Class 6: Relativistic Phenomena

In this class we will apply the laws of relativistic mechanics in a range of settings, and also consider the relativistic Doppler effect

Class 6: Relativistic Phenomena

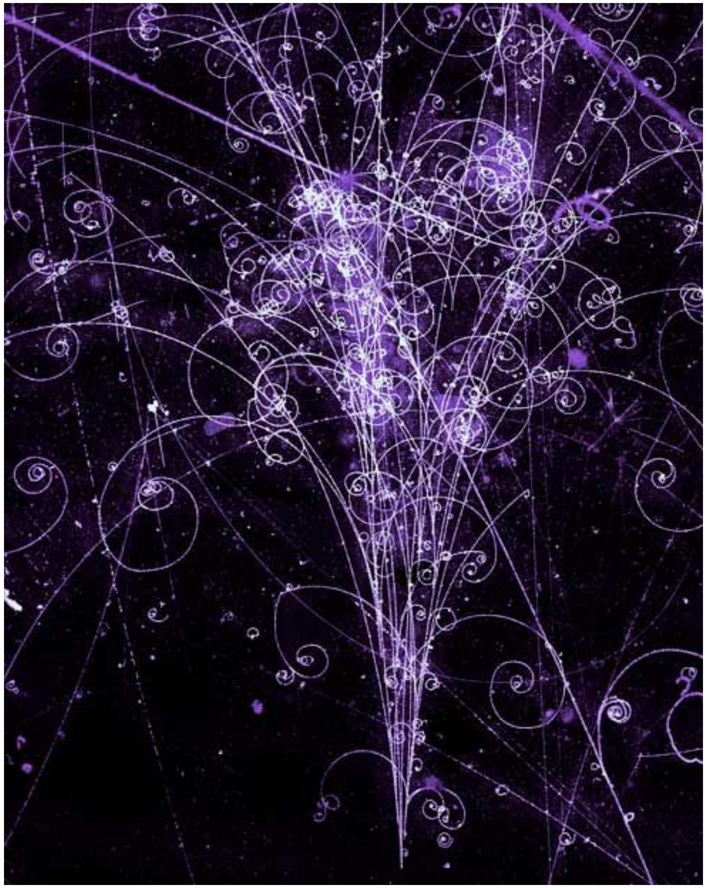
At the end of this session you should be able to …

- … recall the relations that apply in special relativity between **energy, momentum and rest mass**, of particles and photons
- … apply mass-energy equivalence to determine the energy released in **nuclear fission and fusion**
- … apply the conservation of relativistic energy and momentum in **interactions, collisions and annihilations** involving matter and radiation
- … be familiar with the **relativistic Doppler effect**, and the differences compared with the effect at low velocities

Summary of relativistic mechanics

- *Newtonian mechanics is modified at relativistic speeds*

- Relativistic **momentum**, $p = \gamma_u m_0 u$, is conserved $\left[\gamma_u = 1/\sqrt{1 - \frac{u^2}{c^2}}\right]$
- Relativistic **energy**, $E = \gamma_u m_0 c^2$, is conserved
- $E^2 = (pc)^2 + (m_0 c^2)^2$
- There is an equivalent **rest-mass energy** when $u = 0$, $E = m_0 c^2$
- **Photons** have zero rest mass, $E = pc$, where $E = hf = hc/\lambda$

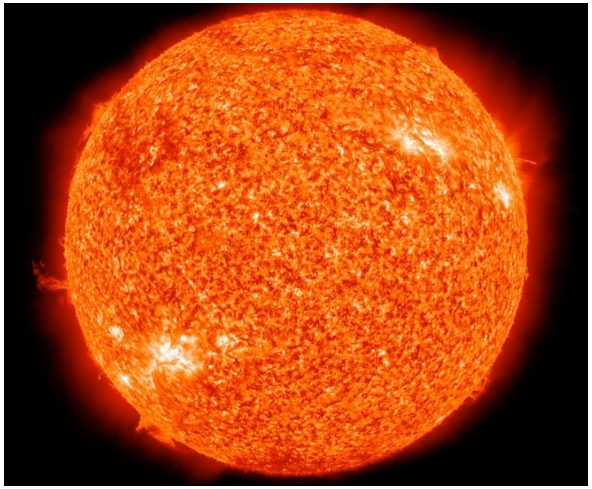


https://www.pinterest.com.au/pin/542120873871190260/

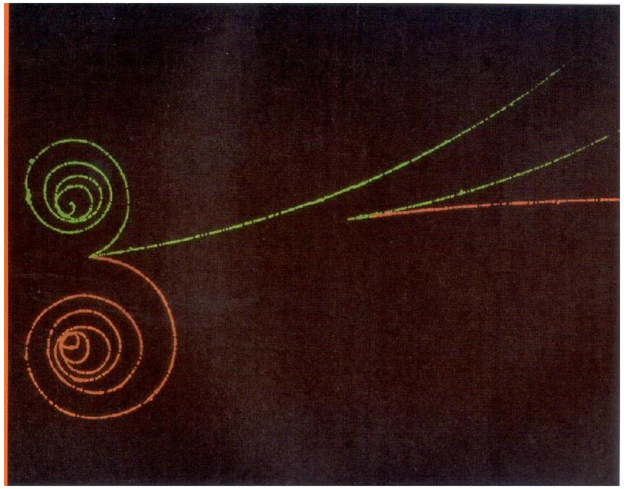
Relativistic phenomena

- We will now explore a set of relativistic phenomena which allow us to calculate some of these effects, in increasing order of difficulty …

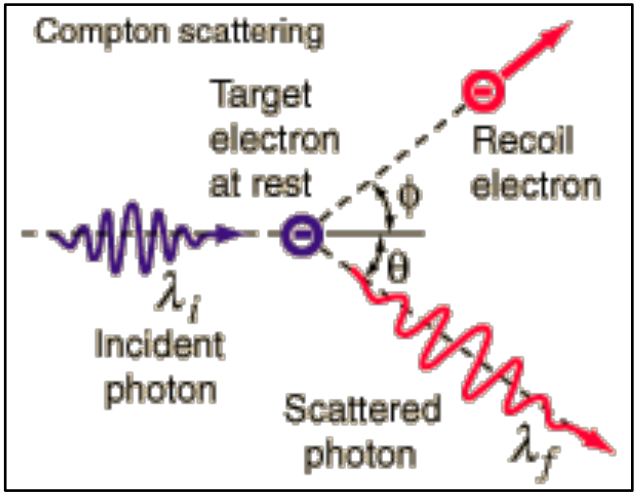
Nuclear energy



Pair production

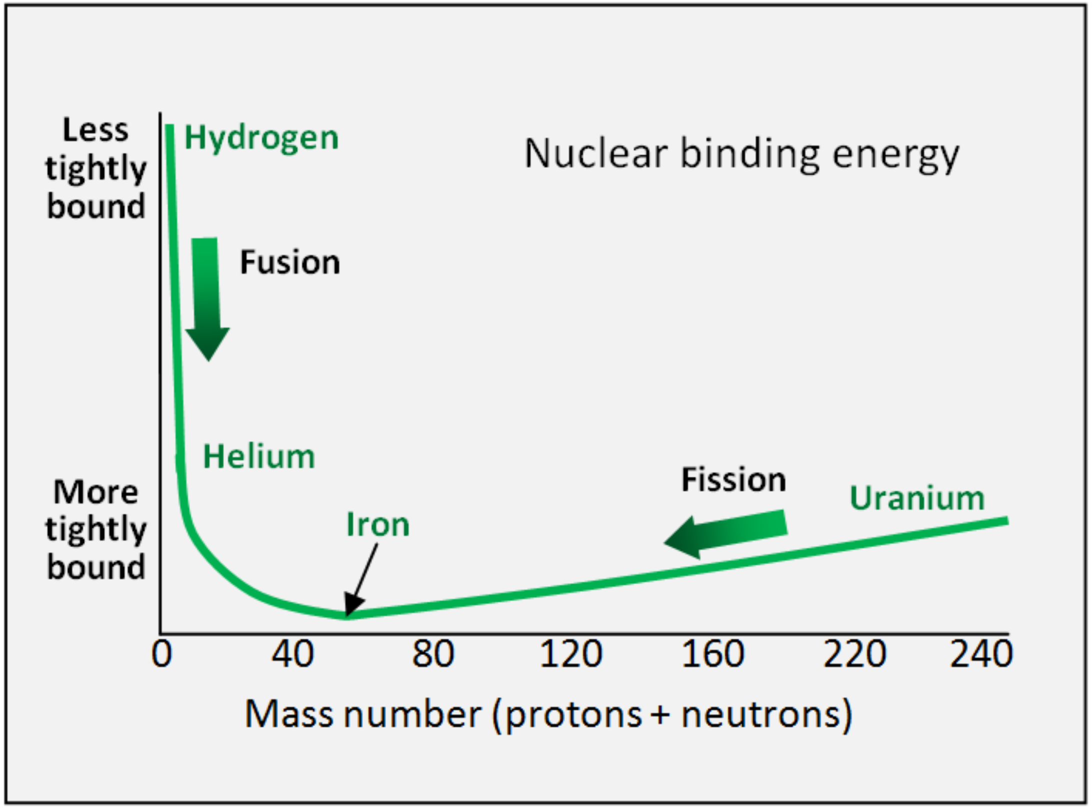


Compton effect



Nuclear energy

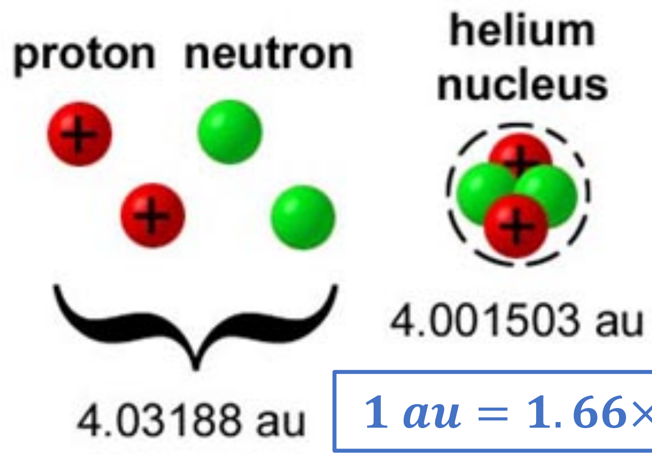
- Energy may be released through the combination **(fusion)** or breaking-up **(fission)** of atomic nuclei



https://marketbusinessnews.com/difference-nuclear-fission-nuclear-fusion/12219/nuclear-binding-energy-curve/

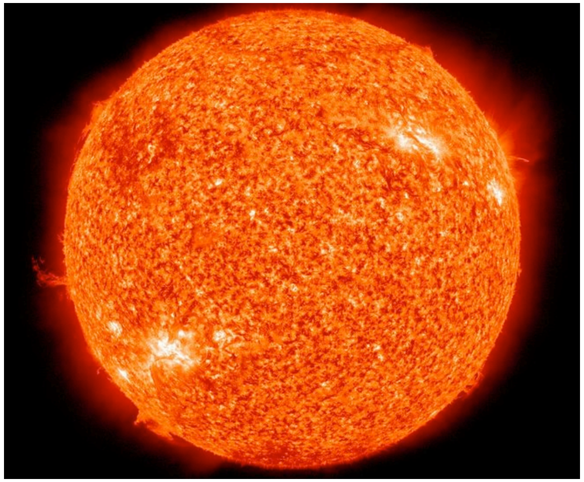
Nuclear energy

- The amount of released energy (or, the binding energy) may be computed from **mass-energy equivalence**, $E = m_0c^2$
- The Sun generates energy by **fusing hydrogen into helium**



$\mathbf{1\ au = 1.66 \times 10^{-27}\ kg}$

http://www.hk-phy.org/articles/fusion/fusion_e.html



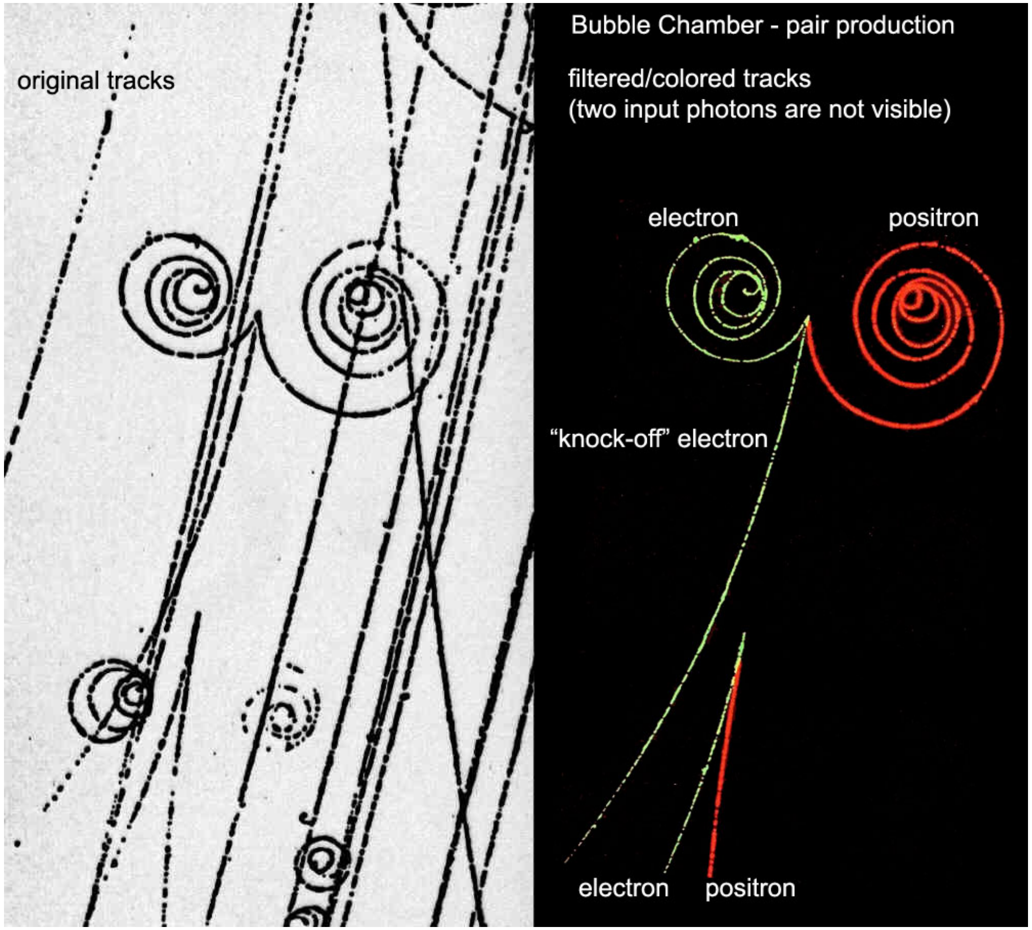
https://www.space.com/19321-sun-formation.html

- The luminosity of the Sun is $3.8\times10^{26}\ W$. What mass of hydrogen is converted in the Sun every second?

Pair production

- **Pair production** is the spontaneous creation of an electron and positron pair from electromagnetic radiation
- It's the main way that high-energy photons interact with matter
- It's a great example of the **interchangeability of matter and energy**

Bubble chamber tracks

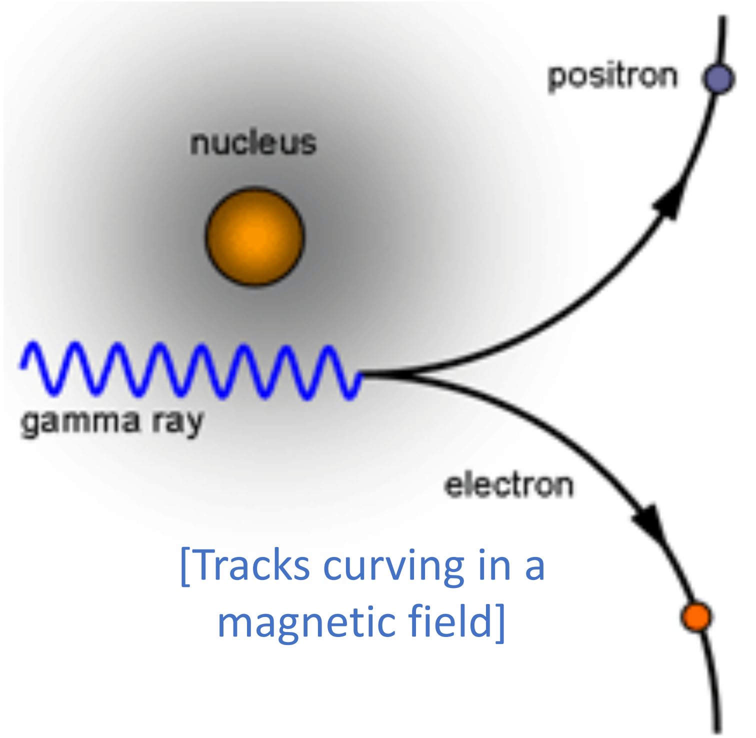


https://www.physics.wisc.edu/undergrads/courses/fall2011/107/images

Pair production

- Let's have a closer look at pair production [approximation!]

Mass of electron $= 9.1\times10^{-31}\ kg$



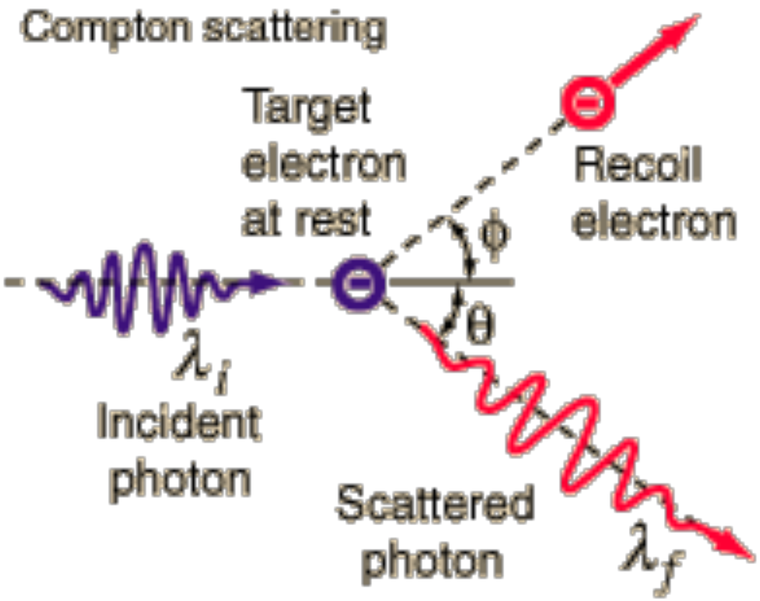
[Tracks curving in a magnetic field]

- Using mass-energy equivalence, what photon **threshold energy** is required to produce the electron/positron pair?
- What **wavelength** (and type) of radiation is required to achieve this?
- Why can this process only take place near an atomic nucleus? [Hint: consider conservation laws]

http://tap.iop.org/atoms/particles/534/page_47331.html

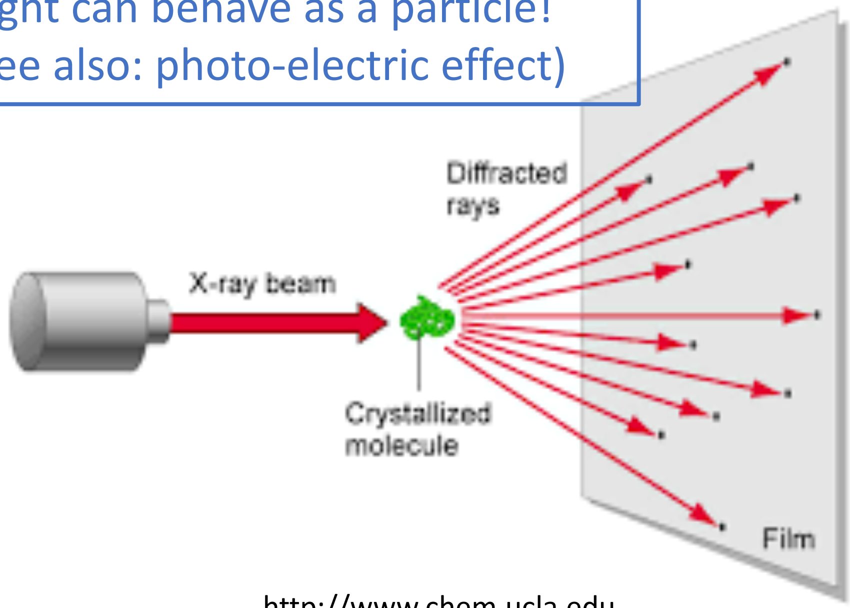
Compton scattering

- The **Compton effect** is the scattering of a photon by a charged particle, resulting in a decrease in the energy (increase in the wavelength) of the radiation



http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/comptint.html

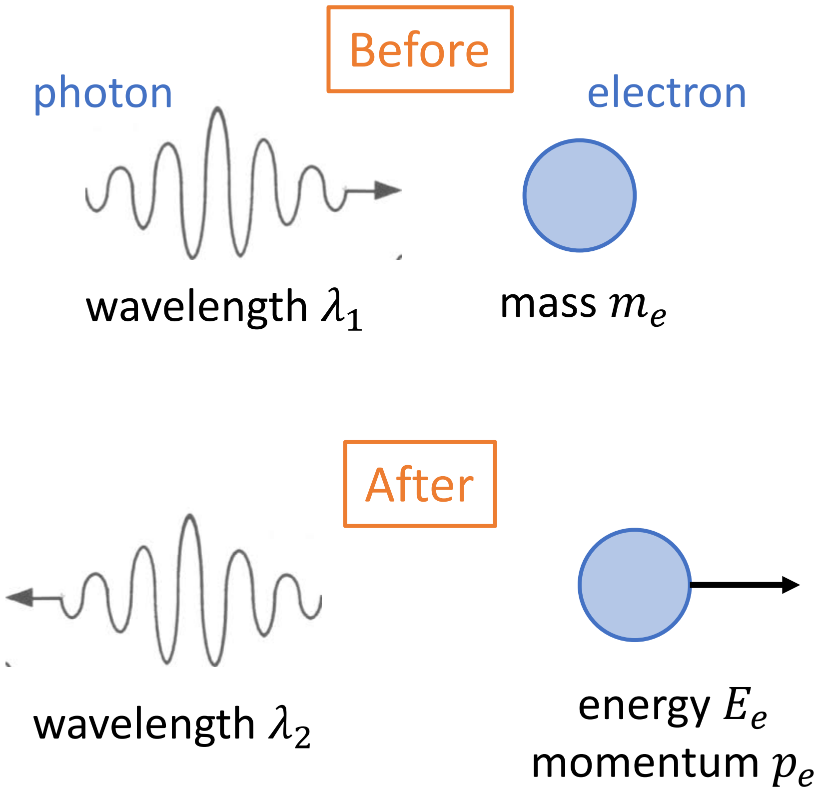
It's a beautiful demonstration that light can behave as a particle!
(see also: photo-electric effect)



http://www.chem.ucla.edu

Compton scattering

- Let's have a closer look at Compton scattering – how much does the wavelength change? [we'll consider a 1D case]



- Write an expression for the **momentum of the electron** after the collision, p_e, in terms of λ_1 and λ_2

- Write an expression for the **energy of the electron** after the collision, E_e, in terms of λ_1, λ_2 and m_e

- Now use the fundamental relation ${E_e}^2 - (p_e c)^2 = (m_e c^2)^2$, to find an expression for $\lambda_2 - \lambda_1$

Compton scattering

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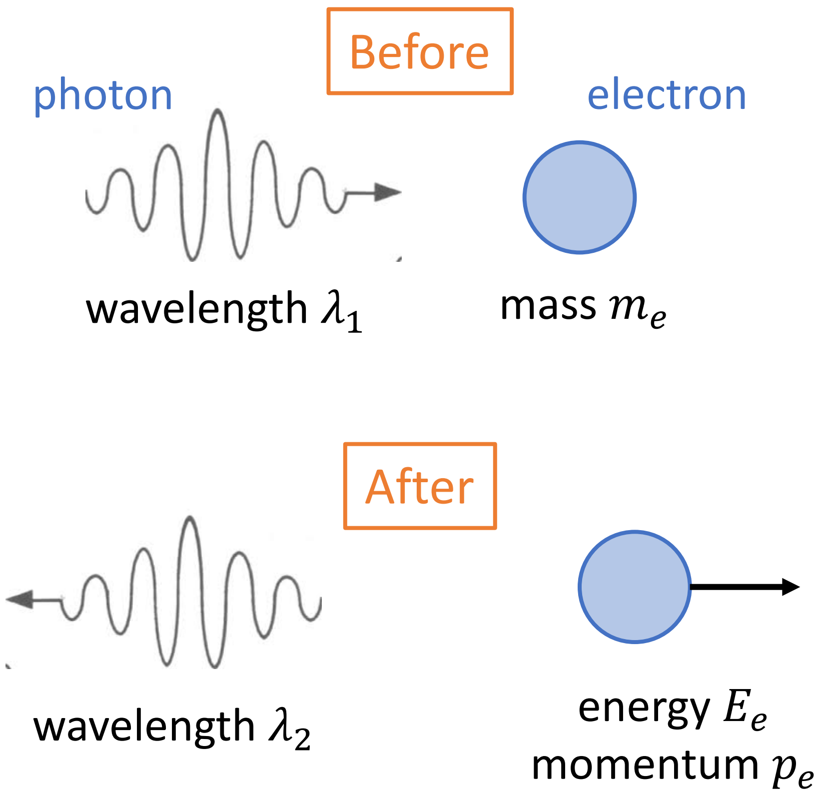


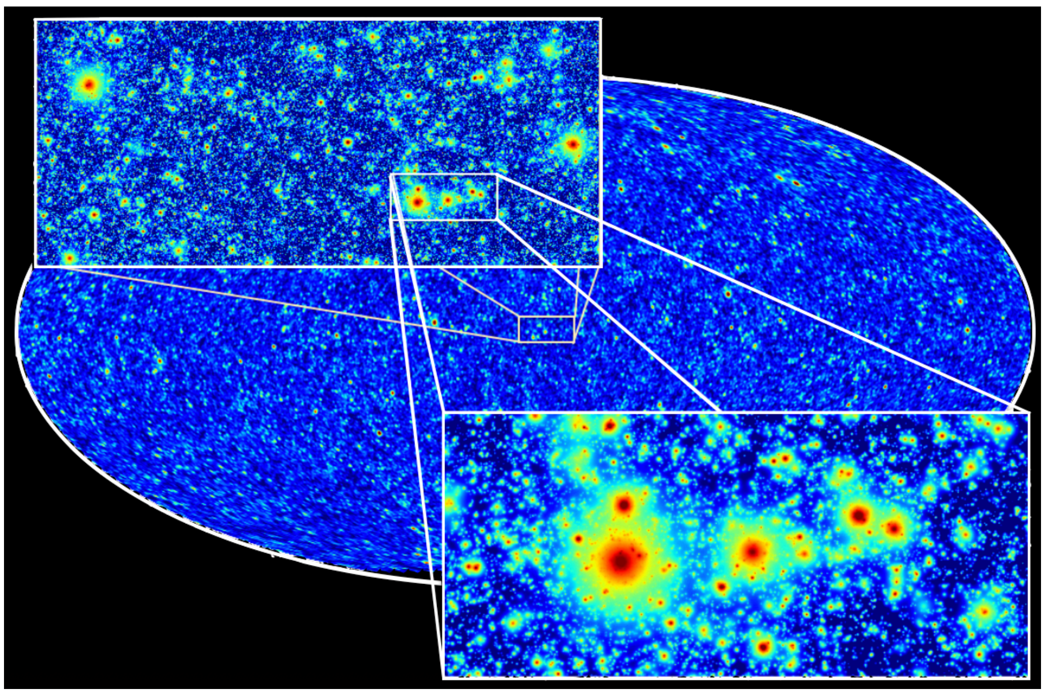
- Momentum of a photon is $p = \frac{E}{c} = \frac{h}{\lambda}$, so $\frac{h}{\lambda_1} = p_e - \frac{h}{\lambda_2}$, or $p_e = \frac{h}{\lambda_1} + \frac{h}{\lambda_2}$

- Energy before $= \frac{hc}{\lambda_1} + m_e c^2$ and energy after $= \frac{hc}{\lambda_2} + E_e$, so we find $E_e = \left(\frac{hc}{\lambda_1} + m_e c^2\right) - \frac{hc}{\lambda_2}$

- Substitute these two expressions in ${E_e}^2 - (p_e c)^2 = (m_e c^2)^2$ and cancel some terms, we find: $\lambda_2 - \lambda_1 = \frac{2h}{m_e c}$

Compton scattering

- **Inverse Compton scattering** – where high-energy electrons impart energy to photons – also occurs in astrophysics, in *clusters of galaxies* and *Active Galactic Nuclei*



http://cita.utoronto.ca/~malvarez/research/

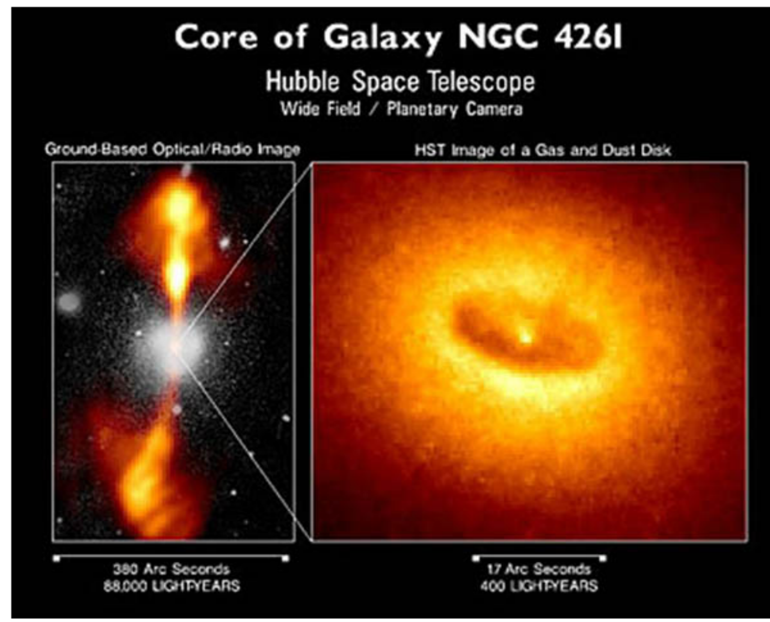
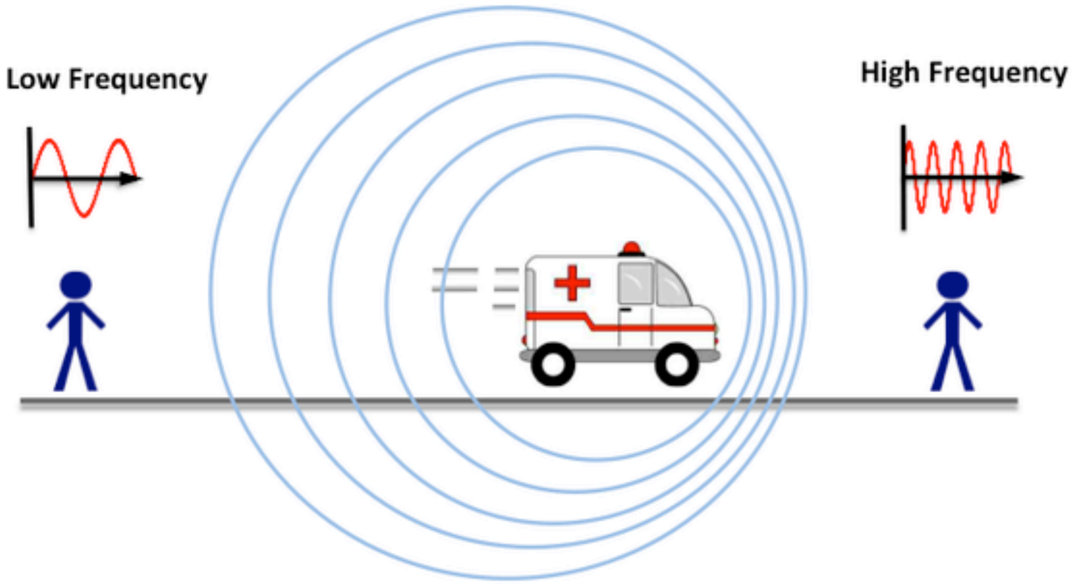


Figure 5: Active galaxy NGC 4261 at radio and optical wavelengths.

http://www.jeffstanger.net/Astronomy/emissionprocesses.html

The Doppler effect

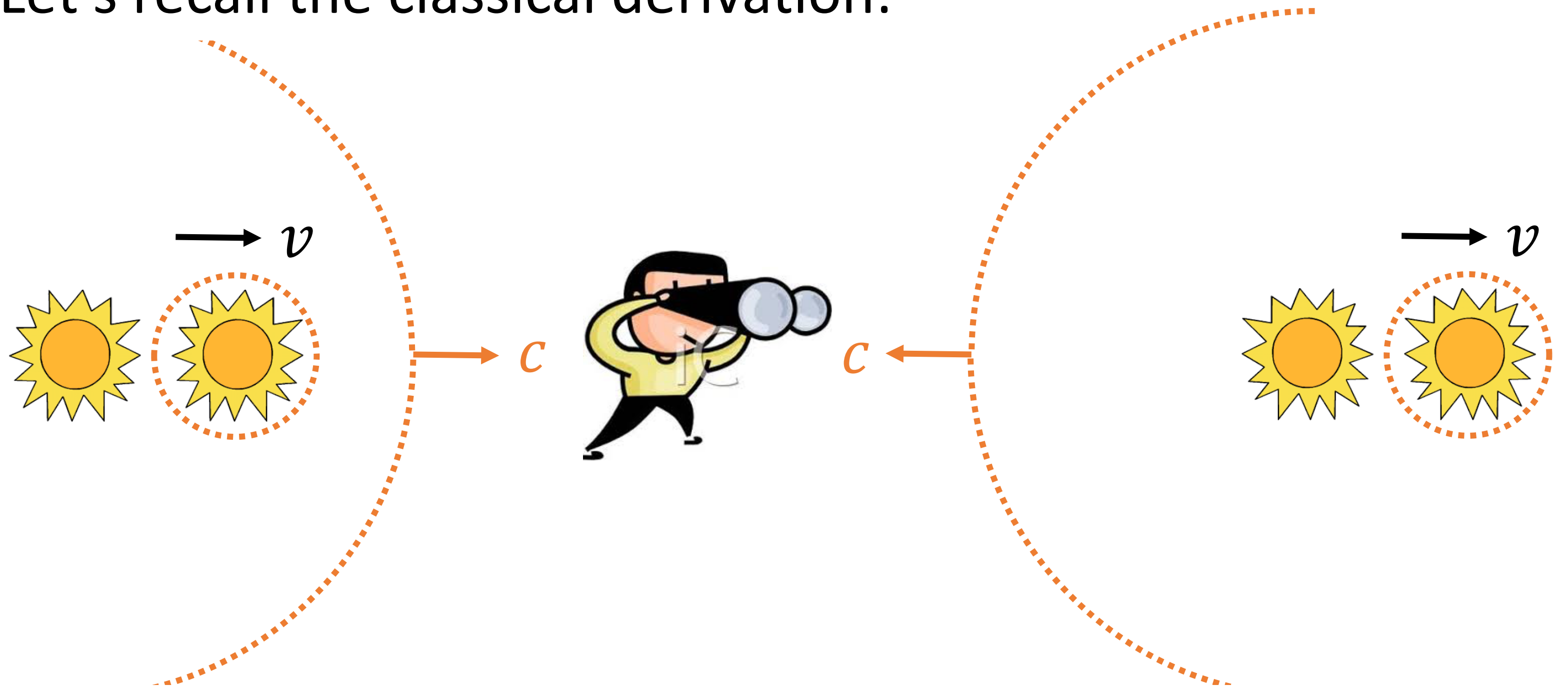
- The **Doppler effect** describes the change in frequency of a wave due to the relative motion of source and observer



- Since times transform differently in relativity, **the Doppler effect is modified at high speeds**

The Doppler effect

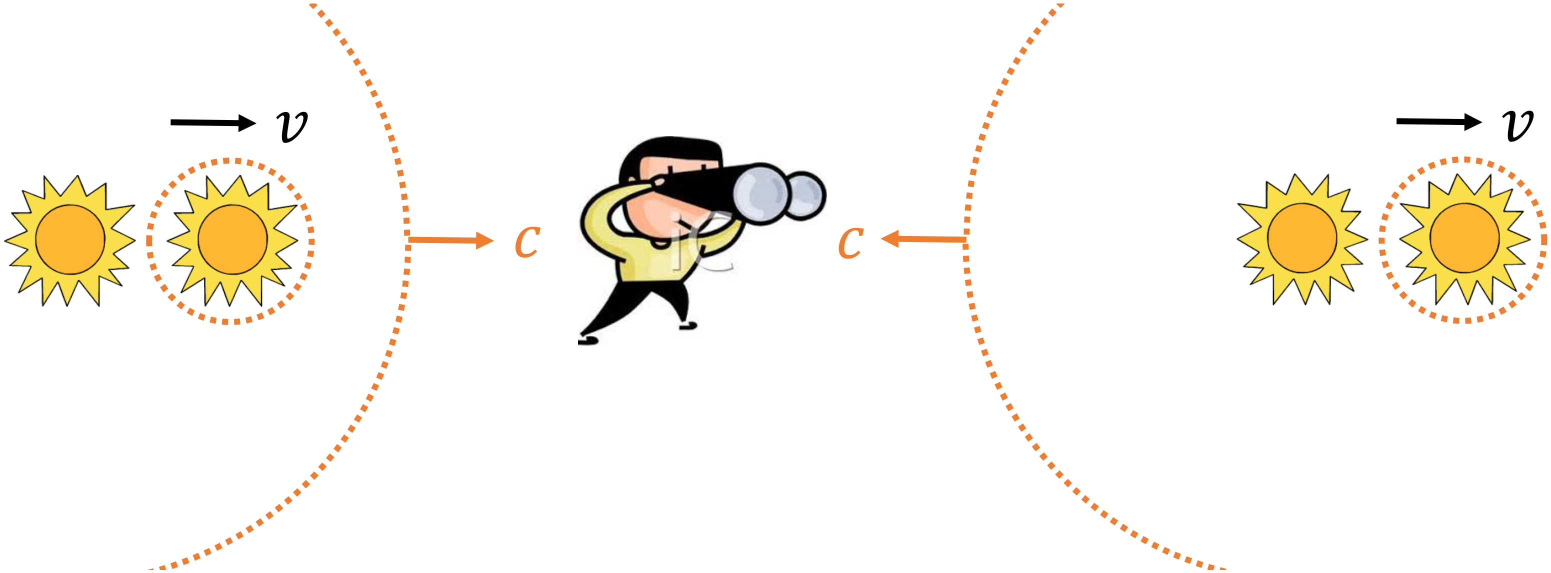
- Let's recall the classical derivation:



- Source pulses every time Δt (frequency $f_e = 1/\Delta t$, wavelength $\lambda_e = c/f_e$)
- In this time, the wave travels distance $c\ \Delta t$ and the source travels $v\ \Delta t$
- Wavelength measured by observer $\lambda_o = c\ \Delta t \pm v\ \Delta t = \lambda_e(1 \pm v/c)$

The Doppler effect

- In relativity, there is additional **time dilation** such that the time interval between pulses in the observer frame is $\gamma\ \Delta t$



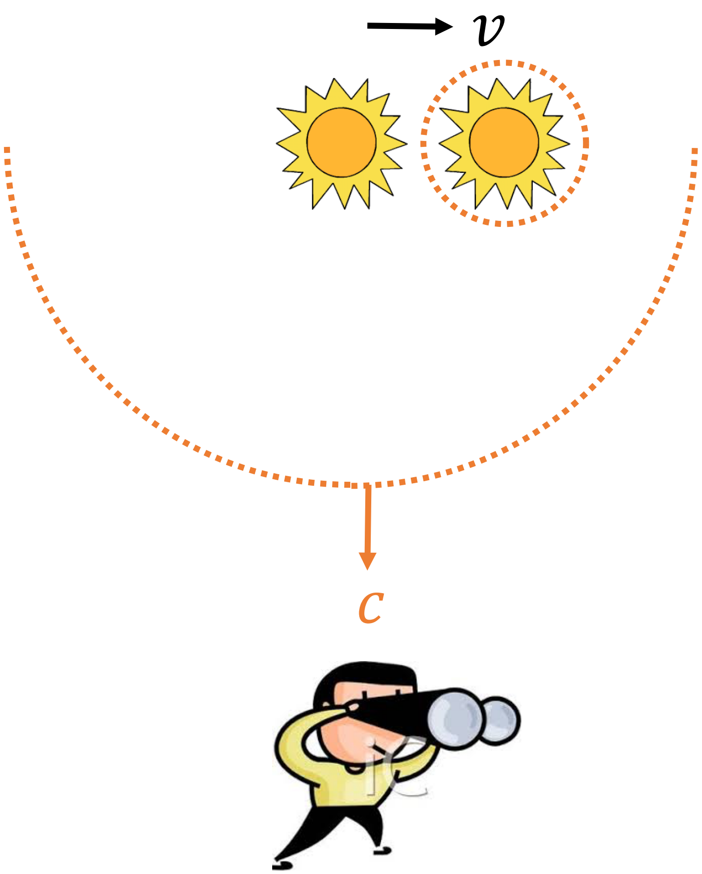
- Wavelength measured by observer $\lambda_o = c\ \gamma\Delta t \pm v\ \gamma\Delta t = \lambda_e \frac{1 \pm v/c}{\sqrt{1 - v^2/c^2}}$

$$\lambda_e = \lambda_o \sqrt{\frac{1 + v/c}{1 - v/c}}\ \text{(receding)} \qquad \lambda_e = \lambda_o \sqrt{\frac{1 - v/c}{1 + v/c}}\ \text{(approaching)}$$

The Doppler effect

- In relativity, time dilation causes a **Doppler effect for transverse motion** which doesn't exist in classical physics!

- $\lambda_e = \lambda_o \frac{1}{\sqrt{1 - v^2/c^2}}$ (transverse)



The Doppler effect

- A relativity student is caught running a red light on the rocket freeway. In court they plead they were driving so fast that the red light ($\lambda = 650$ nm) looked green ($\lambda = 530$ nm). How fast would their rocket car have been travelling?

https://www.fromthegrapevine.com/nature/6-fun-traffic-light-facts

[Problem 8-20 from Taylor & Wheeler, *Spacetime Physics*]

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