Cosmology Week 1 Class Activities

We'll study these activities in our Week 1 tutorial class!

Big Bang Conceptions and Misconceptions

Some curious friends are asking questions about cosmology. How do you answer?

a) If everything is expanding around us, does it mean we're at the centre of the Universe?

b) What is the Universe expanding into?

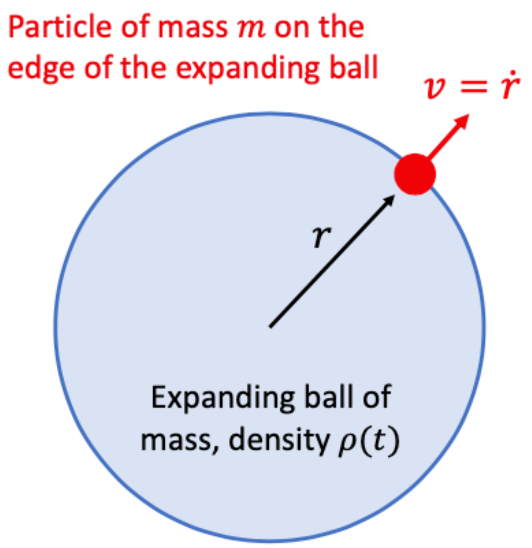
c) What happened before the Big Bang?

d) If the Universe is expanding, does it mean our Galaxy is expanding? Is this room expanding?

The expanding Universe: Newton-style

In this activity we will derive the equation of motion of the Universe!

As a simplistic (but remarkably successful) model of the Universe, consider a particle of mass m on the edge of an expanding ball of mass of radius r and density $\rho(t)$:



a) Focussing on the motion of the particle experiencing the gravity of the ball, explain why:

$$\frac{1}{2}m\dot{r}^2 - \frac{GM(<r)m}{r} = E$$

where $M(<r)$ is the mass enclosed by radius r and E is a constant. What does E represent?

b) What does the resulting motion look like if E is positive, negative or zero?

c) If the ball is uniformly expanding, explain why $\dot{r} = Hr$, where H is a coefficient of proportionality at that moment of time. Hence, by writing an expression for $M(< r)$ in terms of $\rho(t)$, show that the density needed at any given time to produce $E = 0$ is:

$$\rho = \frac{3H^2}{8\pi G}$$

We call this the critical density!

d) We'll now introduce comoving coordinates for the position of the particle by writing the radius in terms of the scale factor: $r(t) = a(t)\, r_0$. Show that the equation of motion then becomes,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\, \rho(t)}{3} - \frac{C}{a^2}$$

where C is a constant.

e) In what ways is this treatment a good and bad analogy for the real expanding Universe?

Miraculously, this is the correct equation of motion for the scale factor of the Universe from General Relativity, which is also known as the Friedmann equation! The only difference is, the real Universe contains dark energy, Λ. The Friedmann equation from General Relativity is:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\, \rho(t)}{3} - \frac{C}{a^2} + \frac{\Lambda}{3}$$

(The constant C is related to the curvature of the Universe in this case.)

f) Let's see what the dark energy term Λ would represent in our Newtonian analogy. By reverting the above equation back to physical coordinates r and differentiating it with respect to time, show that:

$$\ddot{r} = -\frac{GM(< r)}{r^2} + \frac{\Lambda}{3} r$$

Hence, the dark energy term is applying a force which accelerates the particle, opposing the inward force of gravity.

Returning to the expanding ball of mass and now ignoring dark energy, suppose the density is equal to the critical density such that $C = 0$.

g) Explain why $\rho(t) = \rho_0 / a^3$, where ρ_0 is a constant. Hence show that the ball expands such that:

$$a(t) \propto t^{2/3}$$

Olber's Paradox

Olber's paradox is the observation that, in an infinitely large and infinitely old Universe, the sky at night would not be dark. In this activity we will prove this result and demonstrate its resolution!

a) Suppose that the Universe is filled with stars of radius R and number density n. Show that, along any line-of-sight, the average distance travelled before intercepting a star is:

$$D \approx \frac{1}{\pi\, n\, R^2}$$

[Hint: consider the typical volume of a cylinder of radius R that would contain one star!]

b) Suppose each star shines with luminosity L. By adding up the energy flux received from the stars in a series of concentric shells up to $r = D$, show that the total flux at the Earth is:

$$\Phi = \frac{L}{\pi\, R^2}$$

c) We'll now show that this equals the energy flux the Earth would receive if every point on the sky were as bright as the Sun!

If the Sun is located at distance $D_\odot$ and has radius R, explain why the number of Suns to fill the sky would be $4(D_\odot/R)^2$. *[Hint: consider the angular area subtended by the Sun.]* Hence show that the total flux received at the Earth from these Suns would be $\Phi = L/\pi R^2$, as in part b).

d) If the Universe is filled with stars with radius equal to the Sun ($R = 7 \times 10^8$ m) with average separation $l = 1$ pc $= 3 \times 10^{16}$ m, what is the value of D? *[Hint: $n \approx 1/l^3$.]*

e) Judging by the result of part d), give some reasons why every point on the sky is not as bright as the Sun!

Who discovered the expansion of the Universe?

This is a computer-based activity which I recommend we carry out using a Python Jupyter notebook (as widely used in astronomy research) – however, it can be solved using any programming language.

In this activity, we will check who discovered the expansion of the Universe! You have been provided with the original distance-velocity datasets from Lemaitre (1927) and Hubble (1929).

a) Plot these datasets with distance (Mpc) on the x-axis and velocity (km s^{-1}) on the y-axis.

If the Universe is expanding, these data points will be correlated by Hubble's Law.

b) Estimate the correlation coefficient of these data points using the product-moment formula:

$$r = \frac{\sum_{i=1}^{N} x_i\, y_i - N \mu_x \mu_y}{(N-1)\, \sigma_x \sigma_y}$$

where $\mu_x,\ \mu_y$ are the means of the data, and $\sigma_x,\ \sigma_y$ are their standard deviations.
(Note: we can use the scipy.stats.pearsonr function in python to do this.)

c) What is the significance of each correlation (probability that it could arise by random chance)?

d) Fit the best linear relation between these points. What value of Hubble's constant is predicted? *(Note: we can use the scipy.stats.linregress function in python to do this.)*

e) Compare your result to the modern determination, $H_0 = 70$ km s^{-1} Mpc^{-1}.
