Cosmology Week 5 Class Activities

We'll study these activities in our Week 5 tutorial class!

Cosmological horizons

The particle horizon distance $D_H(t)$ at a given cosmic time t is the maximum physical distance between two points that can be linked with a light ray between the Big Bang and that time.

a) Why is $D_H(t) \neq ct$?

Let's say that the light ray moves between comoving coordinates $r = 0$ and $r = r_{\mathrm{max}}$ in this time. We'll assume a flat Universe ($K = 0$) for this calculation.

b) Explain why the physical distance between these points is: $D_H = a(t)\, r_{\mathrm{max}}$.

c) Light rays travel such that $ds = 0$. Use the metric to show that two nearby points along the path of the light ray must be linked by: $dr = c\, dt'/a(t')$.

d) Combining these two equations, show that $D_H = c\, a(t) \int_0^t \frac{dt'}{a(t')}$.

We'll assume now that the Universe is matter-dominated (which is true for most of its history), such that the Friedmann equation can be written as:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\Omega_m\, {H_0}^2}{a^3}$$

(assuming that the radiation, curvature and dark energy terms are negligible). This equation has the solution:

$$a(t) = \left(\frac{3\sqrt{\Omega_m} H_0 t}{2}\right)^{2/3}$$

e) Hence show that $D_H = 3ct$.

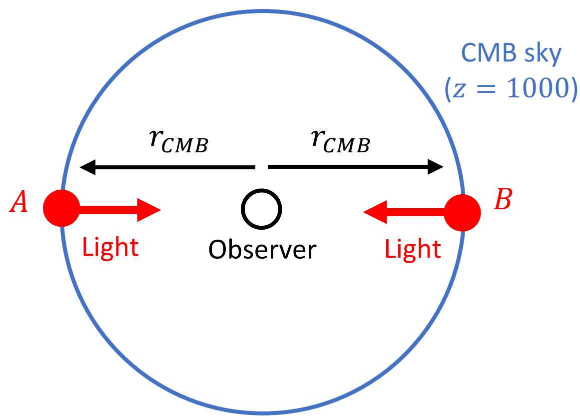
f) Suppose that the Universe is accelerating in its expansion such that $a(t) = e^{H_0(t-t_0)}$. What is the horizon distance in between times t_0 and a later time t?

The horizon problem

Let's now calculate the horizon distance when the CMB was produced.

a) If $\Omega_m = 0.3$ and $H_0 = 70$ km s^{-1} Mpc^{-1}, and the CMB was produced at redshift $z = 1100$, use the formulae in the previous question to find the horizon distance D_H at this moment, assuming a matter-dominated Universe. Convert this distance to a comoving separation in today's Universe.

It's interesting to compare this distance to the physical separation of two points A and B on opposite sides of the sky when the CMB was produced:



b) Assuming $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$, use your cosmology calculator from the previous activity to compute the comoving distance r_{CMB} to redshift $z = 1100$.

Your answers to parts a) and b) will show that the distance between points A and B is much larger than the horizon distance, even though the radiation fields at these points is in mutual equilibrium. This issue is known as the **horizon problem**.

c) Explain how the theory of inflation solves this problem.

When astronomical objects collapse

Let's investigate what type of objects can form at different times in the Universe!

Let's start near recombination, when the temperature of the Universe is $T = 3000\ K$ and the density is given by $\rho = \rho_0\ (1+z)^3 = \Omega_m\ \rho_{crit}\ (1+z)^3$, where $z = 1100$, $\Omega_m = 0.3$ and $H_0 = 70$ km s^{-1} Mpc^{-1}.

a) Evaluate the Jeans length at this epoch, $L_J \approx \sqrt{\frac{k_B T}{G \rho m_p}}$ and hence the Jeans mass $M_J \approx \rho {L_J}^3$. Hence, what type of object is able to collapse at this time?

b) What is the gravitational collapse time $t_G \approx \frac{1}{\sqrt{G\rho}}$ of this object? Hence, comment on the epoch of the Universe by which time this type of object can form.

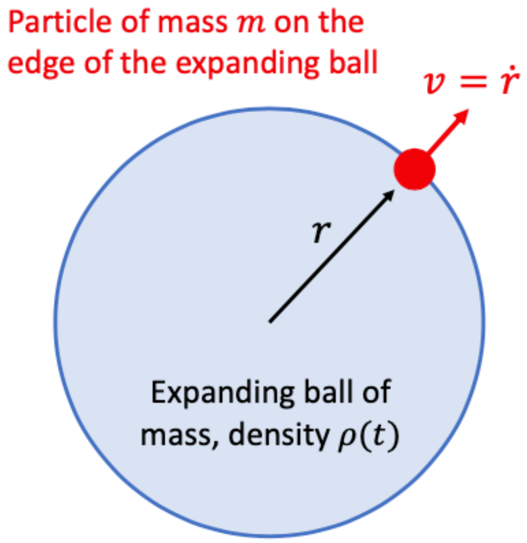
Now let's consider objects on the other end of the scale! Suppose a galaxy cluster has a mass $M = 10^{14}\ M_\odot$ and forms from material within a radius of 3 Mpc.

c) Estimate the gravitational collapse time of the cluster. Hence, comment on the epoch of the Universe when we would expect to see such clusters forming.

Solving spherical collapse

This is a computer-based activity which I recommend we carry out using a Python Jupyter notebook (as widely used in astronomy research) – however, it can be solved using any programming language.

In this activity we'll use Newton's laws to calculate the evolution of a collapsing spherical clump! The situation is exactly like the Newtonian Universe we studied in Week 1:



The edge of the sphere satisfies the equation:

$$\ddot{r} = -\frac{GM}{r^2}$$

The evolution of $r(t)$ satisfies the parametric equations in terms of a variable p:

$$r(p) = \frac{R_0}{2}(1 - \cos p)$$
$$t(p) = \sqrt{\frac{3}{32\pi G\rho_o}}(p - \sin p)$$

where R_0 is the radius of the clump when (if!) it reaches a maximum, and ρ_0 is its density at this time (such that $M = \frac{4}{3}\pi {R_0}^3\rho_0$). You can prove these equations work if you like!

a) Fixing the mass of the clump to be equal to a galaxy cluster, $M = 10^{14}\ M_\odot$, make a series of tracks $r(t)$ choosing different values $R_0 = (1,2,3,4,5,10)$ Mpc. Only plot a range of times corresponding to the age of the Universe, $t_{age} = 4.3 \times 10^{17}$ s.

b) Hence find the maximum size of the region which collapses to form a cluster within the current age of the Universe.