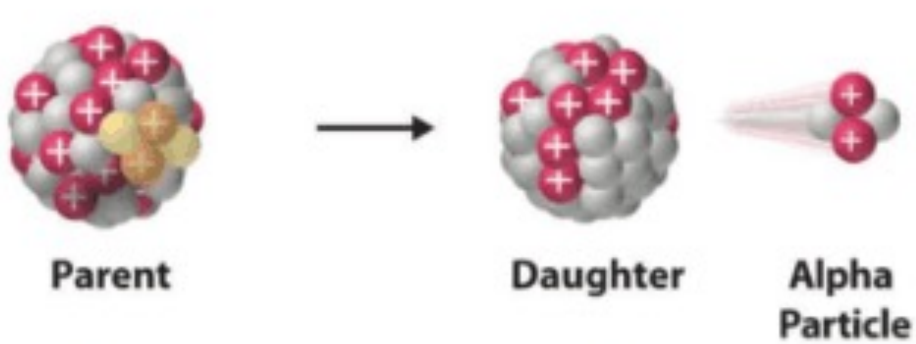
What can we say about $\boldsymbol{\alpha}$**-decay**?

What can we say about $\boldsymbol{\beta}$**-decay**?

What can we say about $\boldsymbol{\gamma}$**-decay**?



Radioactivity



Helium nucleus consists of 2 protons and 2 neutrons

$$^{A}_{Z}X \rightarrow \, ^{A-4}_{Z-2}Y + \, ^{4}_{2}He \quad (+\text{ energy } Q)$$

- α-decay occurs when a nucleus is too large to be stable, and disintegrates to a lower-energy state by **ejecting a helium nucleus**, which is also known as an α-particle
- α-decay requires the parent to have **mass number $A \gtrsim 150$** in order that the decay is spontaneous (i.e. energy $Q > 0$)

Radioactivity

- β-decay transforms the number of protons Z and neutrons N in a nucleus closer to the **line of stability** $Z \sim N$

- There are 3 forms of β-decay:
  - $\boldsymbol{\beta^-}$ **decay**: N is too large for stability, and a neutron becomes a proton involving the emission of an electron and anti-neutrino

    $$n \rightarrow p + e^- + \bar{\nu}_e \qquad {}^{A}_{Z}X \rightarrow {}_{Z+1}^{\ \ A}Y^+ + {}_{-1}^{\ 0}e^- + \bar{\nu}_e$$

  - $\boldsymbol{\beta^+}$ **decay**: Z is too large for stability, and a proton becomes a neutron involving the emission of a positron and a neutrino

    $$p \rightarrow n + e^+ + \nu_e \qquad {}^{A}_{Z}X \rightarrow {}_{Z-1}^{\ \ A}Y^- + {}_{+1}^{\ 0}e^+ + \nu_e$$

  - **Electron capture**: Z is too large for stability, and an atomic electron strays too close to the nucleus and reacts with a proton, producing a neutron (within the nucleus) and a neutrino

    $$p + e^- \rightarrow n + \nu_e \qquad {}^{A}_{Z}X + {}_{-1}^{\ 0}e^- \rightarrow {}_{Z-1}^{\ \ A}Y^- + \nu_e$$

Radioactivity

- γ-decay occurs when a nucleus is in an excited state – often following α- or β-decay – and reverts to the ground state, **emitting a photon** (also known as a γ-ray)

$$^{A}_{Z}X^{*} \rightarrow {}^{A}_{Z}X + {}^{0}_{0}\gamma$$

- γ-decay is similar to atomic de-excitation, but occurs at much higher energy ($\sim$ MeV vs. $\sim$ eV)

Radioactivity Example 1

Sodium-22, which has 11 protons and 11 neutrons, undergoes a radioactive decay into neon-22, which has 10 protons and 12 neutrons. Write the decay equation for this process, including the additional particles produced. What type of radioactive decay does this represent?

The decay equation is ${}^{22}_{11}Na \rightarrow {}^{22}_{10}Ne^{-} + e^{+} + \nu_e$. This process represents β^{+}-decay.

Radioactivity Example 2

Calculate how much energy is released by the radioactive decay in the previous example, in units of MeV:

$$^{22}_{11}Na \rightarrow {}^{22}_{10}Ne^{-} + e^{+} + \nu_e$$

The atomic masses of sodium-22 and neon-22 are $21.99444\ \mathrm{u}$ and $21.99139\ \mathrm{u}$, respectively.

The mass defect for this decay is $21.99444 - 21.99139 = 0.00305\ \mathrm{u}$, which corresponds to an energy of $0.00305 \times 931.5 = 2.84\ \mathrm{MeV}$.

Radioactivity Example 3

Is it possible for the nuclide $^{232}\mathrm{U}$ to spontaneously decay…

… into $^{228}\mathrm{Th}$ by emitting an α-particle?
… into $^{231}\mathrm{U}$ by emitting a neutron?
… into $^{231}\mathrm{Pa}$ by emitting a proton?

Useful data:

$$m\left({}^{232}\mathrm{U}\right) = 232.03717\ \mathrm{u}$$
$$m\left({}^{228}\mathrm{Th}\right) = 228.02875\ \mathrm{u}$$
$$m\left({}^{4}\mathrm{He}\right) = 4.00260$$
$$m\left({}^{231}\mathrm{U}\right) = 231.03627\ \mathrm{u}$$
$$m(n) = 1.00867\ \mathrm{u}$$
$$m\left({}^{231}\mathrm{Pa}\right) = 231.03588\ \mathrm{u}$$
$$m(p) = 1.00727\ \mathrm{u}$$

Radioactivity Example 3

A nuclide can decay if the energy of the new configuration is lower than its current state, or if $Q = (m_{\text{initial}} - m_{\text{final}})c^2$ is positive.

a) $Q = m(^{232}\text{U}) - m(^{228}\text{Th}) - m(^{4}\text{He}) = (232.03717 - 228.02875 - 4.00260)\times 931.5 = 5.42\ \text{MeV}$, hence this decay can be spontaneous.

b) $Q = m(^{232}\text{U}) - m(^{231}\text{U}) - m(n) = (232.03717 - 231.03627 - 1.00867)\times 931.5 = -7.24\ \text{MeV}$, hence this decay cannot be spontaneous.

c) $Q = m(^{232}\text{U}) - m(^{231}\text{Pa}) - m(p) = (232.03717 - 231.03588 - 1.00727)\times 931.5 = -5.57\ \text{MeV}$, hence this decay cannot be spontaneous.

Radioactivity Example 4

Write the complete decay equation for the following processes, using the notation ${}^{A}_{Z}X$ for nuclei:

a) β^- decay of ${}^{60}\mathrm{Co}$
b) β^+ decay of ${}^{50}\mathrm{Mn}$
c) Electron capture by ${}^{7}\mathrm{Be}$
d) α decay of ${}^{239}\mathrm{Pu}$
e) β^- decay producing ${}^{137}\mathrm{Ba}$
f) α decay producing ${}^{228}\mathrm{Ra}$

a) ${}^{60}_{27}\mathrm{Co} \rightarrow {}^{60}_{28}\mathrm{Ni} + e^- + \bar{\nu}_e$

b) ${}^{50}_{25}\mathrm{Mn} \rightarrow {}^{50}_{24}\mathrm{Cr} + e^+ + \nu_e$

c) ${}^{7}_{4}\mathrm{Be} + e^- \rightarrow {}^{7}_{3}\mathrm{Li} + \nu_e$

d) ${}^{239}_{94}\mathrm{Pu} \rightarrow {}^{235}_{92}\mathrm{U} + {}^{4}_{2}\mathrm{He}$

e) ${}^{137}_{55}\mathrm{Cs} \rightarrow {}^{137}_{56}\mathrm{Ba} + e^- + \bar{\nu}_e$

f) ${}^{232}_{90}\mathrm{Th} \rightarrow {}^{228}_{88}\mathrm{Ra} + {}^{4}_{2}\mathrm{He}$

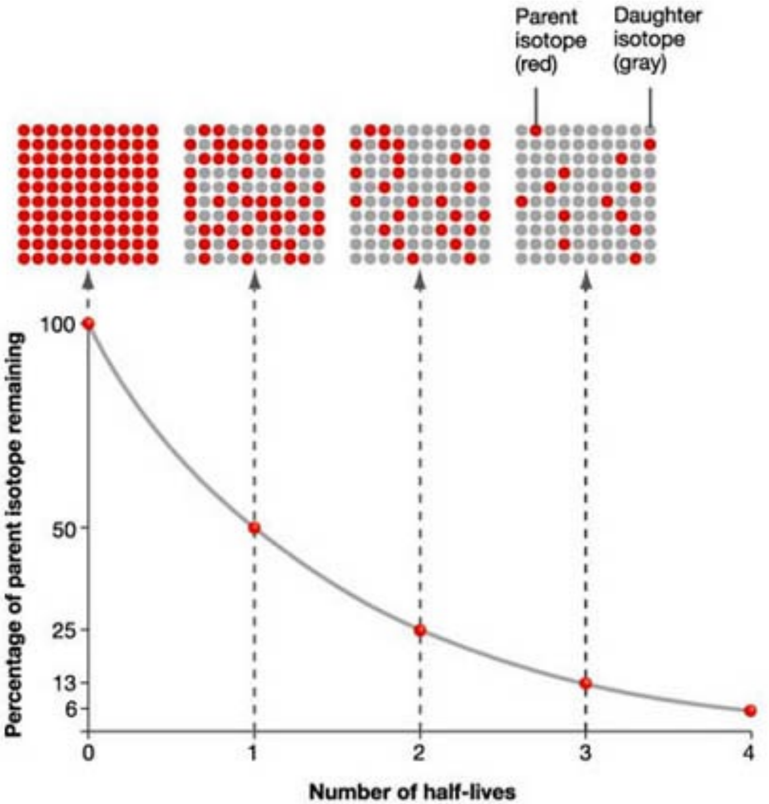
Decay Law:

Concepts & Examples

Decay Law

- Radioactive decay is a random process governed by probability
- We **cannot** predict when an individual nucleus decays, but we **can** predict the average number of decays of a large number

We cannot predict the result of a single dice roll, but we can predict the number of 6's when a large number of dice are rolled



Decay Law

The **decay rate** is given by:

$$\frac{dN}{dt} = -\lambda\, N$$

Solving this equation for the number remaining at time t:

$$N(t) = N(0)\, e^{-\lambda t}$$

The **half-life** $t_{1/2}$ is the time required for *half the nuclei in a given sample to decay*:

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

The **mean lifetime** $\bar{t}$ is the average time taken for a nucleus to decay:

$$\bar{t} = \frac{1}{\lambda}$$

1 **Becquerel** (Bq) of radioactivity is equivalent to 1 decay per second (this is a very small number)

A more practical unit is the **Curie** (Ci), defined such that $1\ \mathrm{Ci} = 3.7\times 10^{10}\ \mathrm{Bq}$

Decay Law Example 1

When the Earth was formed, the isotopes uranium-235 ($^{235}\mathrm{U}$) and uranium-238 ($^{238}\mathrm{U}$) were equally abundant.

Today, natural uranium consists of 99.3% $^{235}\mathrm{U}$ and 0.7% $^{238}\mathrm{U}$.

If the half-lives of $^{235}\mathrm{U}$ and $^{238}\mathrm{U}$ are 4.5×10^9 yr and 7.1×10^8 yr, respectively, what is the age of the Earth?

The number of nuclei remaining after time t is $N(t) = N_0 e^{-\lambda t}$. Hence the number ratio is given by,

$$\frac{N_{235}(t)}{N_{238}(t)} = \frac{N_0 e^{-\lambda_{235} t}}{N_0 e^{-\lambda_{238} t}} = e^{(\lambda_{238}-\lambda_{235})t}$$

which is equal to today's abundance ratio $\frac{99.3}{0.7} = 141.9$. The age is then $t = \frac{\ln 141.9}{\lambda_{238}-\lambda_{235}} = \frac{\ln 141.9}{0.693\times(1/0.71-1/4.5)} = 6.0\ \mathrm{Gyr}.$

Decay Law Example 2

Humans are radioactive!

Living tissue contains carbon-14 ($^{14}\mathrm{C}$) with an abundance of 1.3 parts per trillion (1.3×10^{-12}) of all carbon. $^{14}\mathrm{C}$ decays to $^{14}\mathrm{N}$ via β-decay, with a half-life $t_{1/2} = 5{,}730\ \mathrm{yr}$.

What is the level of radioactivity in Curies of a $70\ \mathrm{kg}$ human associated with this decay, assuming that humans are 18% carbon by mass, and the average carbon atom has mass $12.01\ \mathrm{u}$?

Total number of $^{14}\mathrm{C}$ atoms $N = 1.3\times10^{-12}\times\frac{0.18\times70}{12.01\times1.66\times10^{-27}} = 6.3\times10^{14}$

Decay rate $= \lambda N = \frac{0.693}{t_{1/2}}\cdot N = \frac{0.693}{5730\times3.2\times10^{7}}\times6.3\times10^{14} = 2.4\times10^{3}\ \mathrm{Bq} = 6.5\times10^{-8}\ \mathrm{Ci}$

That’s all for today!