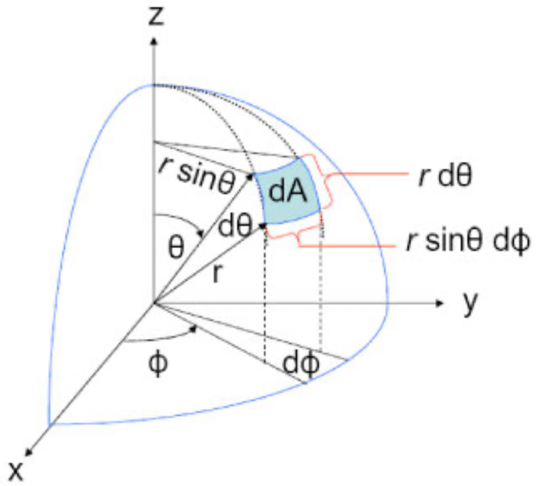
Cosmology Week 2 Class Activities

We'll study these activities in our Week 2 tutorial class!

Metric on the surface of a sphere

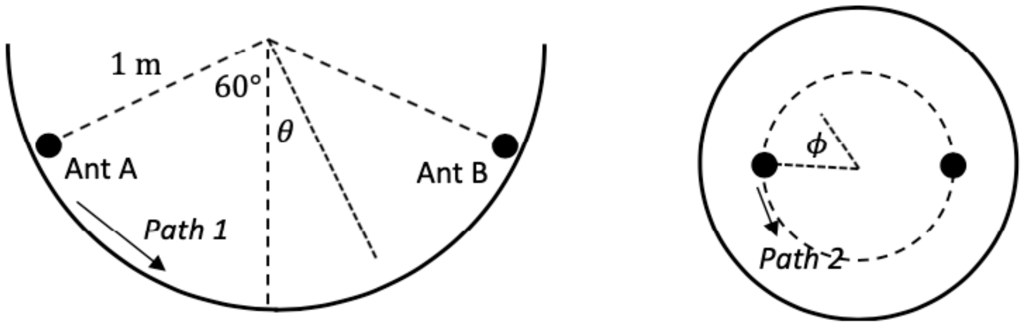
a) Here is a diagram showing an area element on a spherical surface, depicted using spherical polar coordinates (r, θ, ϕ):



Use this diagram to explain why the metric on the surface of the sphere is:

$$ds^2 = r^2(d\theta^2 + \sin^2\theta \, d\phi^2)$$

b) Two ants are sitting opposite each other inside a hemispherical bowl of radius 1 m. The lines joining the ants to the centre of the sphere make angles of 60° with the vertical. The following diagrams show the view from the side and from above:



Ant A is lazy and is wondering, what is the shortest path across the bowl to reach Ant B? Ant A is considering two options. In Path 1, Ant A maintains a fixed ϕ co-ordinate and walks across the bottom of the bowl. In Path 2, Ant A maintains a fixed θ co-ordinate and walks around the bowl at a constant height.

Use the metric of the bowl to find the lengths of Path 1 and Path 2, and determine which is shorter.

Distances in a matter-dominated Universe

The scale factor of a Universe with zero curvature, which only contains matter, evolves according to:

$$a(t) = \left(\frac{3H_0 t}{2}\right)^{2/3}$$

where Hubble's constant is $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

a) What is the age of this Universe in units of Gyr?

b) What is the look-back time of a galaxy whose spectrum has redshift $z = 1$?

The Friedmann-Robertson-Walker (FRW) metric of a Universe with zero curvature takes the form:

$$ds^2 = -c^2 dt^2 + a(t)^2 \left[dr^2 + r^2(d\theta^2 + \sin^2\theta \, d\phi^2)\right]$$

where (r, θ, ϕ) are comoving coordinates.

c) The physical distance D between the origin and a galaxy with radial coordinate r is obtained by integrating the metric along the radial path for a snapshot in time ($dt = 0$). (We're using the term "physical distance" because there are other definitions of distance that we are about to meet!). Use the FRW metric to show that,

$$D = a(t) \, r$$

d) Light travels between two points in space-time such that $ds = 0$. Use the FRW metric to show that the path followed by light travelling from a distant galaxy to the origin satisfies,

$$\frac{dr}{dt} = -\frac{c}{a(t)}$$

e) By integrating this equation, and using the relation for $a(t)$ at the beginning, show that the distance-redshift relation in this Universe is:

$$r = \frac{2c}{H_0}\left(1 - \frac{1}{\sqrt{1+z}}\right)$$

f) What is the comoving coordinate of a galaxy with redshift $z = 1$?

Sizes and fluxes in an expanding Universe

In this activity we will continue to analyse the flat, matter-dominated Universe with metric given in the previous question.

a) A luminous object with comoving coordinate r has an angular diameter $\Delta\theta$ as seen on the sky. Use the metric to show that its physical width is: $W = a(t) \, r \, \Delta\theta$.

b) A galaxy at redshift $z = 1$ has physical size 10 kpc. What is its angular size in units of arcseconds? *(Note: there are* 3600 *arcseconds in* 1 *degree.)*

c) The $z = 1$ galaxy contains a billion Suns, where the solar luminosity is $L_\odot = 3.9 \times 10^{26}$ W. What is the flux of energy received from this galaxy at the Earth?

d) Galaxies at $z = 1$ have an average number density $n = 0.001$ Mpc^{-3}. How many galaxies would we expect to find in a 1 deg^2 patch of sky with redshifts between $z = 0.99$ and $z = 1.01$?

A cosmology calculator

This is a computer-based activity which I recommend we carry out using a Python Jupyter notebook (as widely used in astronomy research) – however, it can be solved using any programming language.

Let's create a function or small program which allows you, for the matter-dominated Universe, to input the

- Value of the Hubble constant in km s^{-1} Mpc^{-1}
- Redshift

and the program will output, for this redshift, the:

- Scale factor
- Look-back time
- Radial comoving coordinate
- Angular diameter distance
- Luminosity distance
- Angular size of a galaxy with physical (proper) width 10 kpc

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