Class 4: Space-time Diagrams

In this class we will explore how space-time diagrams may be used to visualize events and causality in relativity

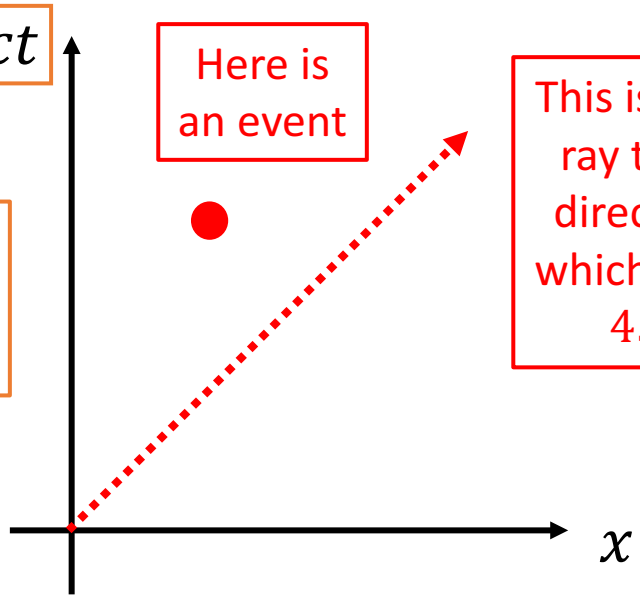
Class 4: Space-time Diagrams

At the end of this session you should be able to …

- … create a **space-time diagram** showing events and world lines in a given reference frame
- … determine geometrically which events are **causally connected**, via the concept of the **light cone**
- … understand that events separated by a constant space-time interval from the origin map out a **hyperbola** in space-time
- … use space-time diagrams to relate observations in different inertial frames, via **tilted co-ordinate systems**

What is a space-time diagram?

- A space-time diagram is a graph showing the **position of objects (events) in a reference frame, as a function of time**
- Conventionally, space (x) is represented in the horizontal direction, and time (t) runs upwards



We have scaled time by a factor of c, so it has the same dimensions as space

Here is an event

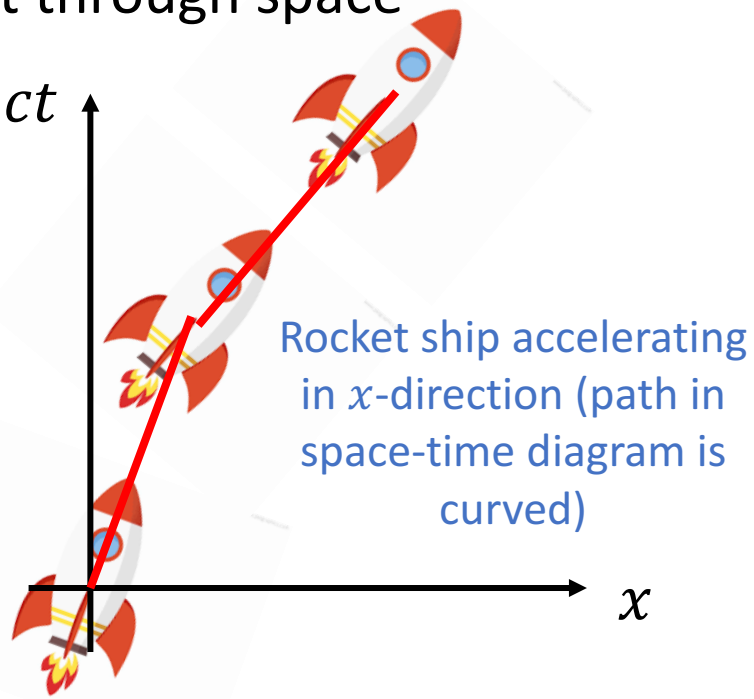
This is the path of a light ray travelling in the x-direction (i.e., $x = ct$), which makes an angle of $45°$ with the axis

What would be the path of an object moving at speed $v < c$?

What is a space-time diagram?

- The path of an object (or light ray) moving through a space-time diagram is called a **world line** of that object, and may be thought of as a chain of many events
- Note that a world line in a space-time diagram **may not be the same shape** as the path of an object through space

Rocket ship accelerating in x-direction (path in space is straight)



Rocket ship accelerating in x-direction (path in space-time diagram is curved)

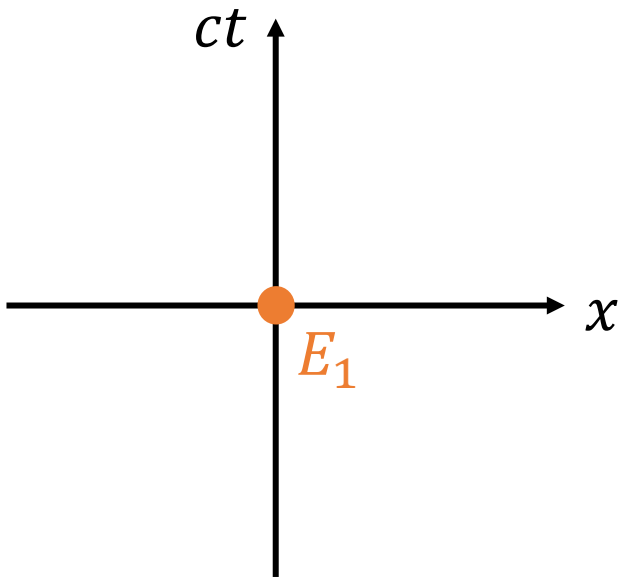
Causality

- In classical physics, **interactions are instantaneous** (change in one thing instantaneously affects another)
- Relativity tells us that **this is not true** – there is a finite maximum speed at which interactions can propagate, which turns out to be the speed of light
- This maximum speed leads to the idea of **cause and effect**

https://www.ihkplus.de/Das_Ende_der_Ketteninsolvenz.AxCMS

Causality

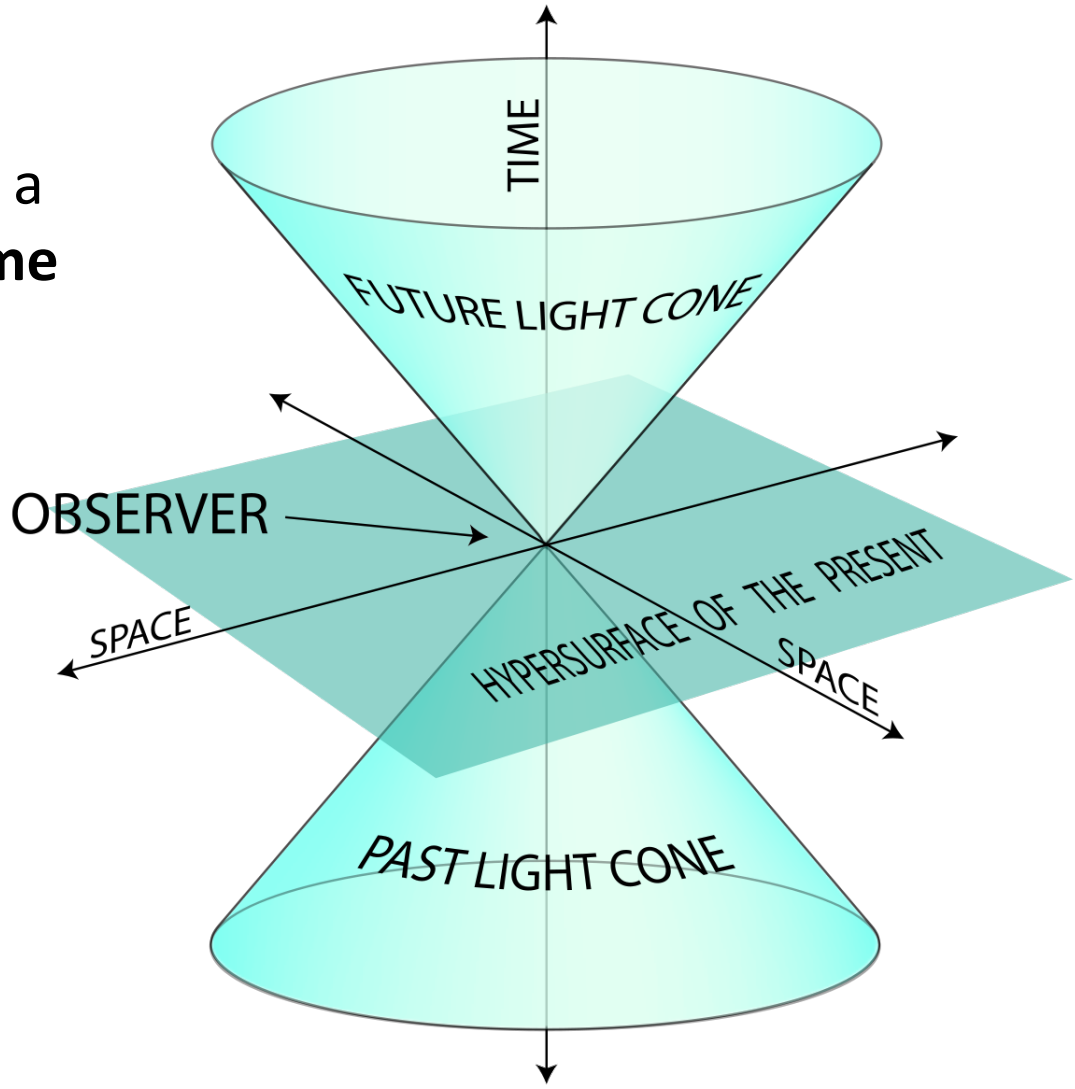
- Consider an event E_1 at the origin of co-ordinates



- Considering that nothing can travel faster than c, what region of the diagram contains events which E_1 can cause?
- What region contains events which can cause E_1?

Causality

- These considerations lead to the concept of a **light cone in space-time** around an event



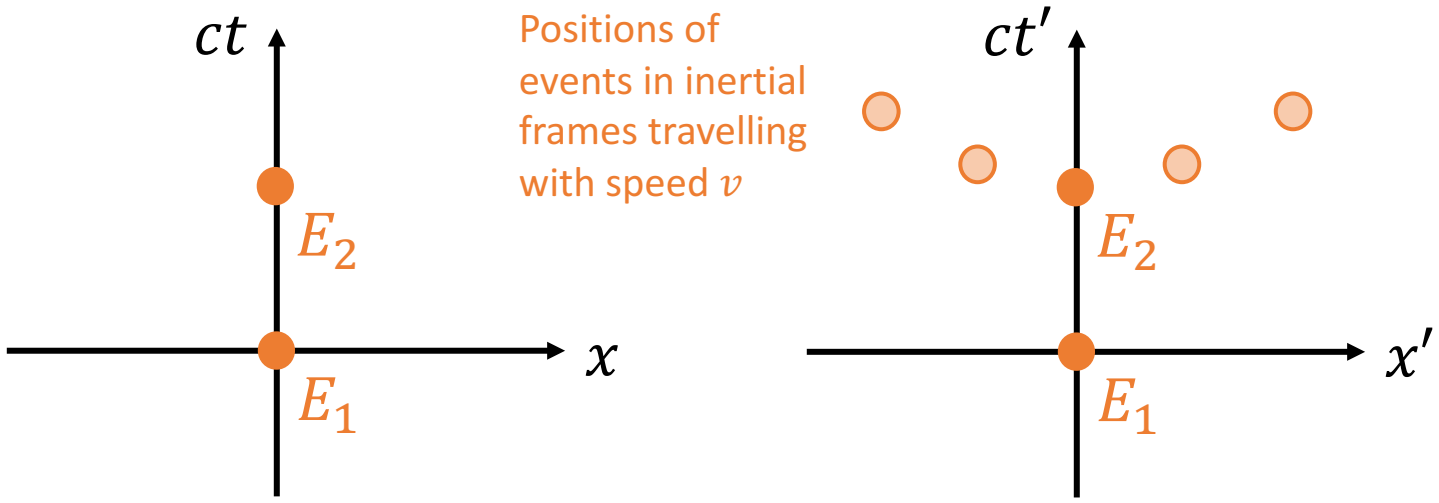
https://en.wikipedia.org/wiki/Light_cone

Causality

- A pair of events can be **causally connected**, or not, depending on their separation in space Δx and time Δt
- To be causally connected, we require $|\Delta x| < c|\Delta t|$, or the **space-time interval $\Delta s^2 = -c^2\Delta t^2 + \Delta x^2$ is negative**
- The space-time interval is an invariant – so **if 2 events are causally connected in one frame, they are in all frames – i.e. relativity preserves cause-and-effect**
- *(Reminder from previous class:) If $\Delta s^2 < 0$, there is a frame in which the events occur at the same place ($\Delta x = 0$) but no frame in which the events occur at the same time ($\Delta t = 0$)*

Space-time geometry

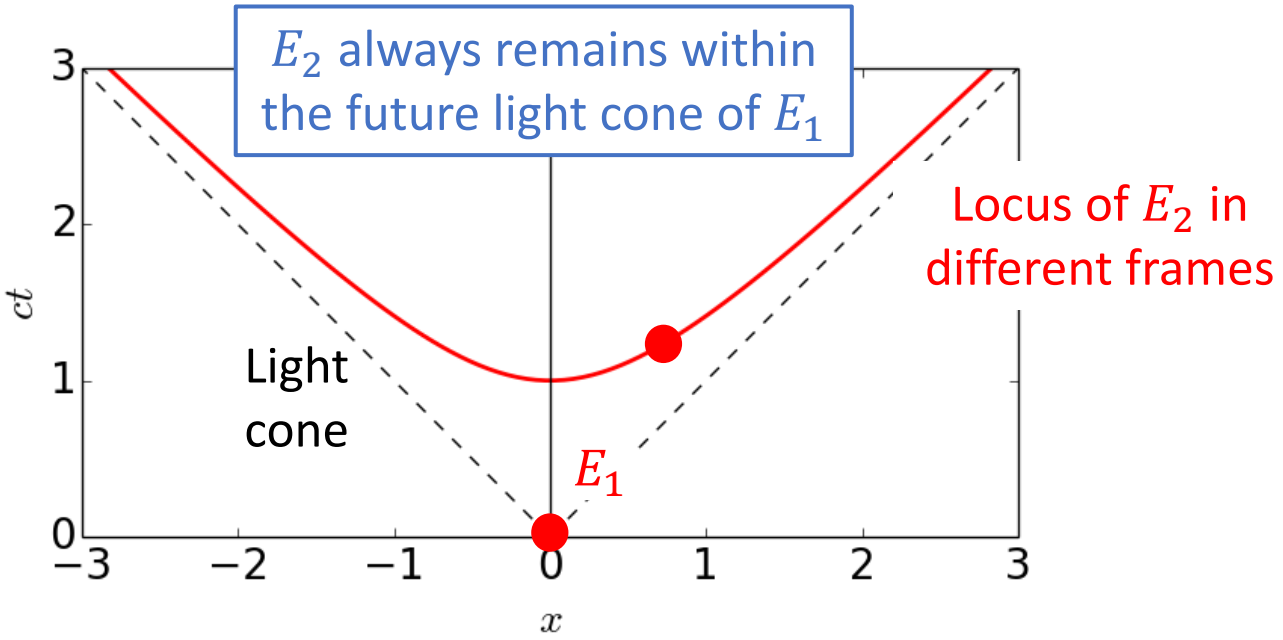
- Consider an event E_2 which takes place at the same spatial location in S as E_1, such as two ticks of a stationary clock



- How would these events appear in another frame S'? We can use the Lorentz transformations with $x = 0$ to deduce that $(x', t') = (-\gamma v t, \gamma t)$, where v is the speed of S'

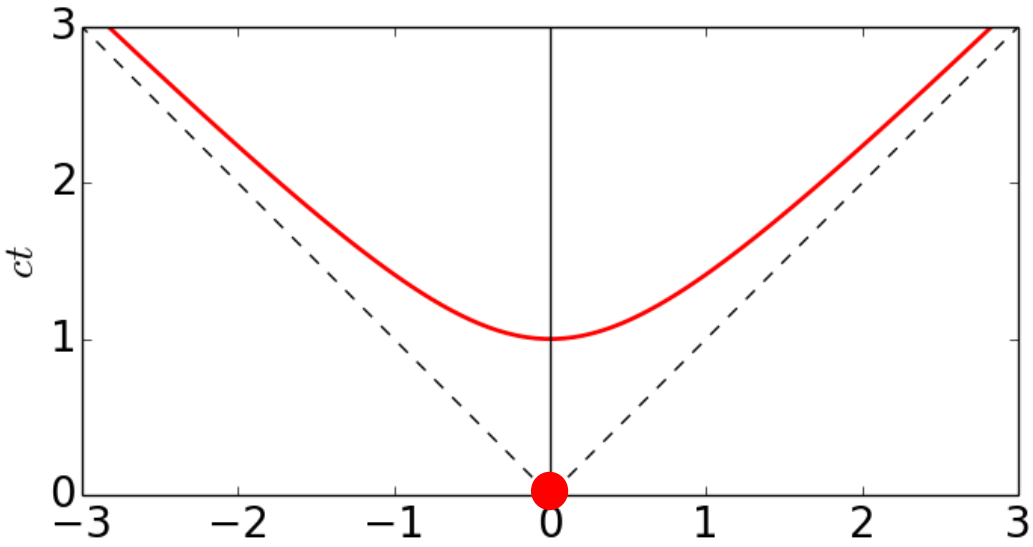
Space-time geometry

- What mathematical curve is the position of E_2 tracing out?
- Recall that the space-time interval between the events, $\Delta s^2 = -c^2\Delta t^2 + \Delta x^2$, is a constant in every frame: "$(ct)^2 - x^2 =$ constant" is a hyperbola

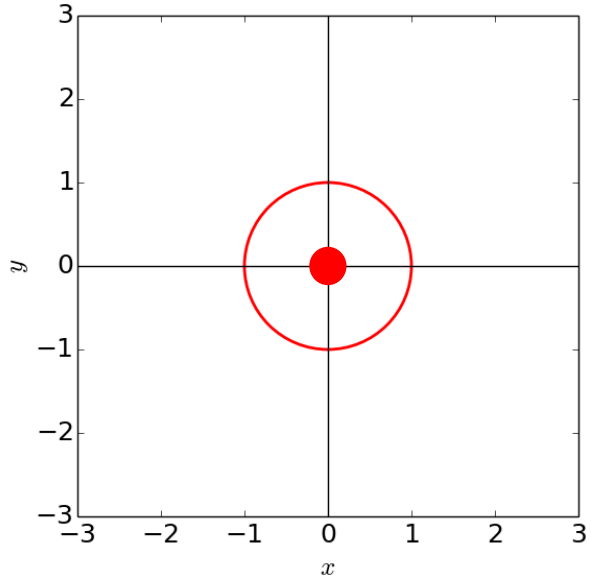


Space-time geometry

Locus of points a **constant space-time interval** away from a point in space-time (trace out by changing frame)

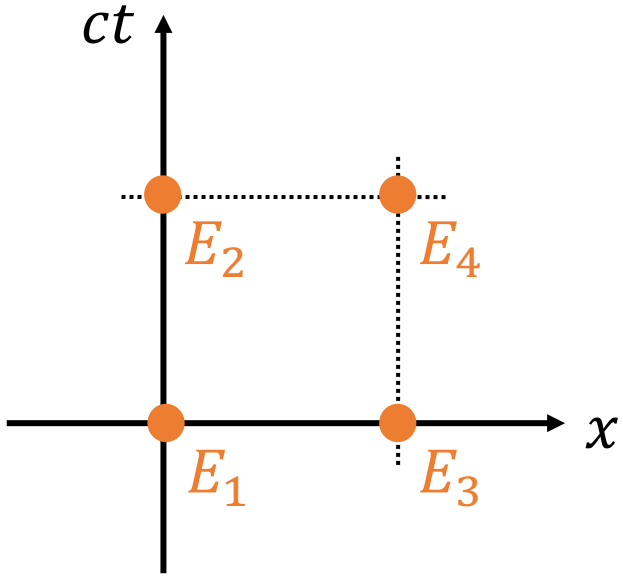


Locus of points a **constant distance** away from a point in Euclidean geometry (trace out by rotation)



Changing frames

- Consider 4 events which take place in frame S at space-time co-ordinates $(x, ct) = (0,0), (1,0), (0,1), (1,1)$

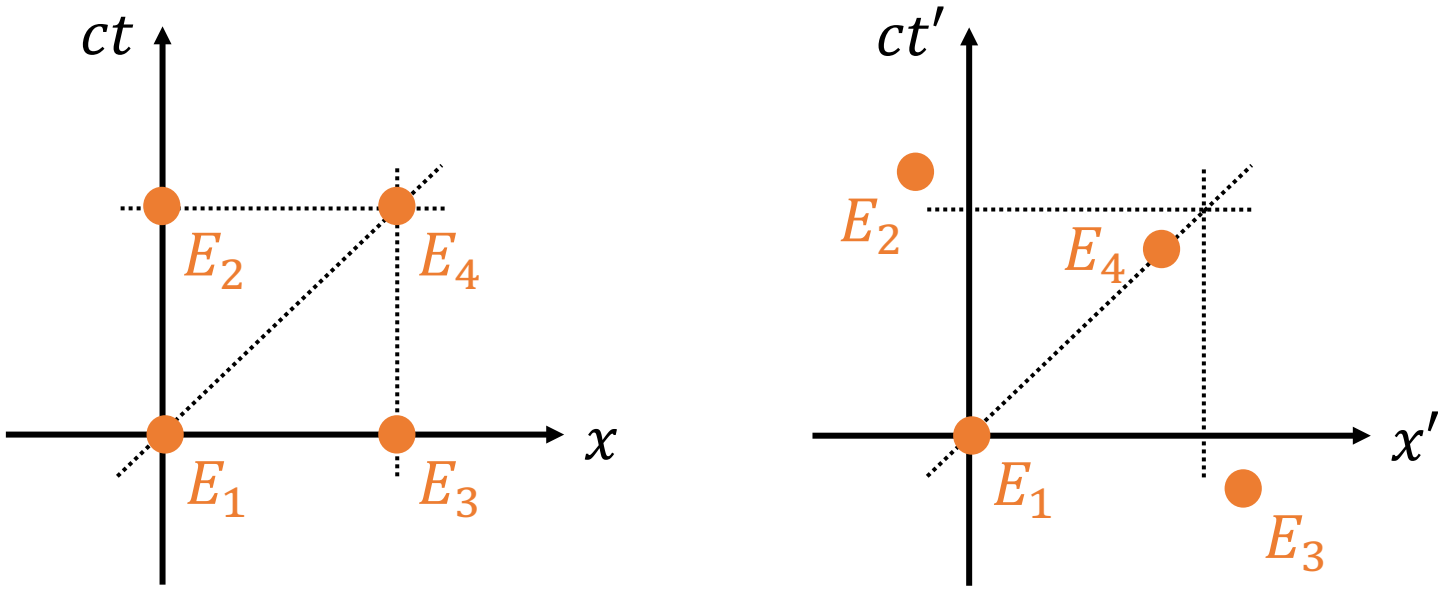


- E_1 and E_2 could represent two ticks of a clock sitting at the origin of S
- E_1 and E_3 could represent the measurement of the length of a ruler in S
- E_1 and E_4 could be connected by a light-signal

- Use the Lorentz transformations to plot these events in a space-time diagram for S' with axes (x', ct'), where $v = 0.6c$

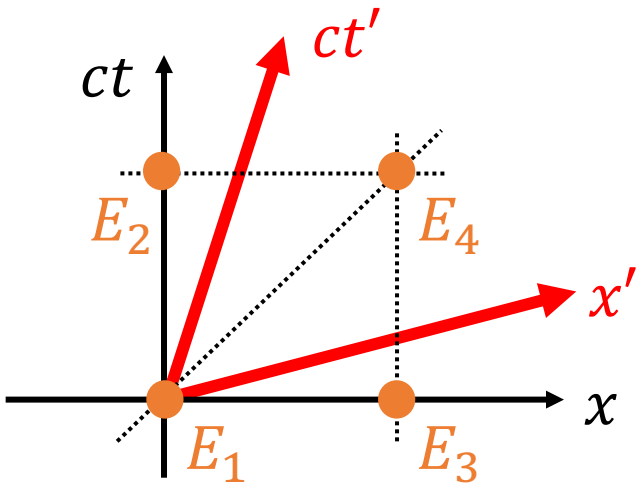
Changing frames

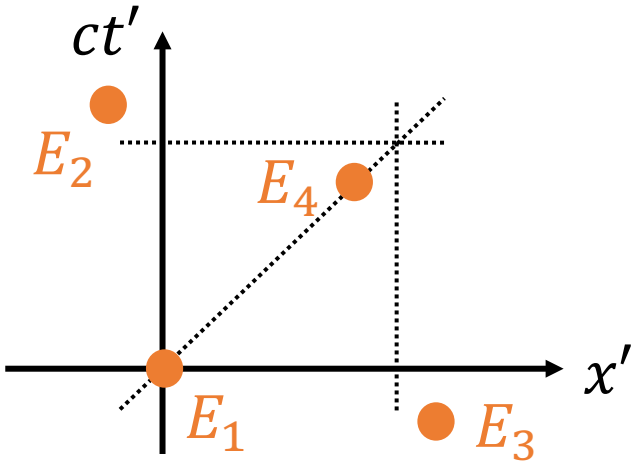
- The events as measured in frame S':



- (E_1, E_3) and (E_2, E_4) are no longer simultaneous in S'
- (E_1, E_4) are still connected by a light signal in S'

Changing frames

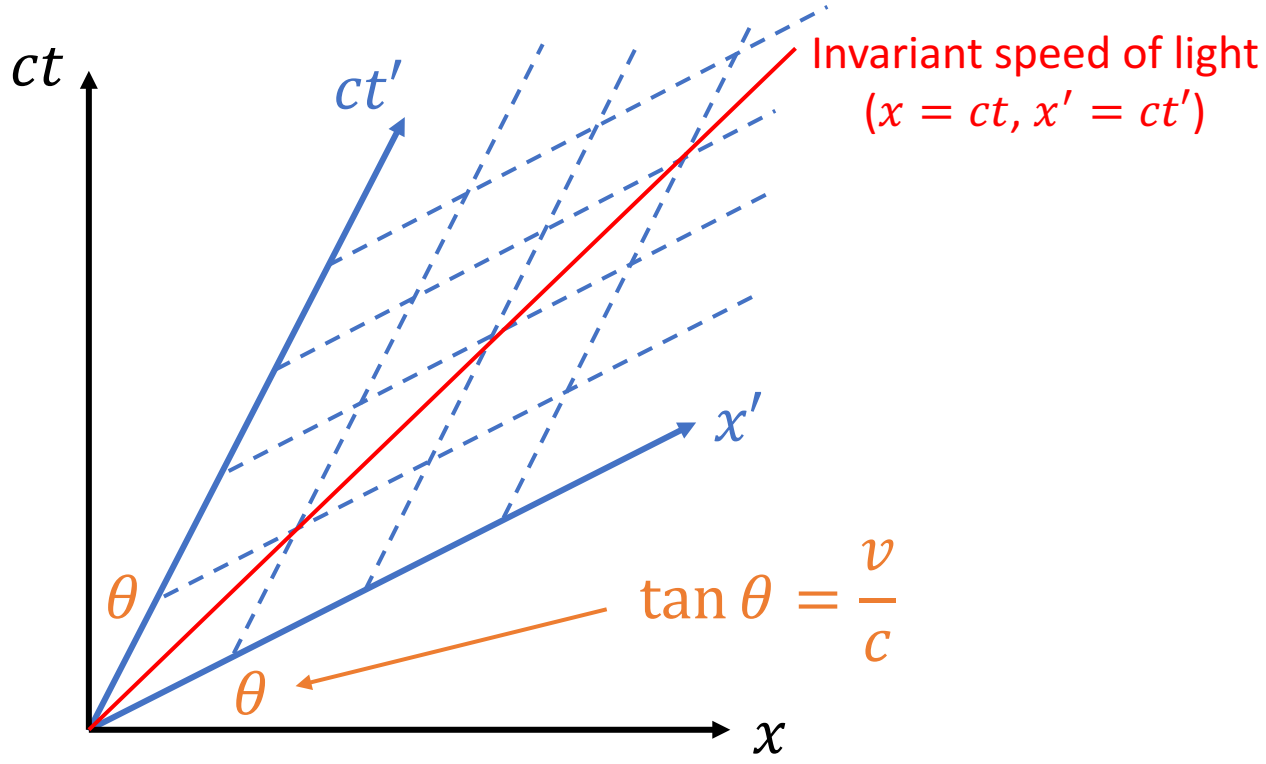




- Rather than change the positions of the events, a nice visualization approach is to **tilt the space-time axes**!
- The x'-axis is the line of $t' = 0$, so can be represented in an (x, ct) diagram by the line $ct = \frac{v}{c}\, x$ [using $t' = \gamma\left(t - \frac{vx}{c^2}\right)$]
- The t'-axis is the line of $x' = 0$, so can be represented in an (x, ct) diagram by the line $ct = \frac{c}{v}\, x$ [using $x' = \gamma(x - vt)$]

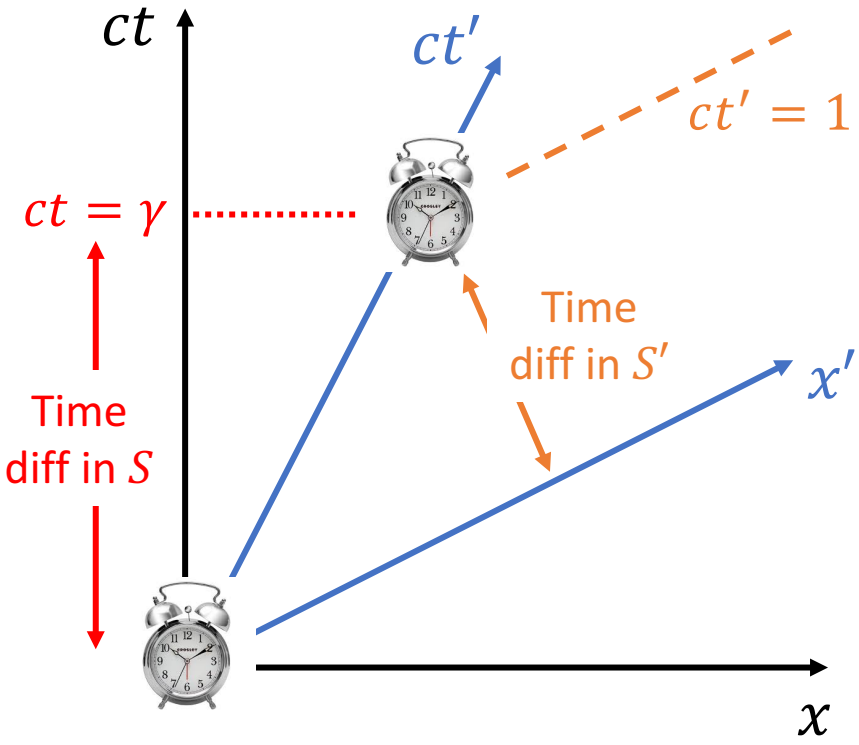
Changing frames

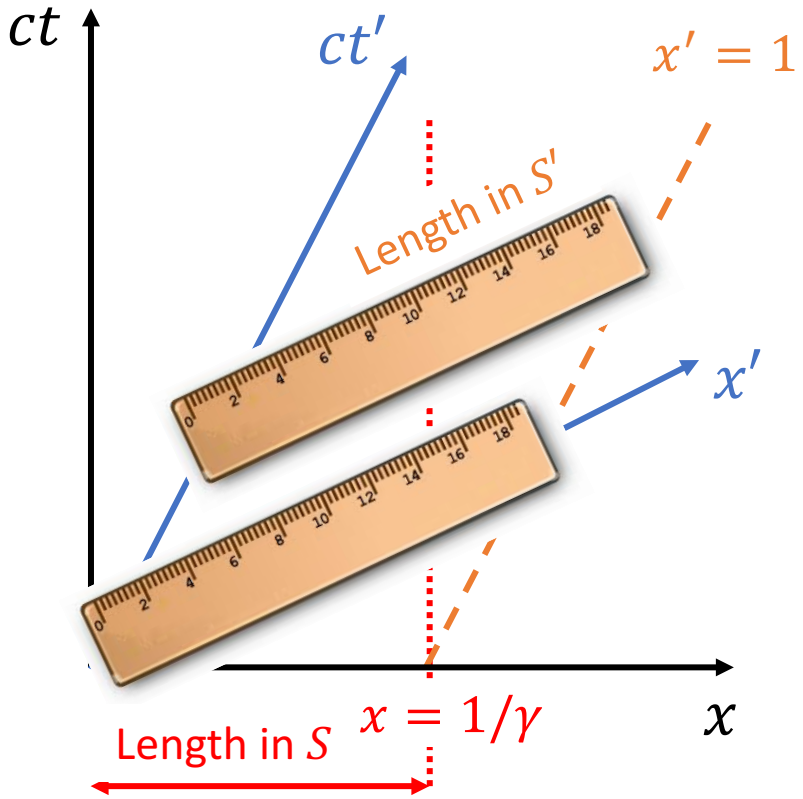
- So the space-time diagram of S' can be overlaid on the space-time diagram of S as a tilted co-ordinate system!



Changing frames

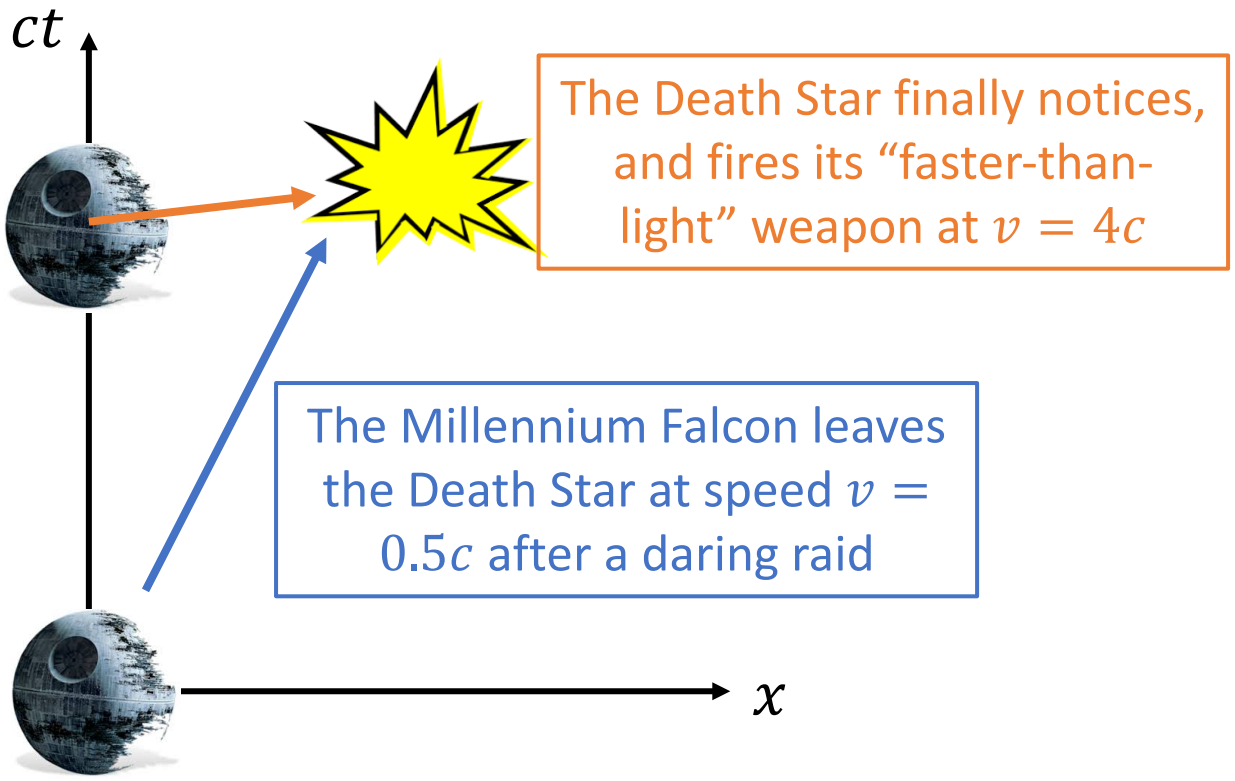
- This provides us with immediate visualizations of the phenomena of **time dilation** and **length contraction**





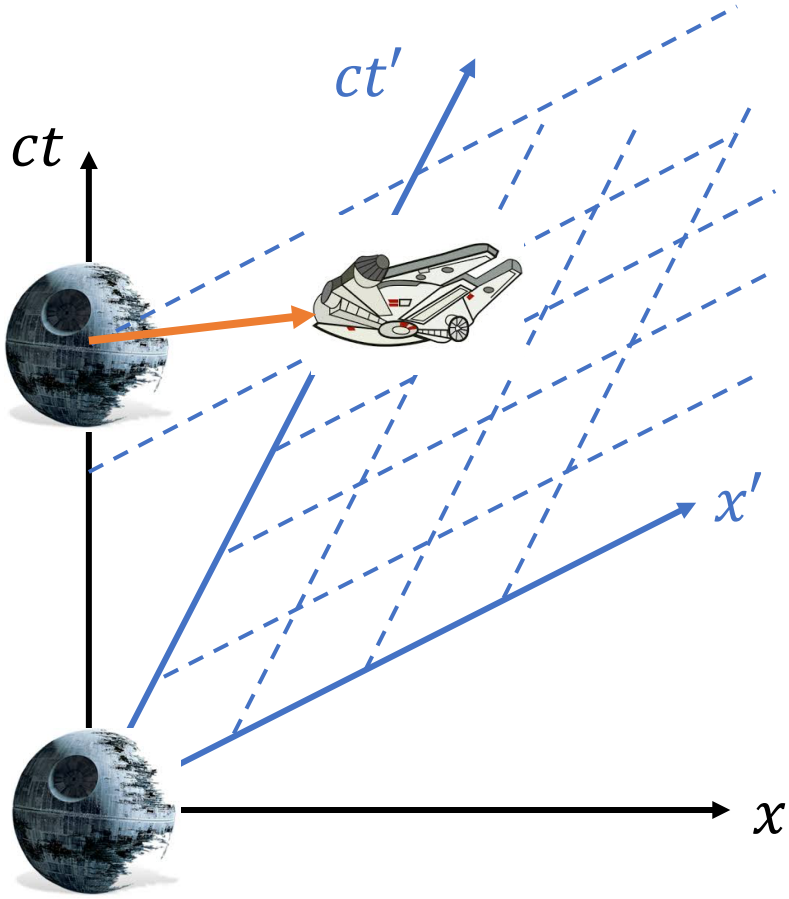
Faster-than-light travel

- We can also use our space-time diagrams to show that **faster-than-light travel contradicts cause-and-effect!**



Faster-than-light travel

- How do these events appear from the Millennium Falcon?



The (x', ct') co-ordinate system is tilted when overlaid on (x, ct)

In the S' frame, the faster-than-light weapon is fired AFTER the Millennium Falcon is destroyed!!

Faster-than-light travel contradicts cause-and-effect!!

Faster-than-light travel

- What other issues make faster-than-light travel difficult to reconcile with standard physics?

Faster-than-light neutrinos could be down to bad wiring

By Jason Palmer
Science and technology reporter, BBC News

23 February 2012 Share

What might have been the biggest physics story of the past century may instead be down to a faulty connection.

In September 2011, the Opera experiment reported it had seen particles called neutrinos evidently travelling faster than the speed of light.

The neutrinos are fired deep under the Italian Apennines to the Gran Sasso lab

The team has now found two problems that may have affected their test in opposing ways: one in its timing gear and one in an optical fibre connection.

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