Cosmology Week 6 Class Activities

We'll study these activities in our Week 6 tutorial class!

Sound waves in the early Universe

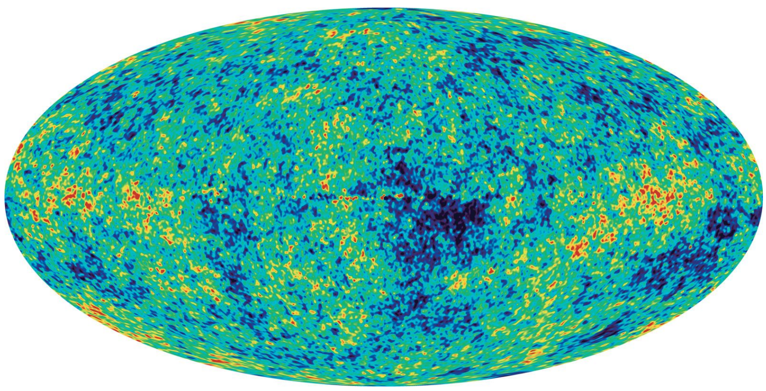
Before recombination, sound waves filled the Universe! These are compression waves propagating in the baryon-photon plasma due to the opposing forces of gravity and radiation pressure. They serve to correlate different points on the CMB sky which are generated from patches separated by this distance, producing the pattern of temperature ripples we detect.

a) The comoving distance travelled by a sound wave between the Big Bang and recombination at redshift $z_{rec} = 1100$ (otherwise known as the sound horizon) is given by,

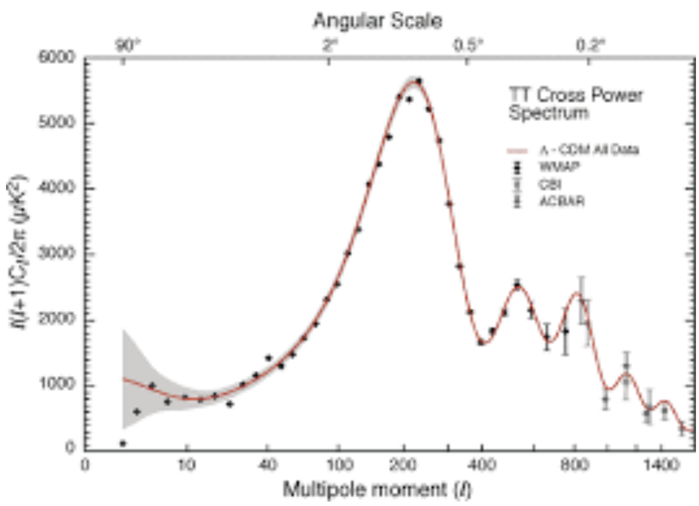
$$r_s = \int_0^{t_{rec}} c_s \, (1+z) \, dt$$

where the $(1+z)$ accounts for expansion. It turns out that sound waves in a relativistic plasma travel rather fast, at speed $c_s = c/\sqrt{3}$! By adapting your calculation of the horizon from Week 5, determine the sound horizon in a matter-dominated Universe, assuming $\Omega_m = 0.3$ and $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ as usual.

b) The CMB temperature pattern shows a strong correlation for points separated by $\theta_s = 1°$ on the sky. It is hard to see this correlation scale in the temperature fluctuations themselves:



But it becomes more obvious when a correlation spectrum with a clear peak is measured:



Use the observation that the sound horizon projects to an angle of 1° to deduce the distance to the last-scattering surface.

c) Using your distance calculator developed in Class 3, compute the comoving distance to the CMB redshift $z = 1100$ in the following scenarios, along with the sound horizon scale r_s and expected correlation angle θ_s:

Ω_m	Ω_Λ	Ω_K
0.2	0.8	0
0.3	0.7	0
0.4	0.6	0

You'll see that the distance to the last-scattering surface and the sound horizon scale vary in a similar way, such that the correlation angle does not change by much. This implies that the CMB provides good evidence that $\Omega_K \approx 0$.

Applying the baryon acoustic peak in galaxies

The sound waves in the early Universe also imprint a preferred separation in the distribution of baryons, which creates a preferred separation between galaxies. You can assume this preferred scale is the same as evaluated above, $r_s \approx 272$ Mpc. (The actual value is a bit different owing to some approximations we made in the last part, but the same principle applies!)

Astronomers create a new galaxy redshift survey at $z = 1$, and measure the angular separations and redshift separations between galaxies.

They find there is a slight excess of galaxies with preferred separations:

$$\Delta\theta \approx 4.7°$$
$$\Delta z \approx 0.112$$

Can these observations distinguish between the three cosmological models listed above?

Mean free path of a photon!

The mean free path of a photon as it propagates through the Universe is given by:

$$l = \frac{1}{n_e \, \sigma_T}$$

where $\sigma_T = 6.65 \times 10^{-29}\ m^2$ is the cross-section for Thomson scattering.

a) The Universe today is fully ionized, meaning that the electron density is equal to the baryon density we computed in Week 4: $n_e = 0.28\ m^{-3}$. What is the mean free path for a photon today? How does this compare to the size of the Universe?

b) Repeat this calculation for redshift $z = 100$.

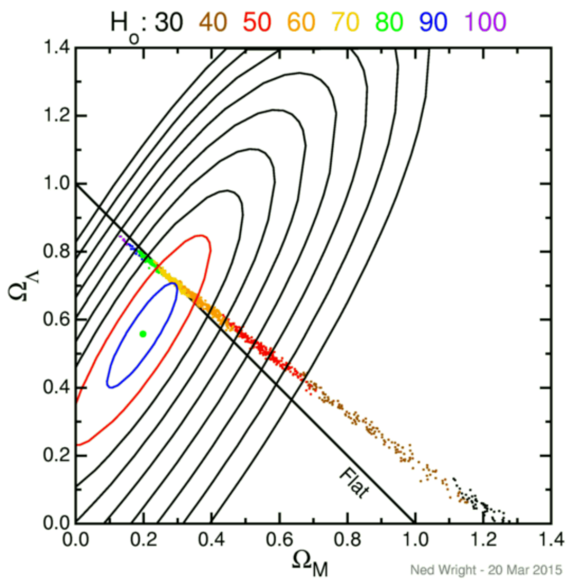
c) Given the result of part b), what process allows the CMB photons to reach us?

d) Estimate the earliest point that reionisation of the Universe can occur without blocking the CMB!

Fitting cosmological parameters to supernovae data

This is a computer-based activity which I recommend we carry out using a Python Jupyter notebook (as widely used in astronomy research) – however, it can be solved using any programming language.

In this activity we will use a recent supernova distance-redshift dataset to find the joint confidence region of the parameters Ω_m and Ω_Λ. Our aim is to learn how to make a graph like this:



1) Create a grid of values in the range $0 < \Omega_m < 1.4$ and $0 < \Omega_\Lambda < 1.4$.

2) For each $(\Omega_m,\ \Omega_\Lambda)$ point, calculate the luminosity distance $D_L(z)$ at the redshift of each supernova.

3) Convert the luminosity distances into the distance modulus $\mu = 5\log_{10} D_L + 25$.

4) Determine the χ^2 value of each model by summing over the data, defined by

$$\chi^2 = \sum_i \left(\frac{\mu_{\rm data}(z_i) - \mu_{\rm model}(z_i)}{\Delta\mu_i} \right)^2$$

5) The 68% confidence region is defined by $\chi^2 = \chi^2_{\rm min} + 2.3$. Plot this contour in the parameter space $(\Omega_m,\ \Omega_\Lambda)$.
