Formula Sheet for Cosmology

Constants	$c = 3.00 \times 10^{8}\ \mathrm{m\,s^{-1}} \qquad G = 6.67 \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$
	$1\ M_\odot = 2 \times 10^{30}\ \mathrm{kg} \qquad 1\ L_\odot = 3.9 \times 10^{26}\ \mathrm{W}$
	$1\ \mathrm{pc} = 3.09 \times 10^{16}\ \mathrm{m}$
	$1\ \mathrm{Gyr} = 3.16 \times 10^{16}\ \mathrm{s}$
	$H_0 = 70\ \mathrm{km\,s^{-1}\,Mpc^{-1}} = 2.27 \times 10^{-18}\ \mathrm{s^{-1}} = 0.0716\ \mathrm{Gyr^{-1}}$
	$1/H_0 = 4.41 \times 10^{17}\ \mathrm{s} = 14.0\ \mathrm{Gyr}\ \ (\text{for } H_0 = 70)$
	$c/H_0 = 1.32 \times 10^{26}\ \mathrm{m} = 4.28\ \mathrm{Gpc}\ \ (\text{for } H_0 = 70)$
FRW metric	$ds^2 = -c^2dt^2 + a(t)^2\left[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$
Light travel	$ds = 0 \rightarrow c\, dt = \pm\frac{a(t)\, dr}{\sqrt{1-Kr^2}}$
Redshifting	$z = \lambda_{\mathrm{obs}}/\lambda_{\mathrm{em}} - 1$
	$a_{\mathrm{em}} = 1/(1+z)$
Hubble parameter	$v = H_0 d$
	$H = \dot{a}/a$
Age of Universe	$t_{\mathrm{age}} = \int_0^1 \frac{da}{a\, H(a)} \qquad t_{\mathrm{lookback}}(a) = \int_a^1 \frac{da'}{a'\, H(a')}$
Distances	$D_{\mathrm{phys}} = a(t)\int_0^r \frac{dr'}{\sqrt{1-Kr'^2}} = \begin{cases} a(t)\sin^{-1}(r\sqrt{K})/\sqrt{K} & (K>0) \\ a(t)\, r & (K=0) \\ a(t)\sinh^{-1}(r\sqrt{K})/\sqrt{K} & (K<0) \end{cases}$
	$D_A = W/\Delta\theta = a(t_{em})\, r = r/(1+z)$
	$D_L = \sqrt{L/4\pi f} = r/a(t_{em}) = r\,(1+z)$
	Distance modulus $\mu = 5\log_{10} D_L(\mathrm{Mpc}) - 25$
Volume element	$dV = \frac{4\pi r^2\, dr}{\sqrt{1-Kr^2}}$
Particle horizon	$D_H(t) = c\, a(t)\int_0^t \frac{dt'}{a(t')}$
Friedmann equation	$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho(t)}{3} - \frac{Kc^2}{a^2} + \frac{\Lambda c^2}{3}$
	Matter: $\rho_m(t) = \rho_{m,0}/a^3$
	Radiation: $\rho_r(t) = \rho_{r,0}/a^4$
	Cosmological constant: $\rho_\Lambda(t) = \text{constant}$
	$\rho_{crit} = \frac{3{H_0}^2}{8\pi G} = 9.20 \times 10^{-27}\ \mathrm{kg\,m^{-3}}\ \ (\text{for } H_0 = 70)$
	Density parameters: $\Omega = \rho/\rho_{crit}$
	$\frac{1}{{H_0}^2}\left(\frac{\dot{a}}{a}\right)^2 = \frac{\Omega_r}{a^4} + \frac{\Omega_m}{a^3} + \frac{\Omega_K}{a^2} + \Omega_\Lambda$
	$\Omega_m = \frac{8\pi G\rho_m}{3{H_0}^2} \qquad \Omega_K = -\frac{Kc^2}{{H_0}^2} \qquad \Omega_\Lambda = \frac{\Lambda c^2}{3{H_0}^2}$
	$\Omega_r + \Omega_m + \Omega_K + \Omega_\Lambda = 1$

Solutions	$a(t) \propto \begin{cases} t^{2/3} & \text{(matter dominated)} \\ t^{1/2} & \text{(radiation dominated)} \\ t & \text{(empty Universe)} \\ e^{Ct} & (\Lambda \text{ dominated}) \end{cases}$
Energy equation	$\frac{d\rho}{dt} + 3H\left(\rho + \frac{P}{c^2}\right) = 0$
Equation of state	$P = w\rho c^2$
	$w = \begin{cases} 0 & \text{(matter)} \\ \frac{1}{3} & \text{(radiation)} \\ -1 & (\Lambda) \end{cases}$
	$\rho(t) \propto a^{-3(1+w)}$
Acceleration equation	$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right) = -\frac{4\pi G\rho}{3}(1 + 3w)$
CMB temperature	$T(z) = (2.73\text{ K})\,(1 + z)$
	Energy density of radiation $U_r = \left(\frac{4\sigma}{c}\right)T^4$
	σ = Stefan Boltzmann constant $= 5.67 \times 10^{-8}\ \text{W m}^{-2}\ \text{K}^{-4}$
	Average energy of photon $= 2.71\ k_B T$
	Baryon to photon ratio $\eta = 6.68 \times 10^{-10}$ (for $\Omega_b = 0.05$)
	Saha equation: $\frac{1-x}{x^2} = \frac{n_b\, h^3}{(2\pi m_e k_B T)^{3/2}}\, e^{E_{\text{bind}}/k_B T}$
	Mean free path $= \frac{1}{n_e \sigma_T}$
	σ_T = Thomson cross section $= 6.65 \times 10^{-29}\ \text{m}^2$
Spherical collapse	Jeans length $L_J = \sqrt{\frac{k_B T}{G\rho_m m_p}}$
	Jeans mass $M_J = \rho {L_J}^3$
	Collapse time $t_G \sim \frac{1}{\sqrt{G\rho_m}}$