Cosmology Assignment 2

Weeks 3 & 4: Contents of the Universe and Big Bang Physics

Q1) Let's compare Universes with $(\Omega_m, \Omega_\Lambda) = (0.3, 0.7)$ and $(\Omega_m, \Omega_\Lambda) = (1, 0)$. You can assume $H_0 = 70\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$.

a) What is the relative apparent brightness of a supernova at $z = 1$ in these Universes? (which case is brighter?)

b) What is the relative angular size of a galaxy at $z = 1$ in these Universes? (which case is larger?)

c) If galaxies are uniformly distributed up to $z = 1$, what is the relative number density of galaxies in these Universes? (which case is denser?)

Q2) If the only kind of matter in the Universe were ordinary baryonic matter, then the matter density parameter would be $\Omega_m \approx 0.04$, which is very small. An approximation to such a Universe would be a completely empty Universe model where $\Omega_m = \Omega_\Lambda = 0$. Assume that the Hubble parameter $H_0 = 70\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$.

a) Use the Friedmann equation to show the scale factor $a(t) = H_0\, t$ in this Universe.

b) Write down the metric for this Universe.

c) Find the age of this model Universe, and compare it with 13.2 Gyr, which is our best estimate of the age of the oldest star in our Galaxy.

d) The Universe became transparent when $z = 1090$. How many years was this after the Big Bang, in this model Universe?

e) Imagine that we observe a galaxy with $z = 1$. How long ago did light leave the galaxy? What is the co-ordinate distance of the galaxy?

[Hint: the integral $\int \frac{dx}{\sqrt{1+x^2}} = \sinh^{-1} x$.

Q3) A positively curved Universe with $\Omega_m = 2$, containing only matter, has entered the phase of contraction. An astronomer discovers that nearby galaxies have blueshifts ($z < 0$) proportional to their distance, with recession velocities $v = -H_0 D$ where $H_0 = 70\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$. How much time is left between these observations and the Big Crunch?

[Hint: the integral $\int_0^1 \sqrt{\frac{a}{2-a}}\, da$ may be solved with the substitution $a = 2\sin^2\theta$.]

Q4) Suppose that the neutron decay half-life was $T_{1/2} = 61$ s rather than $T_{1/2} = 610$ s, with all other physical parameters unchanged. How would the helium abundance relative to hydrogen at the end of Big Bang nucleosynthesis change, assuming all available neutrons are incorporated into helium nuclei?