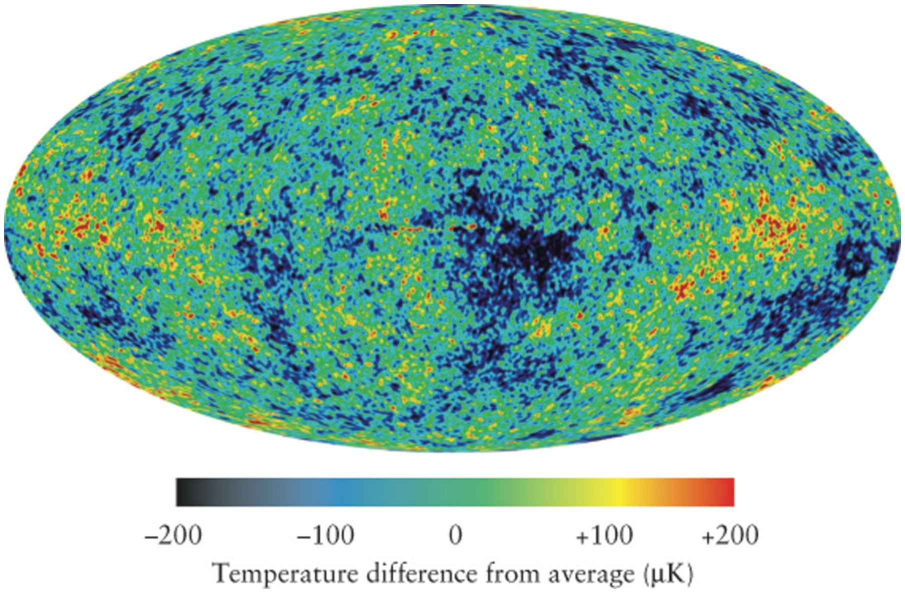
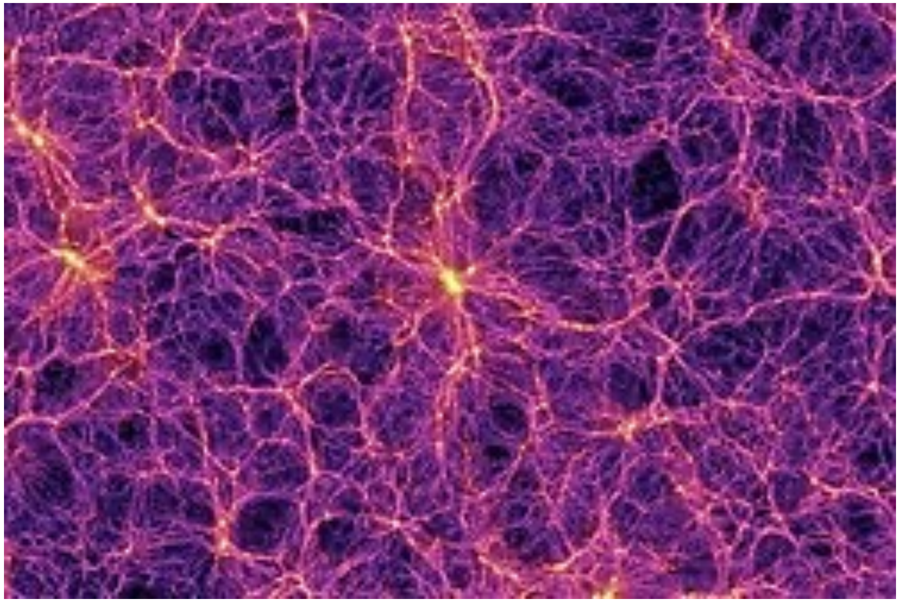
Honours Cosmology Week 5:
Cosmic Structure

This week we will study the processes by which the large-scale structures in the Universe are created and gravitationally grow with time





Cosmic structure

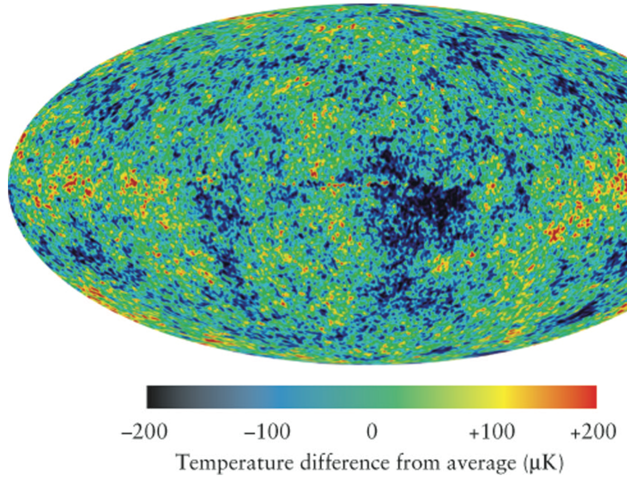
At the end of this week you should be able to …

- … motivate the theory of **inflation**, which might seed the original density fluctuations in the Universe
- … understand how inflation can address the **flatness problem** and **horizon problem**
- … perform calculations for the **gravitational collapse time** of density clumps, and relate them to real cosmic objects
- … describe that structure grows in a **hierarchical fashion**
- … recognise observable effects of the growth of structure, such as **gravitational lensing** and **redshift-space distortions**

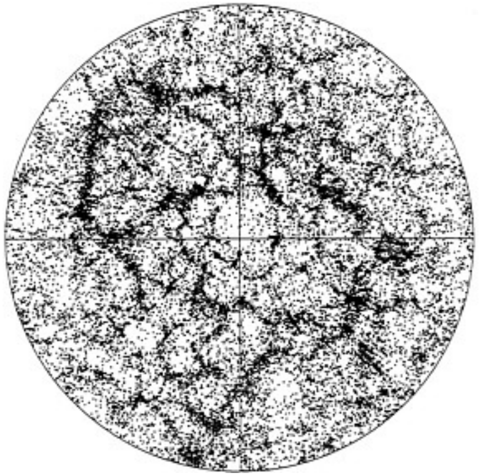
Cosmic structure

- Primordial clumps of matter in the Universe are *gravitationally amplified* to form today's galaxies

CMB fluctuations with $\Delta T/T \sim 10^{-5}$



Distribution of galaxies with $\rho_{\text{galaxy}}/\rho_{\text{crit}} \sim 10^6$



- In this week's class, we'll study the physics and observational consequences of this process

Flatness problem

- We'll start with the question: *what seeded the original fluctuations?* This question is connected with two other issues …
- First, let's consider the *curvature* of the Universe. The Universe today is measured to be flat, $|\Omega_K| < 0.01$. What is the flatness of the Universe at other times?
- Density parameters *as seen by observers at different times* are given by, using $\rho_{\text{crit}}(t) = 3H(t)^2/8\pi G$,

$$\frac{\Omega(t)}{\Omega(t_0)} = \frac{\rho(t)/\rho_{\text{crit}}(t)}{\rho(t_0)/\rho_{\text{crit}}(t_0)} = \frac{\rho(t)}{\rho_0}\frac{{H_0}^2}{H^2}$$

This comes from the Friedmann equation

This depends on the component

Flatness problem

- Assuming the Universe is matter-dominated, $\frac{H^2}{{H_0}^2} \approx 1/a^3$, and for curvature, $\frac{\rho_K(t)}{\rho_{K,0}} = \frac{1}{a^2}$. Hence: $\Omega_K(t) = \Omega_{K,0} \cdot a$

- If the Universe is flat today ($\Omega_{K,0} = 0$), it will always be flat ($\Omega_K(t) = 0$). *A small Ω_k in the past increases $\propto a$!*

- If $|\Omega_K| < 0.01$ today, then $|\Omega_K| < 10^{-12}$ in the first moments of the Universe ($a \sim 10^{-10}$)

- What makes the early Universe flat to this incredible precision? This question is known as the *flatness problem*

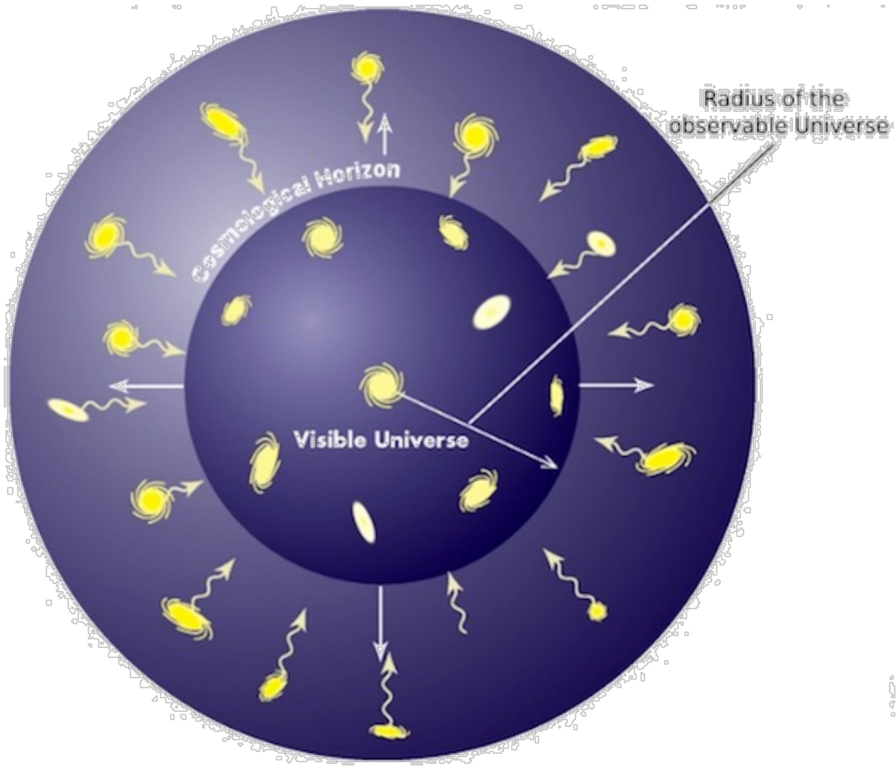
Horizon problem

- The *particle horizon* is the maximum distance, D_H, that a particle can travel between the Big Bang and a given time, and hence gives the *maximum range of causal connection*

- Physical distance between points A and B, measured at a simultaneous coordinate time $= a(t) \int_A^B \frac{dr}{\sqrt{1-Kr^2}}$

- Suppose these points are linked by a light ray, which travels with $ds = 0 \rightarrow c\,dt' = a(t') \frac{dr}{\sqrt{1-Kr^2}}$

- Hence, $D_H(t) = c \cdot a(t) \int_0^t \frac{dt'}{a(t')}$



Horizon problem

- Note that the horizon distance is **not** equal to $D_H(t) = ct$, since the Universe is expanding during this time, so points A and B end up **further apart**!
- Assuming the Universe has $\Omega_m = 1$, we have $a(t) = \left(\frac{3H_0 t}{2}\right)^{2/3}$ and $D_H(t) = c \cdot a(t) \int_0^t \frac{dt'}{a(t')}$ evaluates to:

$$D_H(t) = 3ct = \frac{2c}{H_0} a^{3/2}$$

- Assuming $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, the particle horizon today has size $D_H = 8.6 \text{ Gpc}$, however it's much smaller in the early Universe!

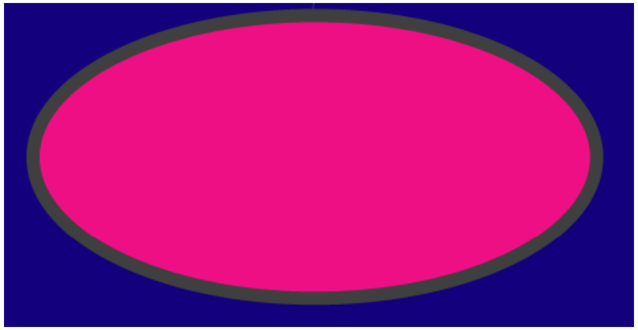
Horizon problem

- The CMB is produced at $a \approx 10^{-3}$ and its temperature is uniform across the sky. The range of *causal connection* must therefore correspond to the size of the sky, $\Delta\theta \sim 180°$!

- However, using the metric to project D_H at $a = 10^{-3}$ to an angular separation on the sky we find,

$$\Delta\theta = \frac{D_H}{r\,a} = \frac{\frac{2c}{H_0}a^{1/2}}{\frac{2c}{H_0}(1-\sqrt{a})} = \sqrt{10^{-3}} = 0.03 \text{ rad} = 1.8°$$

The CMB temperature varies by only 1 part in 10^5 across the sky



However, the causal connection distance is only a tiny angle! This is known as the **horizon problem**

Inflation

- A simple and elegant solution to both of these problems is if the early Universe *had a brief period of accelerated expansion*

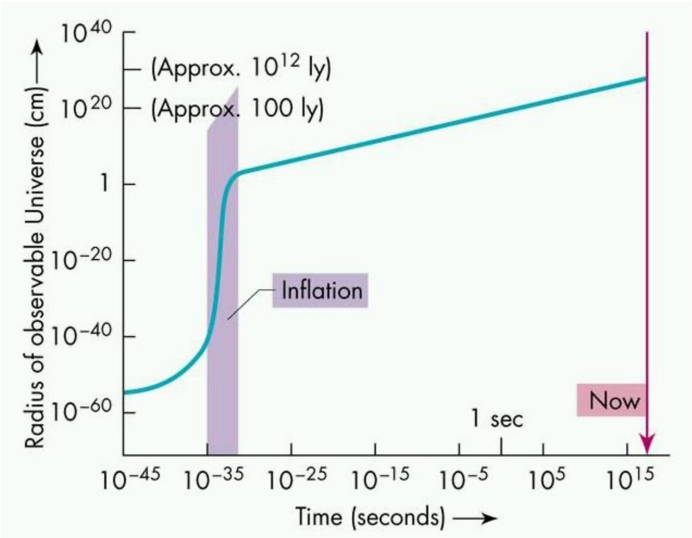
- The flatness problem: accelerated expansion drives Ω_K closer to zero with time [e.g. repeating the calculation on slide 5 for dark energy: $\frac{H^2}{{H_0}^2} = 1$, hence $\Omega_K(t) = \Omega_{K,0}/a^2$ which decreases with time!]

- The horizon problem: accelerated expansion increases the particle horizon by allowing nearby regions to expand to greater separations, bringing the CMB sky into causal contact

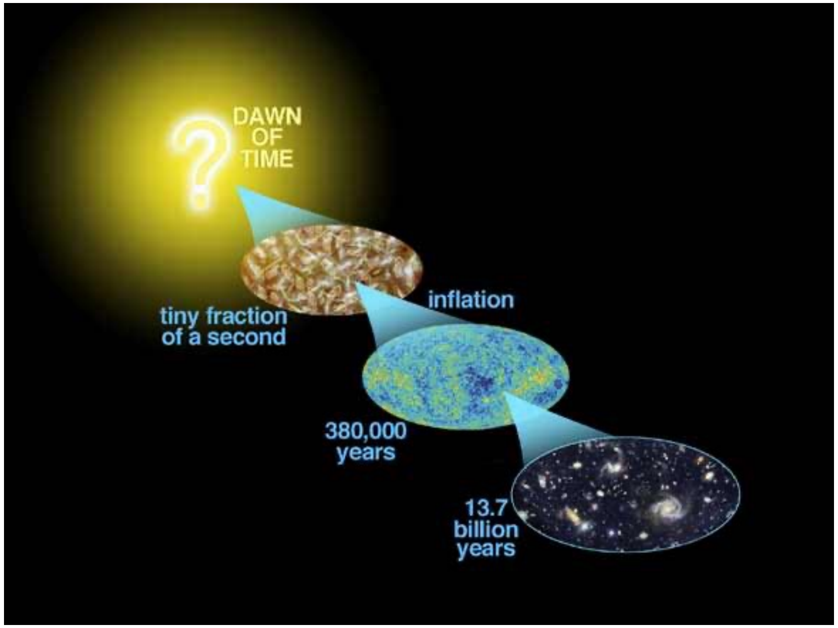
- This period of early accelerated expansion is called *inflation*

Inflation

- Inflation is thought to occur within the first fraction of a second of the Universe, and expand its size by $\sim 10^{40}$

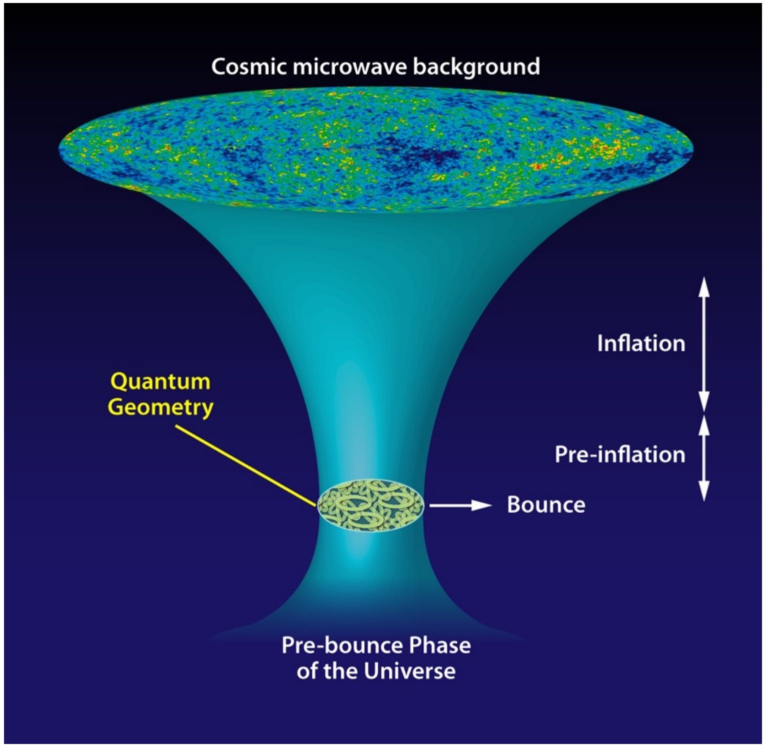


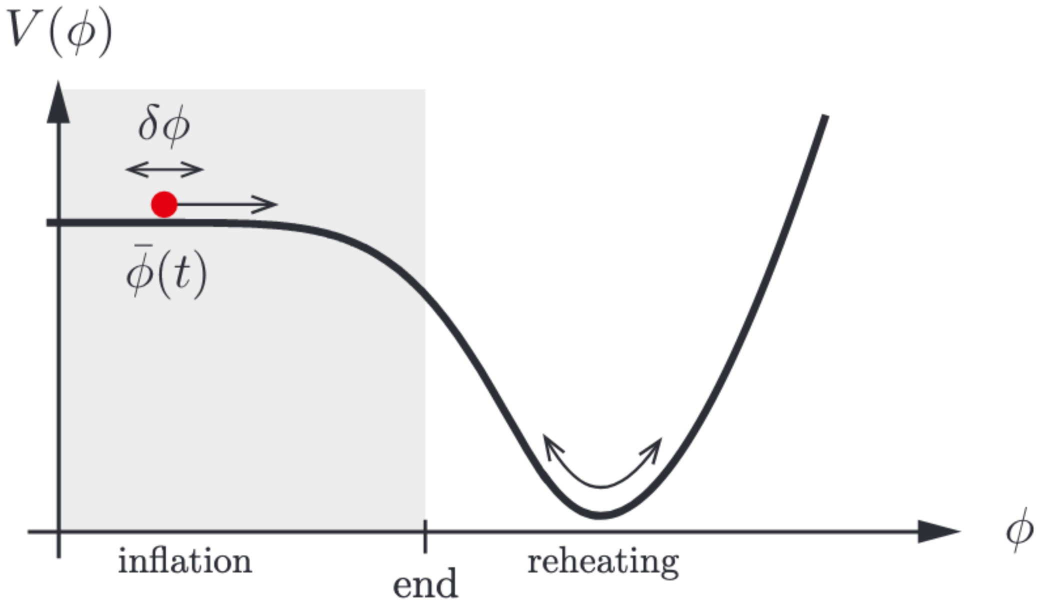
- Inflation can also seed the density fluctuations that are seen in the Universe by *amplifying quantum fluctuations to a macroscopic scale*



Inflation

- The ultimate cause of inflation is currently unknown!
- It would be intriguing if it were related to the other period of accelerating expansion: dark energy





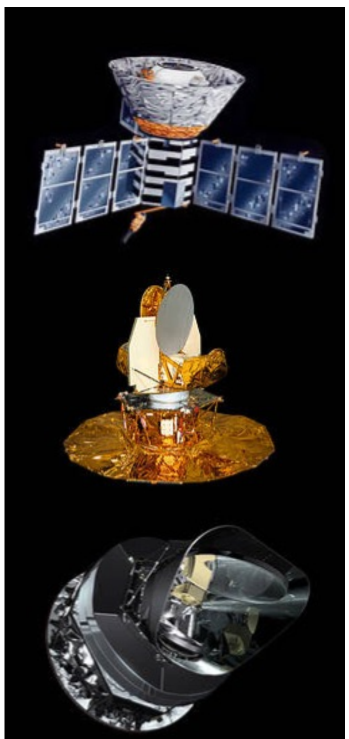
CMB fluctuations

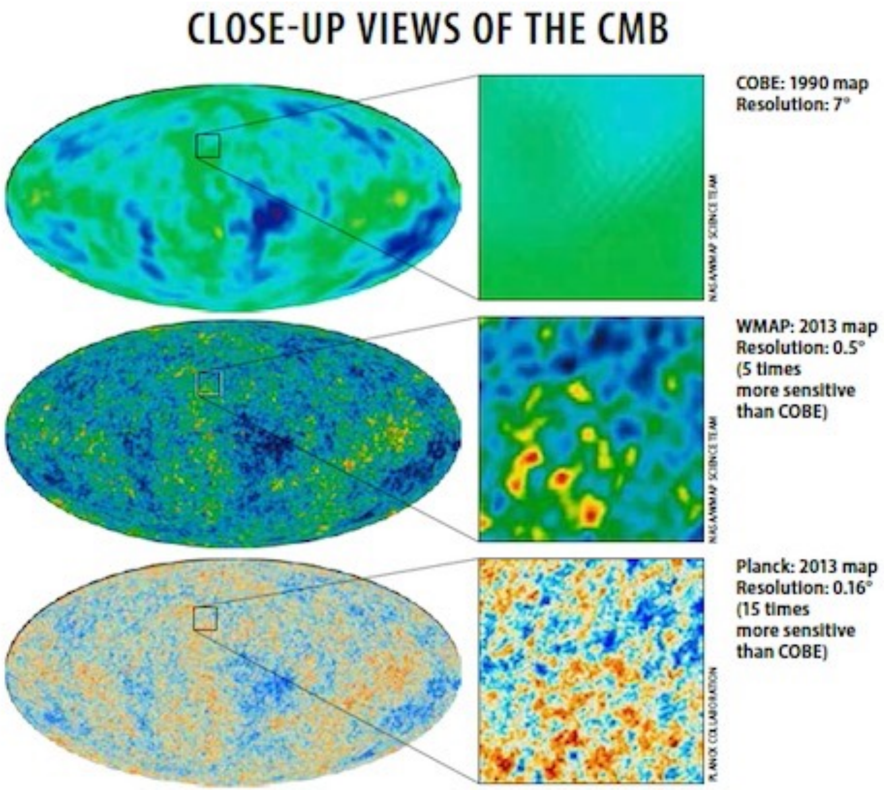
- A series of satellite experiments have mapped out the tiny fluctuations in the CMB with increasing sensitivity and resolution

COBE

WMAP

Planck



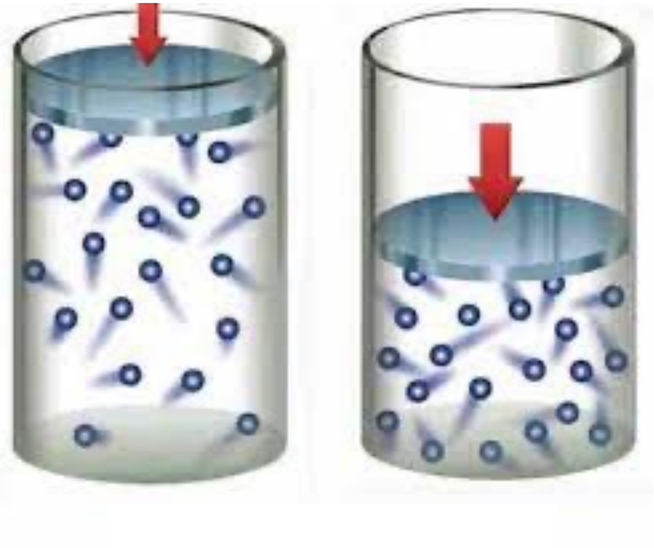


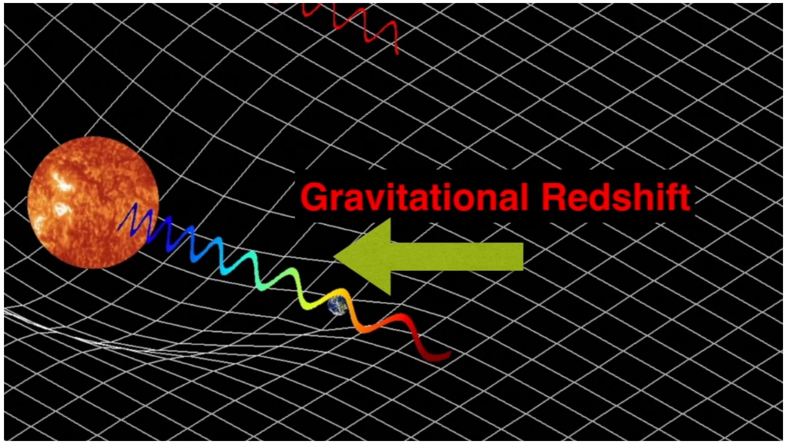
CMB fluctuations

- Two main effects transform the matter fluctuations to variations in temperature

- *Adiabatic compression*: a gravitating clump of matter squeezes the gas, implying a higher temperature

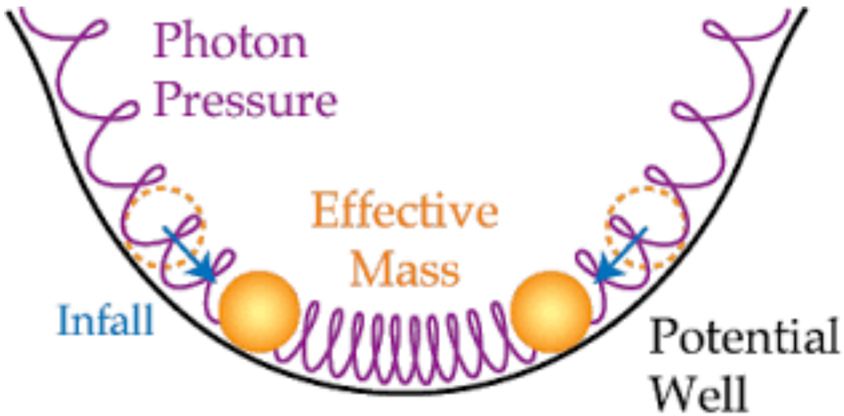
- *Gravitational redshift*: light originating from a gravitating clump loses energy as it escapes, implying a lower temperature

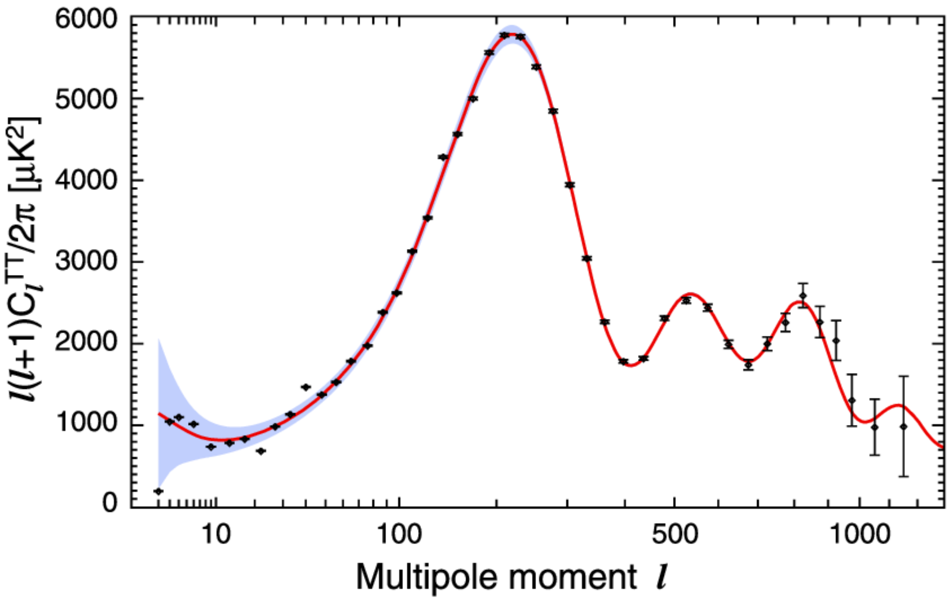




CMB fluctuations

- These competing forces create compression waves or acoustic waves in the plasma, which may be observed today in the CMB temperature spectrum (we'll look at this in more detail next week)

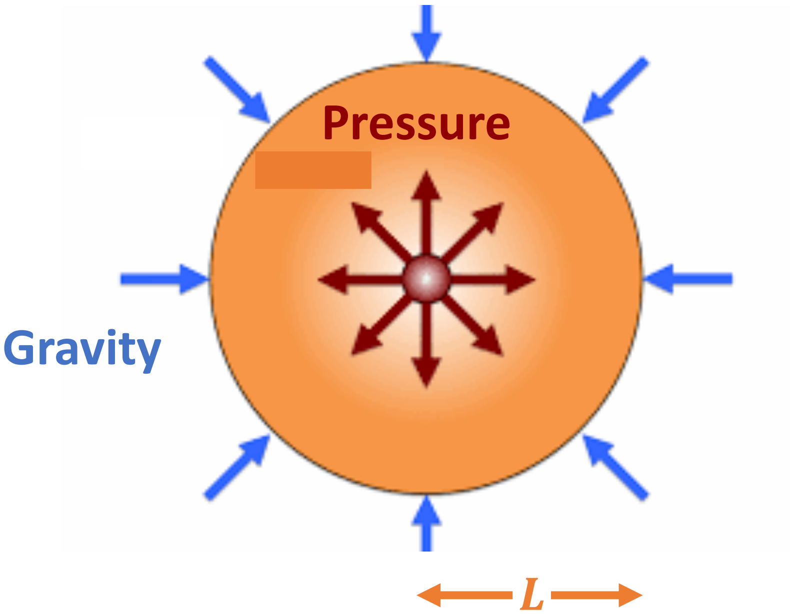




Spherical collapse

- The balance between the inward gravitational force and outward pressure force determines when a clump collapses under its own weight

Clump of N particles of mass m with density ρ and temperature T



Here is an approximate calculation of the minimum collapse size or **Jeans length**:

Number of particles $N \sim \rho L^3/m$

Gravitational potential energy of the clump $E_G \sim -\frac{G(Nm)^2}{L}$

Thermal energy of the clump $E_T \sim N k_B T$

Gravity dominates if $E_G + E_T < 0$

i.e. if $L > \sqrt{\frac{k_B T}{G \rho m}}$ = Jeans length L_J

Spherical collapse

- We can learn from the calculation on the previous slide that large regions ($L > L_J$) collapse under gravity whilst small regions ($L < L_J$) oscillate between pressure and gravity

- The *collapse time* t_G can be estimated assuming a constant gravitational acceleration $\frac{GM}{L^2} \sim \frac{L}{{t_G}^2} \rightarrow t_G \sim \frac{1}{\sqrt{G\rho}}$

- The minimum mass for collapse, or *Jeans mass* M_J, can be estimated as $M_J \sim \rho {L_J}^3 = \rho \left(\frac{k_B T}{G\rho m}\right)^{3/2} \propto T^{3/2}\, \rho^{-1/2}$

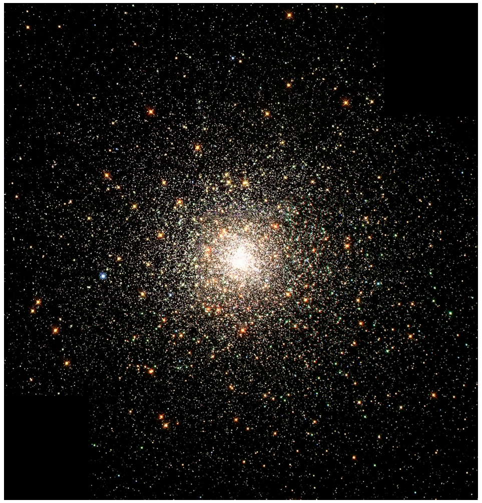
- We learn that the mass needed for collapse reduces with lower temperature or higher density

Spherical collapse

- We can calculate the *gravitational collapse time* of some typical objects in the Universe:

Globular cluster

$t_G \sim 2\ \mathrm{Myr}$



Can form very early in the history of the Universe

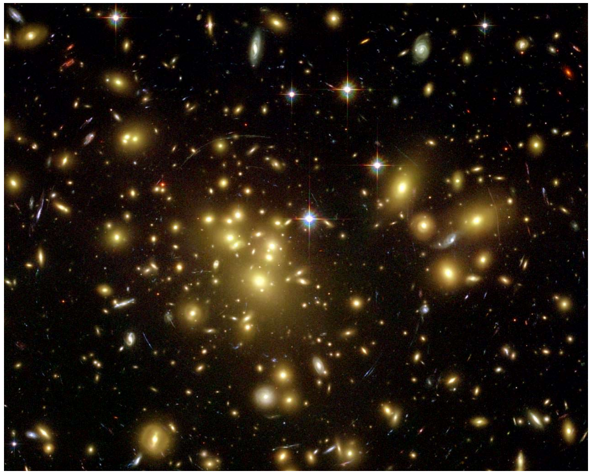
Galaxy

$t_G \sim 300\ \mathrm{Myr}$

In place by $z \sim 10$

Galaxy cluster

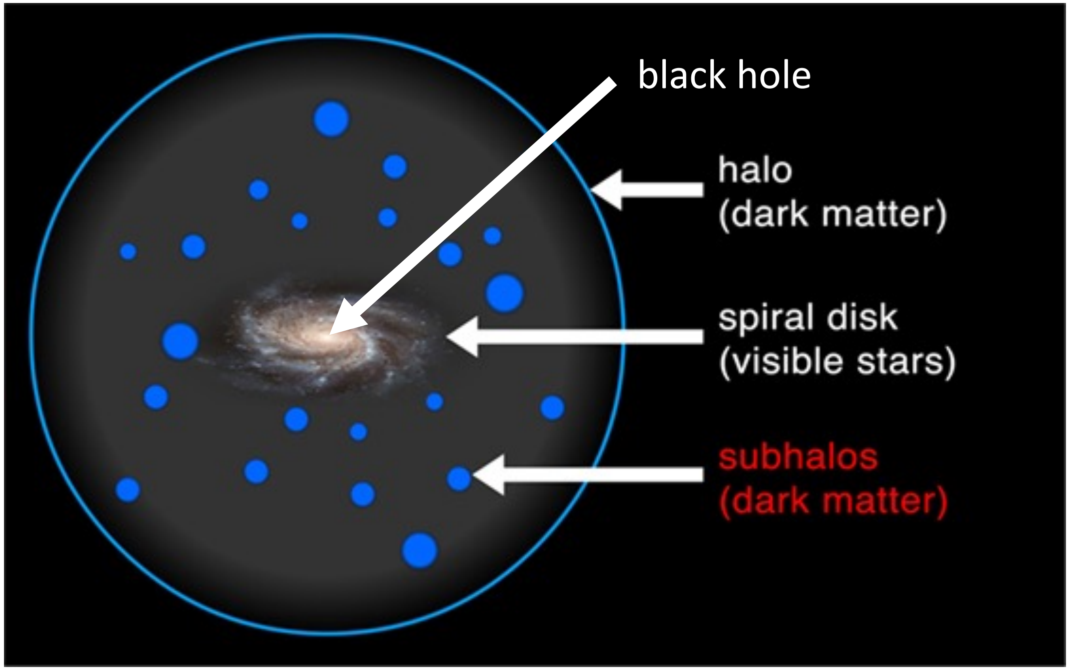
$t_G \sim 7\ \mathrm{Gyr}$



In place by $z \sim 1$

Hierarchical structure formation

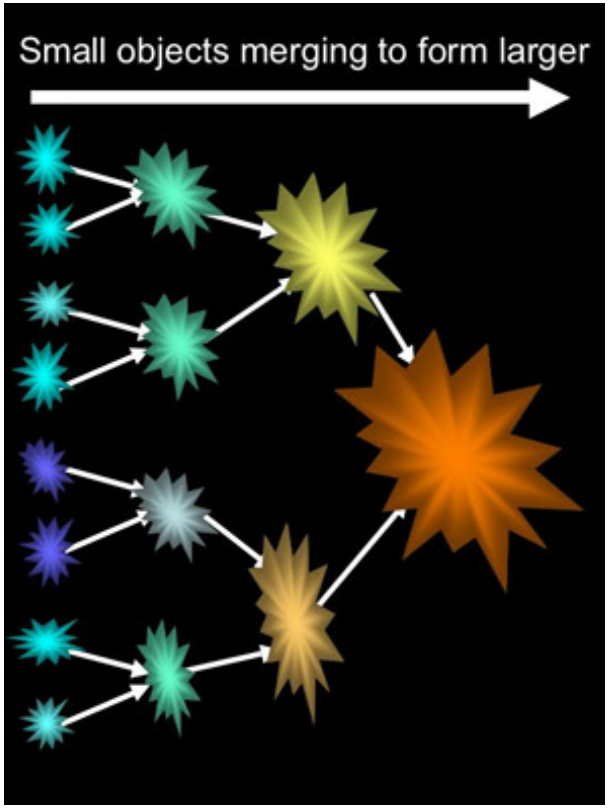
- Dark matter clumps collapse early into *“dark matter halos”* and form the gravitating structure of the Universe
- Baryons fall into these haloes and form galaxies, which contain *supermassive black holes* at their centre



- Dark matter halos have a characteristic “NFW” (Navarro-Frenk-White) radial mass profile
- Dark matter halos contain substructure which may host satellite galaxies

Hierarchical structure formation

- The large-scale distribution of galaxies forms in a *hierarchical fashion* – smaller structures form first, and agglomerate into larger ones!

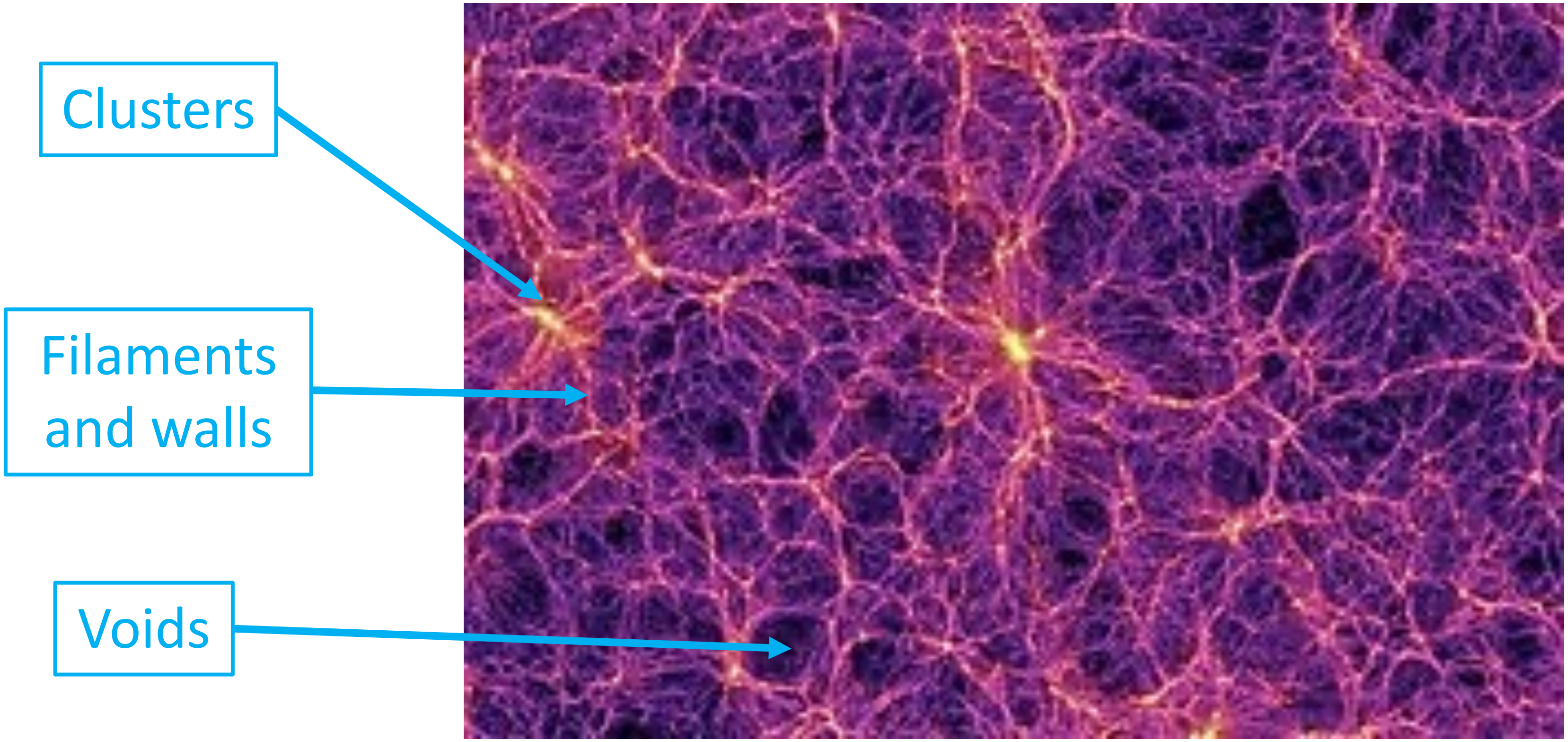


We can observe galaxies in the process of merging!

Hierarchical structure formation

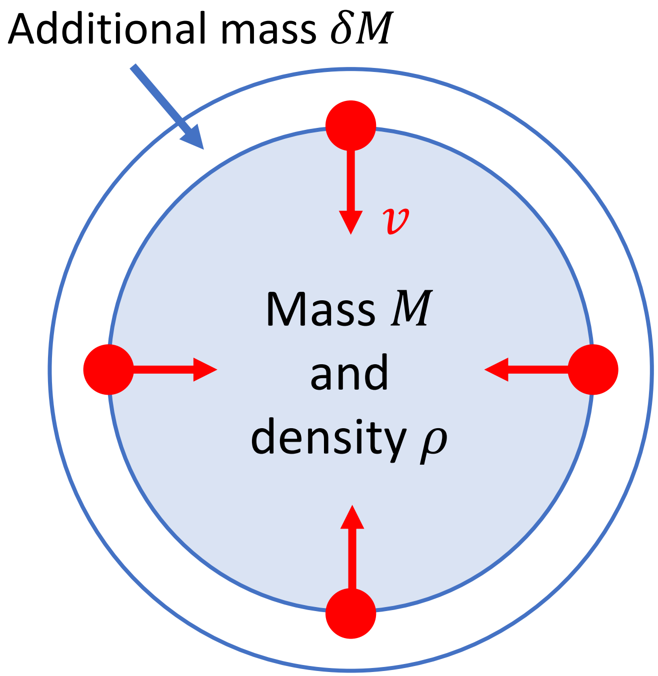
- A *sponge-like topological structure* of galaxies results, featuring clusters, walls, filaments and voids!

Simulation of the matter distribution within a large volume



Infall velocity

- The gravitational collapse of a structure will create an *infall velocity* in the surrounding matter



We can relate the increase in mass δM in time δt to the infall velocity v by the **continuity equation**. The additional mass is in a slice of thickness $v \times \delta t$:

$$\delta M = (v\,\delta t) \cdot 4\pi r^2 \cdot \rho$$

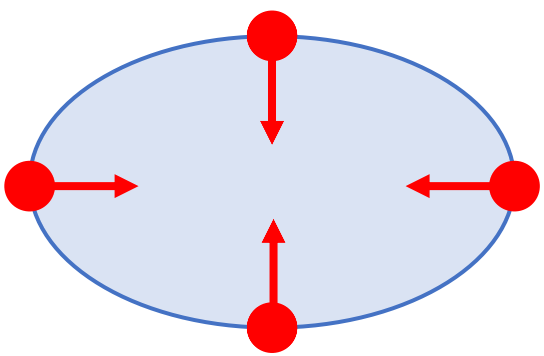
Since $M = \frac{4}{3}\pi r^3 \rho$, we have

$$v = \frac{1}{M}\frac{dM}{dt} \cdot \frac{r}{3}$$

Redshift-space distortions

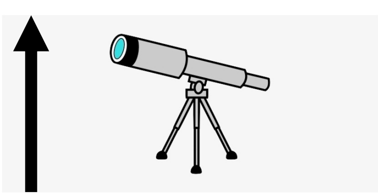
- This infall velocity is observable because it systematically changes the galaxy redshifts in an affect known as *redshift-space distortions*

This galaxy is moving towards the observer so is **blueshifted**



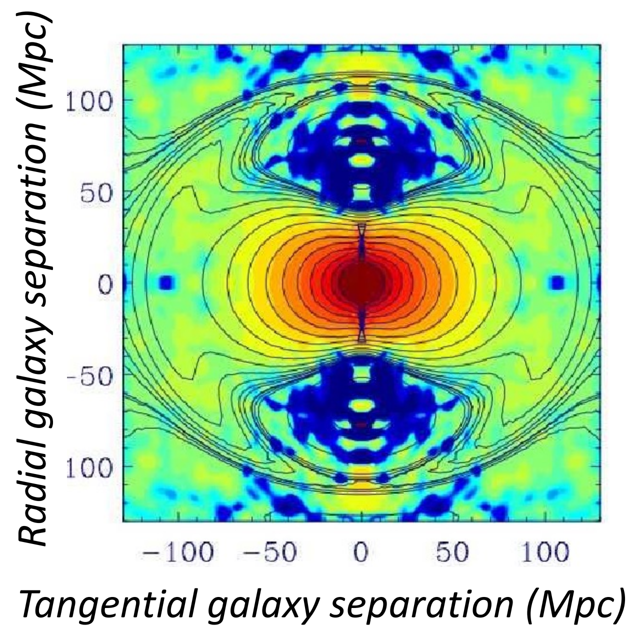
This galaxy is moving away from the observer so is **redshifted**

Observer is looking from here



The "average structure" appears squashed when mapped out with galaxy redshifts

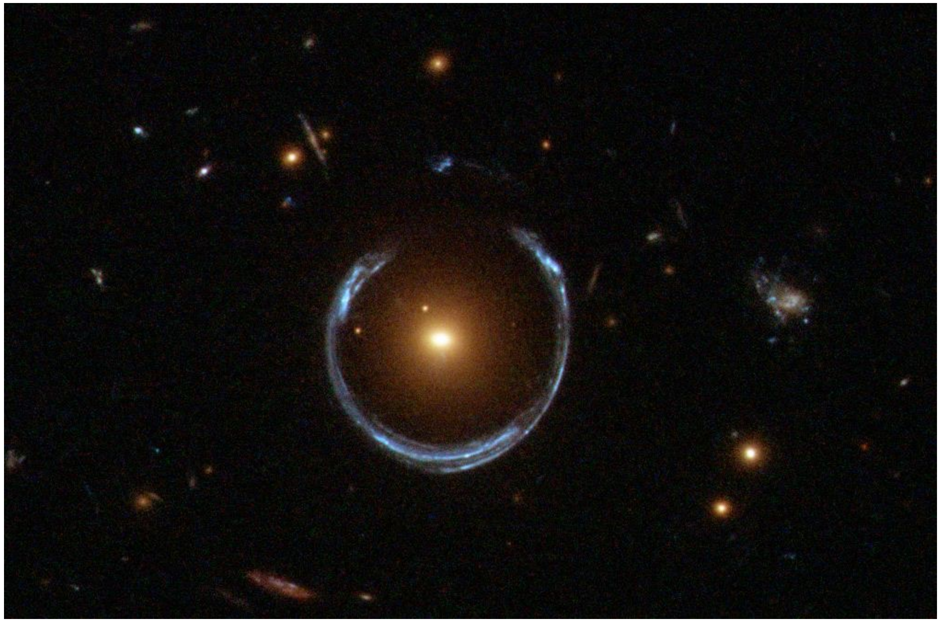
This is a real observation made by stacking up structures in redshift-space:

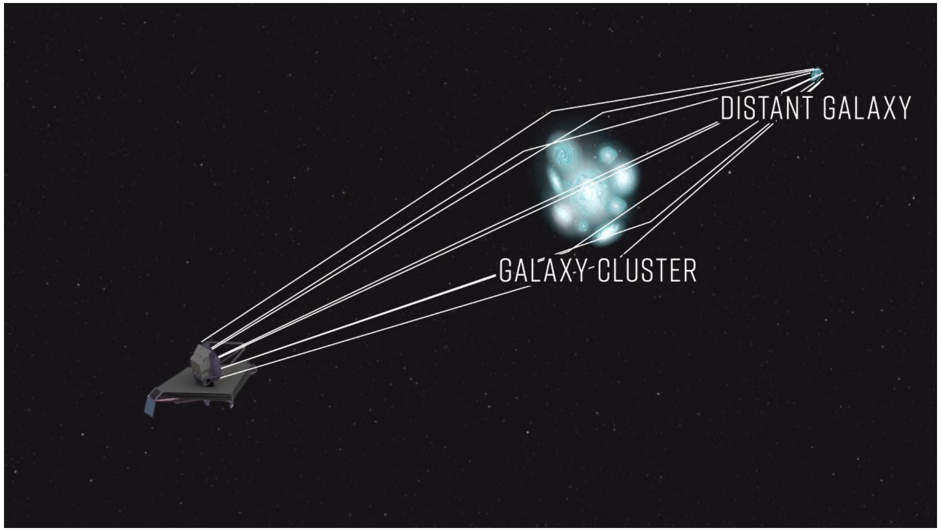


Gravitational lensing

- Massive clumps cause *gravitational lensing* of light from background galaxies, which can be observed through characteristic arcs

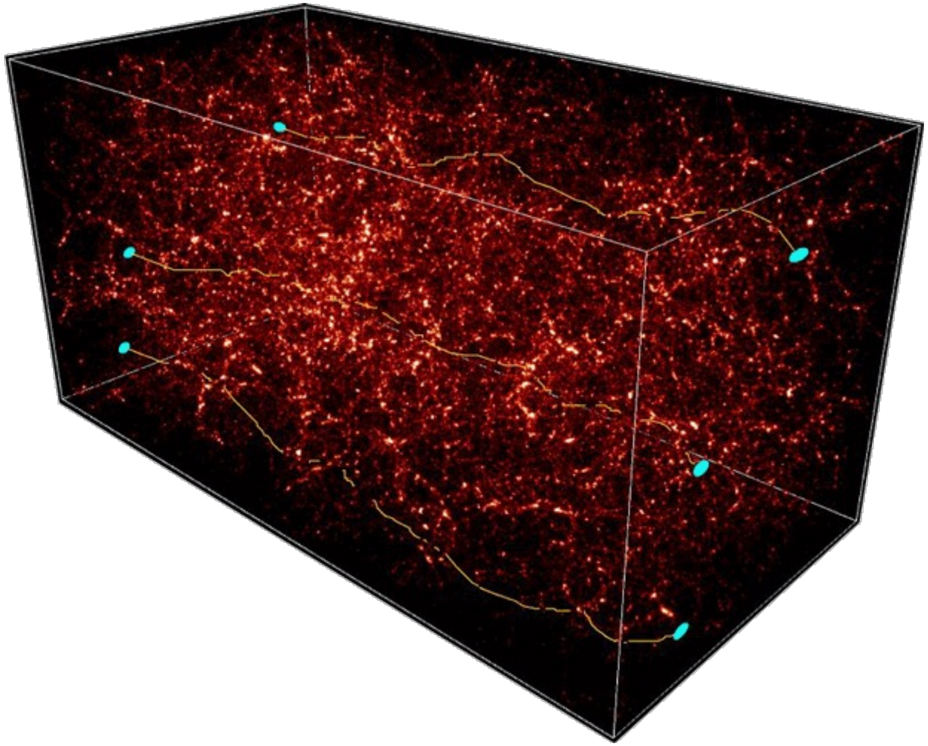
- The space-time around the masses is *distorted* according to General Relativity, bending the paths of the light rays

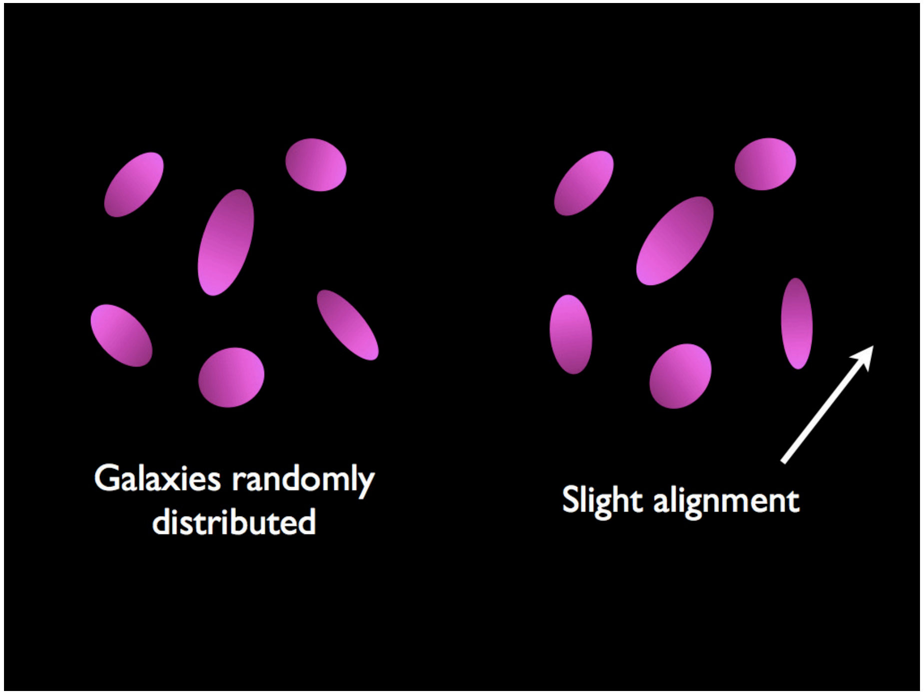




Gravitational lensing

- An important technique called *cosmic shear* uses the small lensing distortions in background galaxy shapes to trace out the large-scale structure





Key take-aways

- According to our current theories, the density fluctuations in the Universe were created by a very early period of rapid expansion known as **inflation**
- Inflation is an attractive theory because it can solve both the **flatness problem** and **horizon problem**
- The density fluctuations can be traced as **temperature fluctuations in the CMB** and by the **growth of galaxies**
- Cosmic structure grows in a **hierarchical manner**, with smaller objects forming early and merging into larger objects
- The growth of structure may be measured by effects such as **gravitational lensing** and **redshift-space distortions**