Formula Sheet for Relativity

Special relativity	$t' = \gamma(t - vx/c^2) \quad x' = \gamma(x - vt) \quad y' = y \quad z' = z$
	$t = \gamma(t' + vx'/c^2) \quad x = \gamma(x' + vt') \quad y = y' \quad z = z'$
	$\Delta l = \Delta L/\gamma \quad \Delta t = \gamma\Delta\tau \quad \gamma = 1/\sqrt{1 - v^2/c^2}$
	$ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2$
	$u' = (u - v)/(1 - uv/c^2)$
	$m(v) = \gamma m_0 \quad p = \gamma m_0 v \quad E = \gamma m_0 c^2$
4-vectors	$x^\mu = (ct, x, y, z)$
	$dx^\mu = (cdt, dx, dy, dz)$
	$\partial_\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$
	$u^\mu = \frac{dx^\mu}{d\tau} = (\gamma c, \gamma u_x, \gamma u_y, \gamma u_z)$
	$p^\mu = m_0 u^\mu = (E/c, p_x, p_y, p_z)$
Energy-momentum tensor	$T^{\mu\nu} = \frac{dp^\mu\, dx^\nu}{dV\, dt} \qquad T^{\nu\mu} = T^{\mu\nu}$
	T^{00} = energy density
	$T^{0i} = T^{i0}$ = flux of energy in i-direction
	$T^{ij} = T^{ji}$ = flux of i-momentum in the j-direction
Energy-momentum tensor for perfect fluid	$T^{\mu\nu} = \begin{pmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix}$
Energy-momentum tensor for electromagnetism	$T^{\mu\nu} = \frac{1}{\mu_0}\left(F^\mu{}_\lambda F^{\nu\lambda} - \frac{1}{4}\eta^{\mu\nu}F_{\kappa\lambda}F^{\kappa\lambda}\right)$
Energy-momentum conservation	$\partial_\mu T^{\mu\nu} = 0$
Index operations in special relativity for A^μ and $B^{\mu\nu}$	$\eta_{\mu\nu} = \eta^{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
	$A_\mu = \eta_{\mu\nu}A^\nu \quad A^\mu = \eta^{\mu\nu}A_\nu$
	$B_\lambda{}^\nu = \eta_{\lambda\mu}B^{\mu\nu} \quad B^\mu{}_\lambda = \eta_{\lambda\nu}B^{\mu\nu} \quad B_{\kappa\lambda} = \eta_{\kappa\mu}\eta_{\lambda\nu}B^{\mu\nu}$
	$B^\lambda{}_\nu = \eta^{\lambda\mu}B_{\mu\nu} \quad B^\lambda{}_\mu = \eta^{\lambda\nu}B_{\mu\nu} \quad B^{\kappa\lambda} = \eta^{\kappa\mu}\eta^{\lambda\nu}B_{\mu\nu}$
	$A_\mu A^\mu$ and $B_{\mu\nu}B^{\mu\nu}$ are invariants
Lorentz (L) and inverse ($\tilde{L}$) transformations between inertial frames in special relativity	$A'^\mu = L^\mu{}_\nu A^\nu \quad A'_\mu = L_\mu{}^\nu A_\nu$
	$A^\mu = \tilde{L}^\mu{}_\nu A'^\nu \quad A_\mu = \tilde{L}_\mu{}^\nu A'_\nu$
	$B'^{\mu\nu} = L^\mu{}_\kappa L^\nu{}_\lambda B^{\kappa\lambda} \quad B^{\mu\nu} = \tilde{L}^\mu{}_\kappa \tilde{L}^\nu{}_\lambda B'^{\kappa\lambda}$
	$L^\mu{}_\nu = \begin{pmatrix} \gamma & -v\gamma/c & 0 & 0 \\ -v\gamma/c & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \tilde{L}^\mu{}_\nu = \begin{pmatrix} \gamma & v\gamma/c & 0 & 0 \\ v\gamma/c & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

4-vectors for electromagnetism	$J^\mu = \frac{dQ}{dV}\frac{dx^\mu}{dt} = (\rho c, J_x, J_y, J_z)$
	$A^\mu = \left(V/c, A_x, A_y, A_z\right)$
Maxwell field tensor	$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$
	$F^{\mu\nu} = \begin{pmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & B_z & -B_y \\ -E_y/c & -B_z & 0 & B_x \\ -E_z/c & B_y & -B_x & 0 \end{pmatrix}$
Maxwell's equations	$\partial_\mu F^{\mu\nu} = -\mu_0 J^\nu$
	$\partial^\lambda F^{\mu\nu} + \partial^\mu F^{\nu\lambda} + \partial^\nu F^{\lambda\mu} = 0$
Charge conservation	$\partial_\mu J^\mu = 0$
Lorentz force law	$dp^\mu/d\tau = qF^\mu{}_\nu u^\nu$
Space-time metric in general relativity	$ds^2 = g_{\mu\nu}dx^\mu dx^\nu \qquad g_{\nu\mu} = g_{\mu\nu}$
	$ds^2 = -c^2 d\tau^2 \qquad d\tau =$ proper time interval
Stationary clock in a gravitational field	$d\tau = \sqrt{-g_{tt}}\, dt$
Proper distance in radial direction	$dL = \sqrt{g_{rr}}\, dr$
Weak-field limit	$g_{tt} = -1 - 2\phi/c^2 \qquad \phi =$ gravitational potential
Index operations in general relativity	$A_\mu = g_{\mu\nu}A^\nu \qquad A^\mu = g^{\mu\nu}A_\nu$
	$g^{\mu\nu}$ is the inverse of $g_{\mu\nu}$
	$B_\lambda{}^\nu = g_{\lambda\mu}B^{\mu\nu} \qquad B^\mu{}_\lambda = g_{\lambda\nu}B^{\mu\nu} \qquad B_{\kappa\lambda} = g_{\kappa\mu}g_{\lambda\nu}B^{\mu\nu}$
	$B^\lambda{}_\nu = g^{\lambda\mu}B_{\mu\nu} \qquad B^\lambda{}_\mu = g^{\lambda\nu}B_{\mu\nu} \qquad B^{\kappa\lambda} = g^{\kappa\mu}g^{\lambda\nu}B_{\mu\nu}$
	$A_\mu A^\mu$ and $B_{\mu\nu}B^{\mu\nu}$ are invariants
Co-ordinate transformations in general relativity	$A'^\mu = \frac{\partial x'^\mu}{\partial x^\nu}A^\nu \qquad A'_\mu = \frac{\partial x^\nu}{\partial x'^\mu}A_\nu$
	$A^\mu = \frac{\partial x^\mu}{\partial x'^\nu}A'^\nu \qquad A_\mu = \frac{\partial x'^\nu}{\partial x^\mu}A'_\nu$
	$B'^{\mu\nu} = \frac{\partial x'^\mu}{\partial x^\kappa}\frac{\partial x'^\nu}{\partial x^\lambda}B^{\kappa\lambda} \qquad B'_{\mu\nu} = \frac{\partial x^\kappa}{\partial x'^\mu}\frac{\partial x^\lambda}{\partial x'^\nu}B_{\kappa\lambda}$
Geodesic equations	$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu_{\kappa\lambda}\frac{dx^\kappa}{ds}\frac{dx^\lambda}{ds} = 0$
	$g_{\mu\nu}\frac{d^2x^\nu}{ds^2} + \left(\partial_\lambda g_{\mu\kappa} - \frac{1}{2}\partial_\mu g_{\kappa\lambda}\right)\frac{dx^\kappa}{ds}\frac{dx^\lambda}{ds} = 0$
	For light ray: $ds = 0$, so use an affine parameter instead
Christoffel symbols	$\Gamma^\mu_{\kappa\lambda} = \frac{1}{2}g^{\mu\nu}(\partial_\lambda g_{\nu\kappa} + \partial_\kappa g_{\lambda\nu} - \partial_\nu g_{\kappa\lambda})$
Metric on the surface of a sphere	$ds^2 = R^2\, d\theta^2 + (R\sin\theta)^2\, d\phi^2$
Schwarzschild metric	$ds^2 = -\left(1 - \frac{R_S}{r}\right)c^2dt^2 + \frac{dr^2}{1 - \frac{R_S}{r}} + r^2[d\theta^2 + (\sin\theta)^2 d\phi^2]$
Schwarzschild radius	$R_S = 2GM/c^2$
Gravitational redshift	$1 + z = 1/\sqrt{1 - R_s/r}$

Radial free-fall in the Schwarzschild metric	$\frac{dt}{d\tau} = \frac{K}{A} \qquad \frac{1}{c}\frac{dr}{d\tau} = \sqrt{K^2 - A} \qquad A = 1 - \frac{R_S}{r}$
FRW metric	$ds^2 = -c^2 dt^2 + a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + (\sin\theta)^2 d\phi^2)\right]$
Riemann tensor	$dA^\kappa = R^\kappa{}_{\lambda\mu\nu} A^\lambda dx^\mu dx^\nu$
	$R^\kappa{}_{\lambda\mu\nu} = \partial_\mu \Gamma^\kappa_{\lambda\nu} - \partial_\nu \Gamma^\kappa_{\lambda\mu} + \Gamma^\kappa_{\mu\alpha}\Gamma^\alpha_{\lambda\nu} - \Gamma^\kappa_{\nu\alpha}\Gamma^\alpha_{\lambda\mu}$
	$R_{\mu\nu\kappa\lambda} = R_{\kappa\lambda\mu\nu}$
	$R_{\lambda\kappa\mu\nu} = -R_{\kappa\lambda\mu\nu}$
Ricci tensor	$R_{\mu\nu} = R^\lambda{}_{\mu\lambda\nu}$
	$R_{\mu\nu} = \partial_\lambda \Gamma^\lambda_{\mu\nu} - \partial_\nu \Gamma^\lambda_{\mu\lambda} + \Gamma^\kappa_{\kappa\lambda}\Gamma^\lambda_{\mu\nu} - \Gamma^\kappa_{\nu\lambda}\Gamma^\lambda_{\mu\kappa}$
	$R_{\mu\nu} = 0$ in empty space
Ricci scalar	$R = R^\mu{}_\mu = g^{\mu\nu} R_{\mu\nu}$
Einstein tensor	$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$
Einstein equation	$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$
	$R_{\mu\nu} = \frac{8\pi G}{c^4}\left(T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu}\right) \qquad T = T^\mu{}_\mu = g^{\mu\nu} T_{\mu\nu}$
Friedmann equation	$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G \rho(t)}{3} - \frac{kc^2}{a^2}$