Cosmology Assignment 3

Weeks 5 & 6: Cosmic Structure and Cosmological Observations

Q1) In a Universe with $\Omega_m = 0.3$ and $H_0 = 70$ km s^{-1} Mpc^{-1}, an early galaxy at $z = 20$ can be considered as a sphere containing total mass $10^{10}\ M_\odot$, which is a factor 200 more dense than the Universe as a whole.

a) What is the radius of this proto-galaxy?

b) Estimate the dynamical formation time of the galaxy.

c) Assuming the Universe is matter-dominated, by what redshift will the galaxy have formed?

Q2) The galaxy M81 at 3.5 Mpc is one of the nearest galaxies not bound to the "Local Group". Assuming a cosmology with $\Omega_m = 0$ and $\Omega_\Lambda = 1$, which is a good approximation of our own Universe in the distant future, show that in 100 billion years' time, light from M81 will not be able to reach us! You can assume $H_0 = 70$ km s^{-1} Mpc^{-1}.

Q3) According to a slightly weird theory, cold dark matter consists of lead balls the size of the Earth randomly distributed across the Universe ($\rho_{\mathrm{Lead}} = 11{,}300$ kg m^{-3} and $R_{\mathrm{Earth}} = 6378$ km).

a) If $\Omega_{DM} = 0.25$, how many balls will there be per cubic Mpc?

b) If the lead balls are optically thick, what would be the mean free path of light through the Universe? Can we detect the dark matter in this way?

c) How would the mean free path depend on the radius of the balls, and what radius would make the Universe opaque to light?

d) Why is this not a good model for dark matter?

Q4) The process where electrons and positrons annihilate to form photons ($e^+ + e^- \leftrightarrow \gamma + \gamma$) "freezes out" when the temperature of the Universe is $T = 5 \times 10^9$ K. Assuming that the Universe is radiation-dominated at this time, calculate the age and redshift of this epoch. You can assume the radiation temperature today is 2.73 K and corresponds to a density $\Omega_r = 5.1 \times 10^{-5}$.