Cosmology Week 4 Class Activities

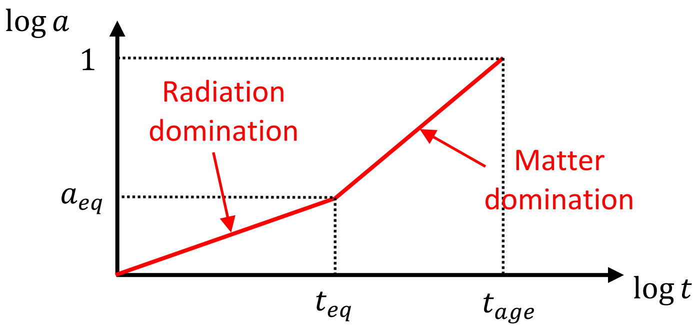
We'll study these activities in our Week 4 tutorial class!

The temperature is falling!

As the Universe expands, it cools! The temperature varies with scale factor as $T(t) \propto 1/a(t)$. Since the CMB temperature is $T = 2.73$ K today ($a = 1$), we can write:

$$T(t) = \frac{2.73 \text{ K}}{a(t)}$$

Now we need to find the relation $a(t)$! For the purposes of this calculation, let's ignore dark energy and assume that the Universe switches from a period of radiation domination ($a(t) \propto t^{1/2}$) to matter domination ($a(t) \propto t^{2/3}$) at a time t_{eq} and scale factor a_{eq}:



a) The energy density of a gas of blackbody radiation is given by $U_r = \left(\frac{4\sigma}{c}\right) T^4$ where $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4} is the Stefan-Boltzmann constant. What is the value of the density parameter in radiation, Ω_r, in today's Universe, if $H_0 = 70$ km s^{-1} Mpc^{-1}?

b) Find the scale factor a_{eq} at which the energy density in radiation equals the energy density in matter, assuming $\Omega_m = 0.3$.

c) The age of the Universe is $t_{age} = 13.7$ Gyr. Assuming $a(t) \propto t^{2/3}$ for $t > t_{eq}$, find t_{eq}.

d) What is the temperature at the time of matter-radiation equality?

e) Assuming $a(t) \propto t^{1/2}$ for $t < t_{eq}$, find the relation between temperature and time in the radiation-dominated phase.

To check your result, a very good approximation (which we can use in the following activity) is:

$$k_B T = \frac{1.14 \text{ MeV}}{\sqrt{t \text{ [s]}}}$$

(our calculation neglects the impact of dark energy on the expansion, and the contribution of neutrinos to the radiation density).

The story of a neutron

Big Bang nucleosynthesis and stellar nucleosynthesis both fuse hydrogen into helium. The big difference is that in the early Universe, there are plenty of free neutrons available! In this activity we'll explore what difference this makes.

The free protons and neutrons in the hot, early Universe are kept in thermal equilibrium by the enormous sea of electrons, positrons, neutrinos and anti-neutrinos. Protons and neutrons have slightly different rest mass energies, $m_n c^2 = 939.6$ MeV and $m_p c^2 = 938.3$ MeV. In thermal equilibrium, the ratio of the numbers of neutrons and protons is given by:

$$\frac{N_n}{N_p} \approx \exp\left[-\frac{(m_n - m_p)c^2}{k_B T}\right]$$

When the temperature falls to $k_B T \approx 0.8$ MeV, thermal equilibrium can no longer be maintained because the reaction rate of forming electrons and neutrinos drops too much, hence neutrons decouple from protons.

a) What is the ratio N_n/N_p at neutron decoupling? What is the age of the Universe at this point?

Left alone, neutrons are unstable to beta decay into protons with a half-life of $T_{1/2} = 610$ s. Free protons and neutrons can also combine to form deuterium through the process $p + n \rightarrow D + \gamma$. However, the deuterium nucleus can only remain bound when the temperature reaches $k_B T \approx 0.06$ MeV.

b) What is the age of the Universe when deuterium starts forming? Allowing for the neutron beta decays into protons up to this point, what is the ratio N_n/N_p at deuterium formation?

c) Suppose all the neutrons present at deuterium formation end up in helium-4 nuclei, which each consist of 2 neutrons and 2 protons. What would be the relative abundance of helium-4 to hydrogen?

It's lucky that deuterium can start forming *before* all the neutrons disappear through beta decay!

Deducing the baryon-to-photon ratio

In this activity we'll deduce the number of photons for every baryon in the Universe!

We'll start with today's estimate of the baryon density parameter: $\Omega_b = 0.05$.

a) Assuming the Hubble constant is $H_0 = 70$ km s^{-1} Mpc^{-1}, what is the critical density?

b) Hence find the mass density in baryons, ρ_b.

c) Assuming each baryon has an average mass of $m_b = 1.67 \times 10^{-27}$ kg, what is the number density of baryons, n_b, in today's Universe?

We'll now use the formula given in the previous activity for the energy density of blackbody radiation.

d) The mean energy of a blackbody radiation photon is 2.71 $k_B T$. Evaluate the number density of photons, n_γ, for the CMB radiation in today's Universe.

e) Hence, estimate the baryon-to-photon ratio, $\eta = n_b/n_\gamma$. Would your answer depend on redshift?

We'll see that there are approximately a billion photons for every baryon!!

When does recombination happen?

This is a computer-based activity which I recommend we carry out using a Python Jupyter notebook (as widely used in astronomy research) – however, it can be solved using any programming language.

In this activity we'll determine how long after the Big Bang recombination occurs.

The fraction of free electrons, x, is related to the temperature of the plasma by the Saha equation:

$$\frac{1-x}{x^2} = \frac{n_b\, h^3}{(2\pi m_e k_B T)^{3/2}} e^{E_{\mathrm{bind}}/k_B T}$$

where n_b is the baryon density, h is Planck's constant, m_e is the electron rest-mass, and $E_{\mathrm{bind}} =$ 13.6 eV is the binding energy of hydrogen.

For a given scale factor, we may evaluate the right-hand side of the equation using the results from the previous activities:

$$n_b = (0.28\ \mathrm{m}^{-3})/a^3$$
$$T = (2.73\ K)/a$$

and determine the ionisation fraction x by solving a quadratic equation!

a) Write a program to make a plot of x against a.

b) Find the value of a and T for which $x = 0.1$ (i.e. 90% of electrons are in atoms).

c) Hence determine the time after the Big Bang at which recombination occurs.
