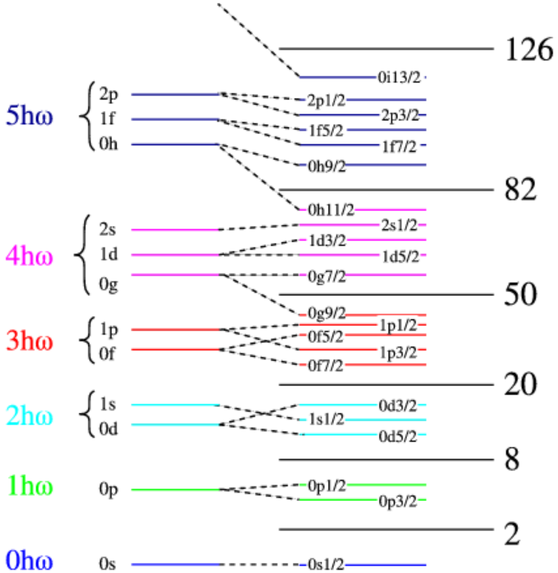
PHY20004 Nuclear Physics Class 2:

Modelling the Nucleus

In this class we'll learn how different aspects of the physics of the nucleus can be captured by two models: the liquid-drop model and the shell model



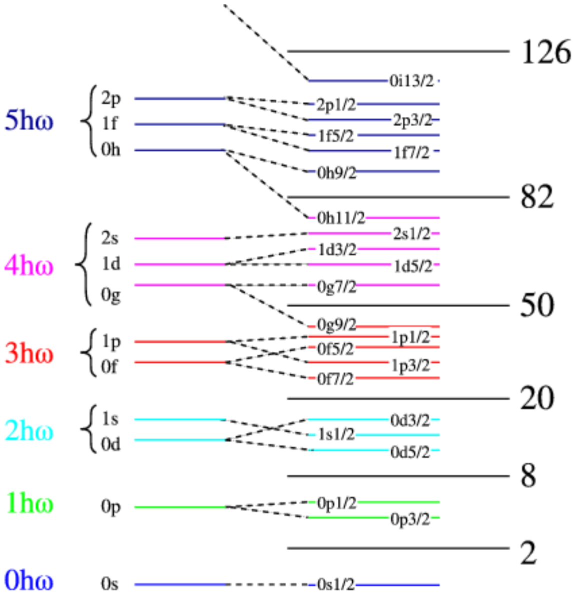
Modelling the nucleus

- There are two very useful physical models of the nucleus:

- **Liquid-drop model**: **nucleons act collectively as a single object** held together by the strong nuclear force

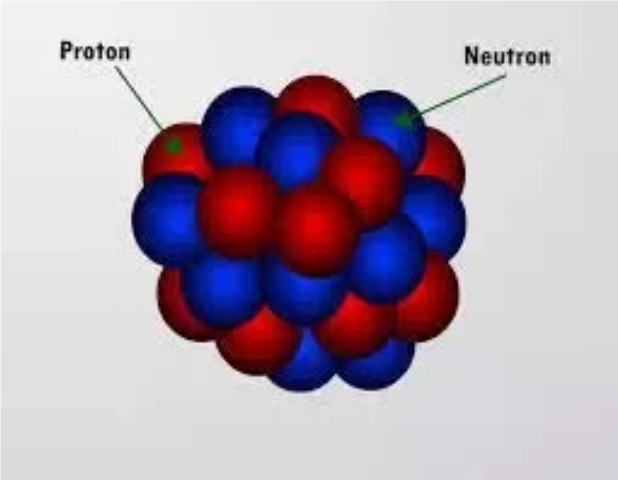
- **Shell model**: **nucleons move as independent particles** in an overall nuclear potential well

- Aspects of *each* of these models are needed to explain the observed phenomena of nuclear physics



The nucleus as a liquid drop

- The **liquid drop model** is a description of nuclei in which protons & neutrons behave *like molecules in a drop of liquid*



- Each system is held together by **short-range forces** and **surface tension**, and density is independent of size

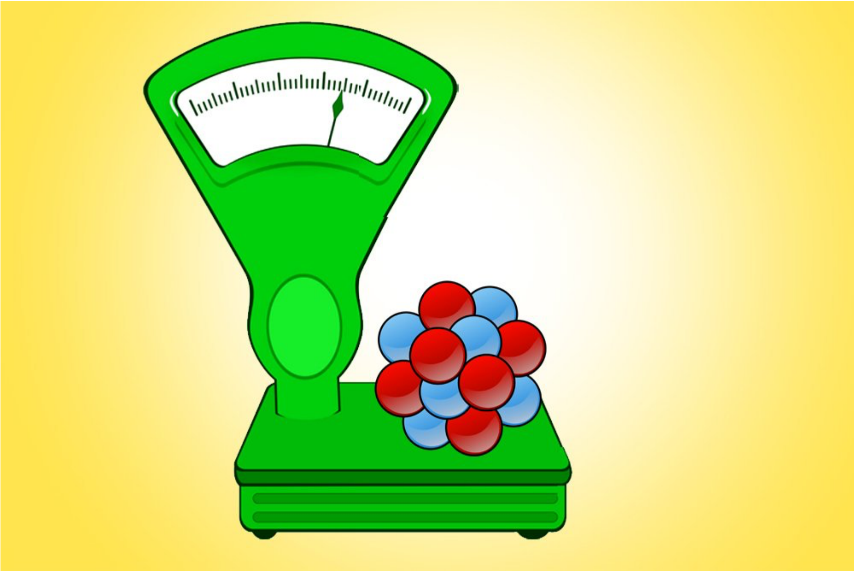
- The liquid drop model provides *excellent estimates of the average properties of nuclei*

Semi-empirical mass formula

- The liquid drop model motivates a model for the mass (hence binding energy) of a nucleus, in terms of Z, N and A, called the **semi-empirical mass formula** (or SEMF for short)

- The SEMF represents the binding energy of a nucleus by a **sum of a few terms** corresponding to different physical effects

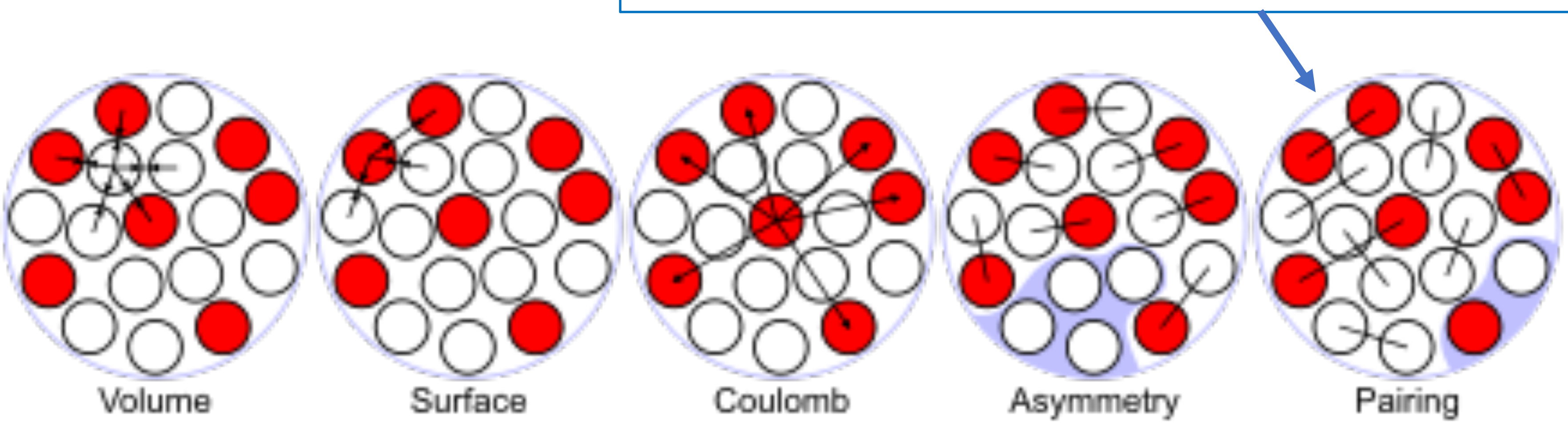
The model is called ***semi-empirical*** because these terms are motivated by theory, but their coefficients are adjusted to give the best experimental fit to observed masses



Semi-empirical mass formula

- Before meeting the formula, let's summarise the effects it's representing:

We're going to neglect this one in our discussion



Credit: https://en.wikipedia.org/wiki/Semi-empirical_mass_formula

Volume effect: the binding energy increases with the number of nucleons

Surface effect: nucleons on the surface have a reduced binding

Coulomb effect: there is electrostatic potential energy between protons

Asymmetry effect: nuclei prefer to have roughly balanced numbers of protons and neutrons

Semi-empirical mass formula

- The SEMF for the **nuclear binding energy** B in terms of the mass number A, atomic number Z and neutron number N:

$$B = a_V\, A - a_S\, A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_A \frac{(N-Z)^2}{A}$$

(we'll neglect the "pairing term" here)

Volume term: the strong force increases B by a constant amount per nucleon

Surface term: nucleons on the surface are not as strongly bound, decreases B as the surface area $\propto R^2 \propto A^{2/3}$

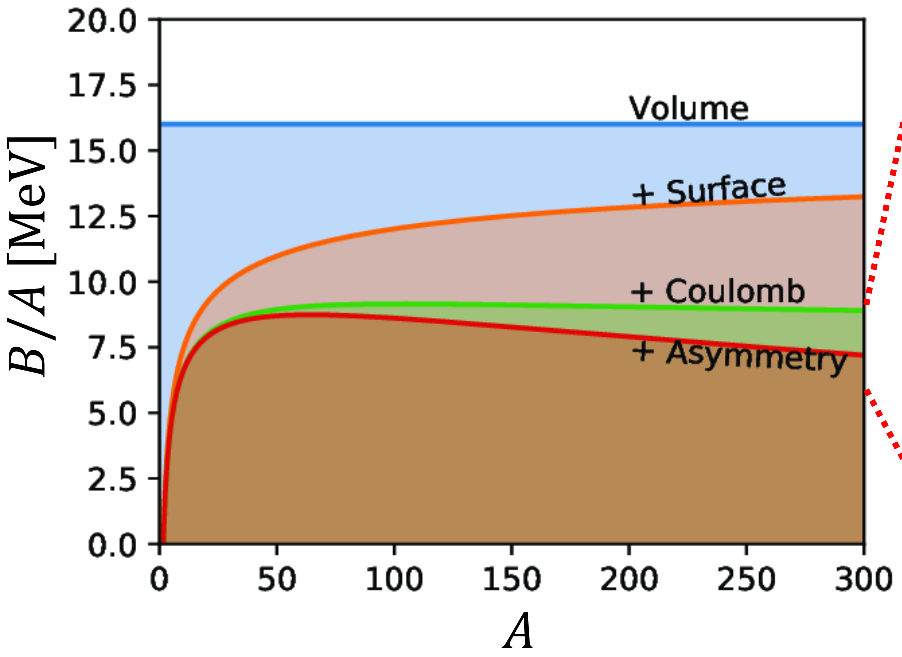
Coulomb term: protons repel each other, decreases B by electrostatic potential energy $\propto \frac{Q^2}{R} \propto \frac{Z^2}{A^{1/3}}$

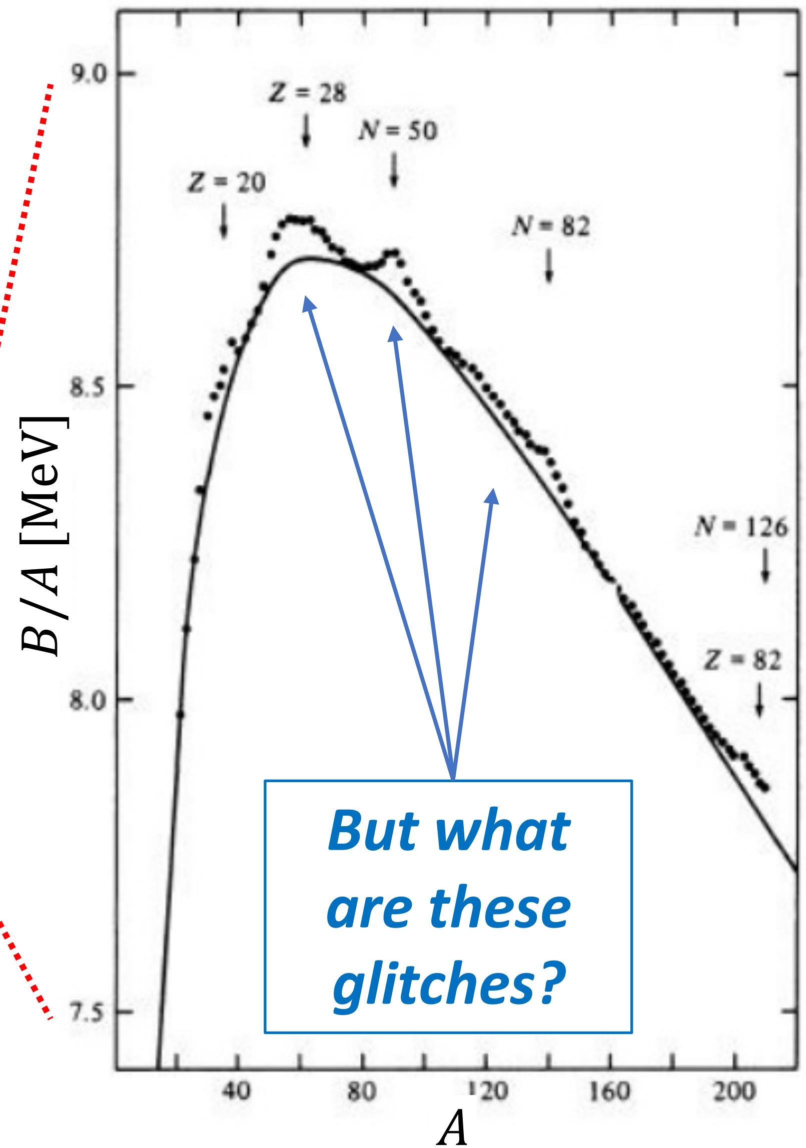
Asymmetry term: accounts for the tendency of nuclei to have $N \sim Z$

- The **coefficients** in the SEMF formula are found to be: $a_V = 15.8\text{ MeV}$, $a_S = 18.0\text{ MeV}$, $a_C = 0.72\text{ MeV}$, $a_A = 23.5\text{ MeV}$

Semi-empirical mass formula

- The SEMF successfully predicts the masses of many hundreds of nuclei with considerable ($\sim 1\%$) accuracy



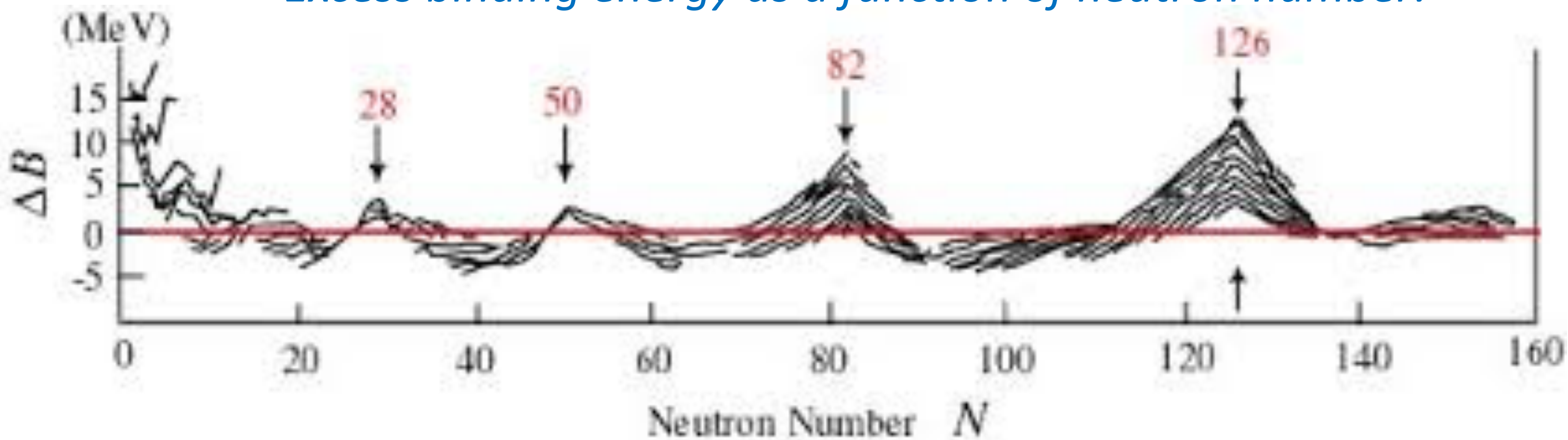


Credit: https://arxiv.org/abs/1911.03559

What's the magic number?

- The SEMF predicts the *average* nuclear binding energy, but **some nuclei are significantly more stable** (i.e. have higher binding energy) than calculated by the model
- These cases happen for nuclei with so-called **"magic numbers"** of protons and/or neutrons: Z and/or $N = 2, 8, 20, 28, 50, 82, 126$ (effects are more pronounced for "doubly magic")

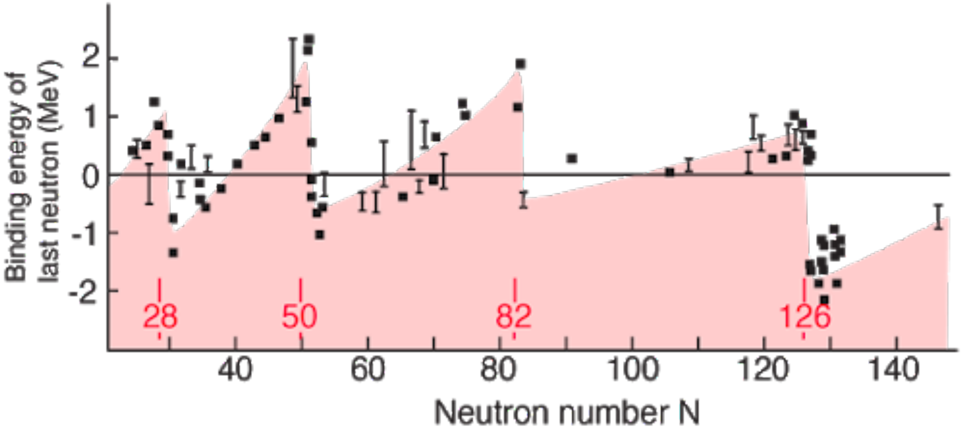
Excess binding energy as a function of neutron number:



Credit: http://ne.phys.kyushu-u.ac.jp

Properties of magic-number nuclei

- How do nuclei with magic numbers of protons and/or neutrons differ from other nuclei?
  - There are **a greater number of stable nuclei** with magic Z, N (e.g. 6 for $Z = 20$, compared to ~2 for similar Z)
  - There's a **greater natural abundance** of isotopes with magic Z, N
  - The **energy needed to remove** the first proton or neutron is unusually high for nuclei with magic Z, N



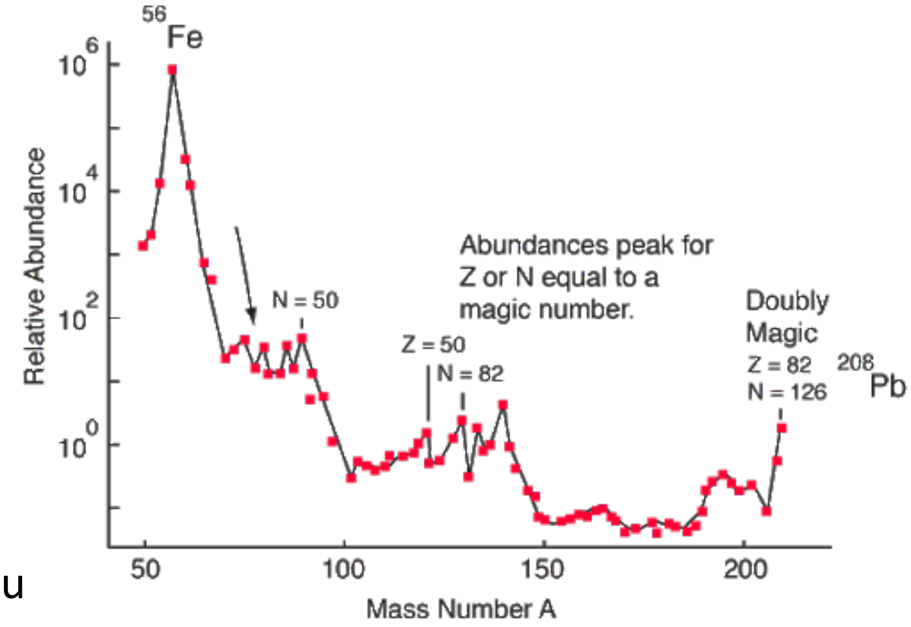
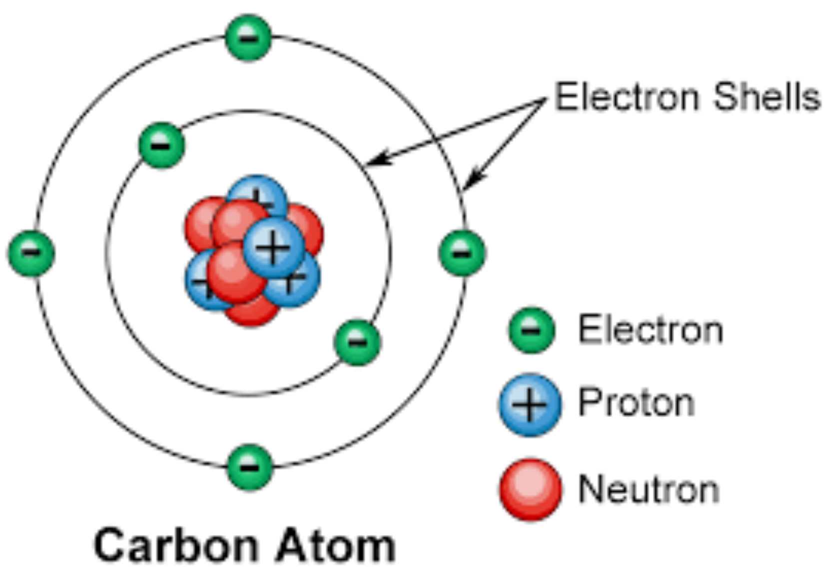


Image credit: http://hyperphysics.phy-astr.gsu.edu

The shell model

- The presence of magic numbers suggests a **shell model** – similar to the model of the atomic electrons – where the *magic numbers correspond to the closing of shells*
- Let's first recall the shell model for atomic structure:



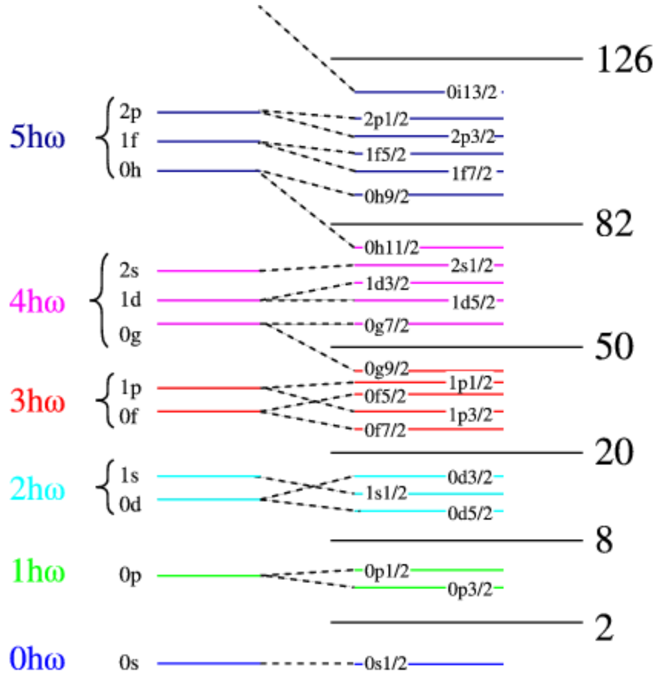
- Electrons **move independently** in an effective Coulomb potential
- Electrons occupy "shells" (energy levels) because of **quantum mechanics** and the **Pauli exclusion principle**
- **Shell closure** gives the most inert/stable atoms (Nobel gases)

The shell model

- The presence of magic numbers suggests a **shell model** – similar to the model of the atomic electrons – where the *magic numbers correspond to the closing of shells*

- This model can also describe nuclear structure:

(Requires detailed quantum calculations which we won't cover here!)



- Nucleons **move independently** in an effective strong-force potential

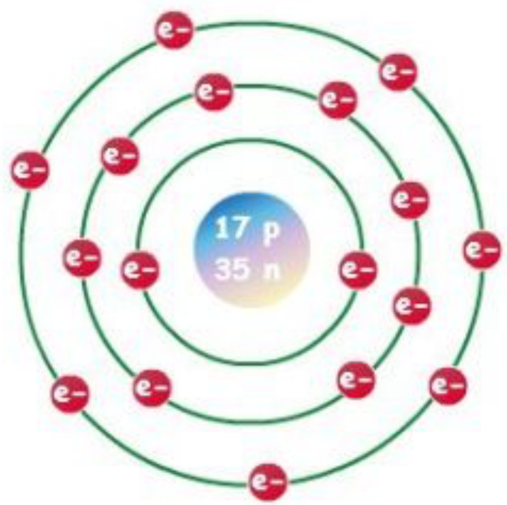
- Nucleons also obey the laws of **quantum mechanics** and the **Pauli exclusion principle**, and likewise occupy shells

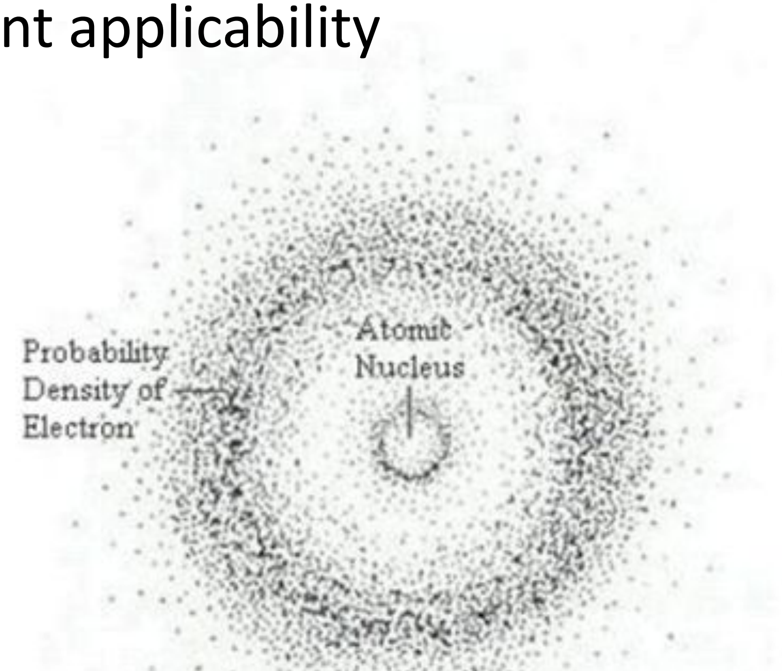
- **Shell closure** gives the most stable nuclei

Credit: https://oer.physics.manchester.ac.uk

Two models for the same thing?!

- It's quite common in physics that we have different models in our tool box, with different applicability





Credit: https://tracingcurves.wordpress.com

- Another example is atomic structure, where we can use Bohr's shell model, or the electron-cloud model of quantum mechanics!

Key take-aways

- There are **two important physical models** of the nucleus
- In the **liquid-drop model**, nucleons interact collectively
- This leads to the **semi-empirical mass formula**, which describes the nuclear binding energy as a sum of different effects depending on Z, N, A
- In the **shell model**, nucleons are independent particles moving in an overall nuclear potential
- Shell closure effects explain nucleon **magic numbers** – the unusually stable, tightly-bound and abundant nuclei that occur at Z and/or $N = 2, 8, 20, 28, 50, 82, 126$