Class 1: Special Relativity

In this class we will review some important concepts in Special Relativity, that will help us build up to the General theory

Class 1: Special Relativity

At the end of this session you should …

- … **be familiar with the basic ingredients of Special Relativity**: reference/inertial frames, the Lorentz transformations, Einstein's postulate and its consequences

- … **understand the significance of the space-time interval** ds^2 between two events

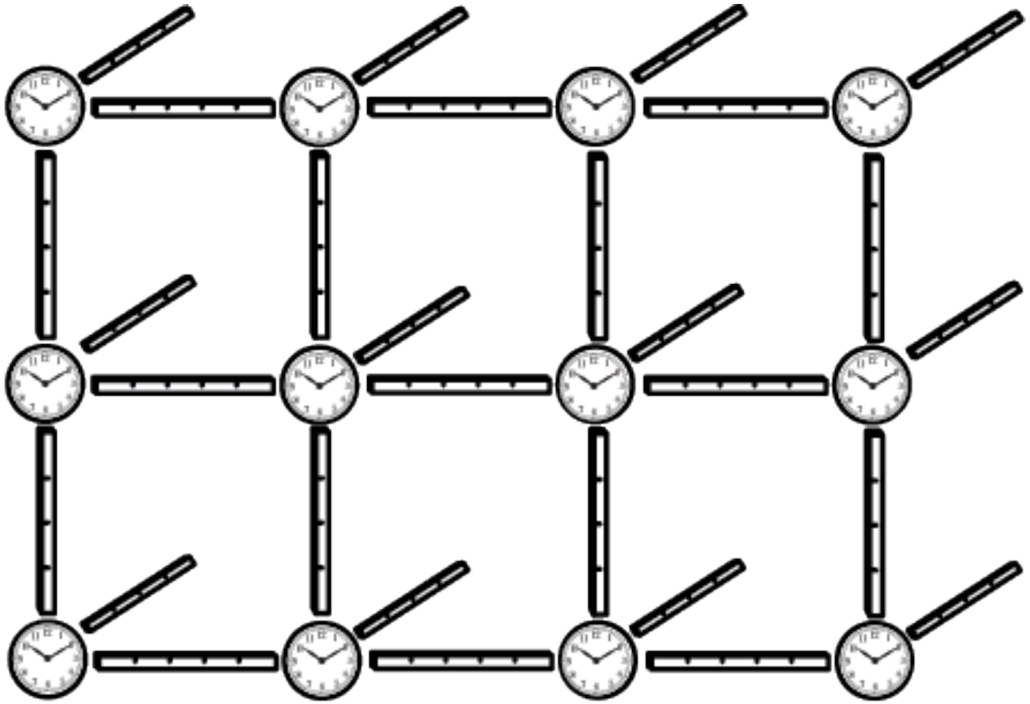
- … **be able to use space-time diagrams** to describe some simple visualizations of events in space-time

- … **understand how the definitions of classical momentum and energy** must be modified in Special Relativity

Reference frames

- How do we describe occurrences in nature?

- A **reference frame** is an information-gathering system consisting of a field of synchronized clocks on a co-ordinate grid

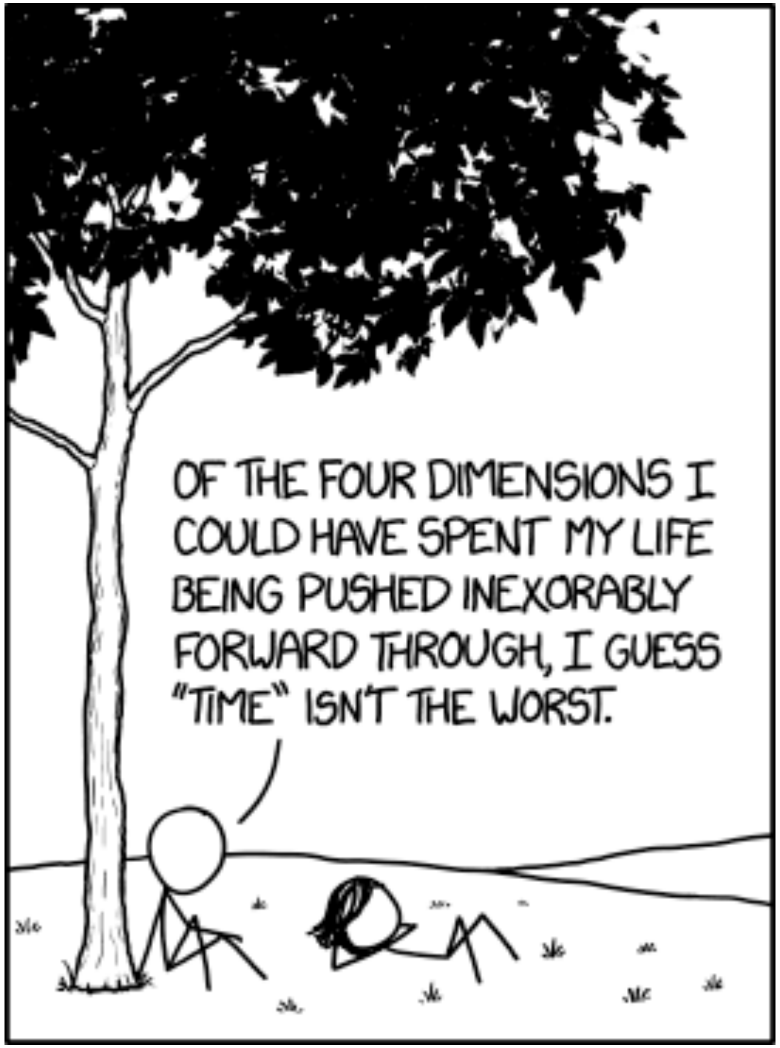


http://www.pitt.edu/~jdnorton/teaching/HPS_0410/chapters/Special_relativity_principles

- An **event** is an occurrence with definite location/time

Reference frames

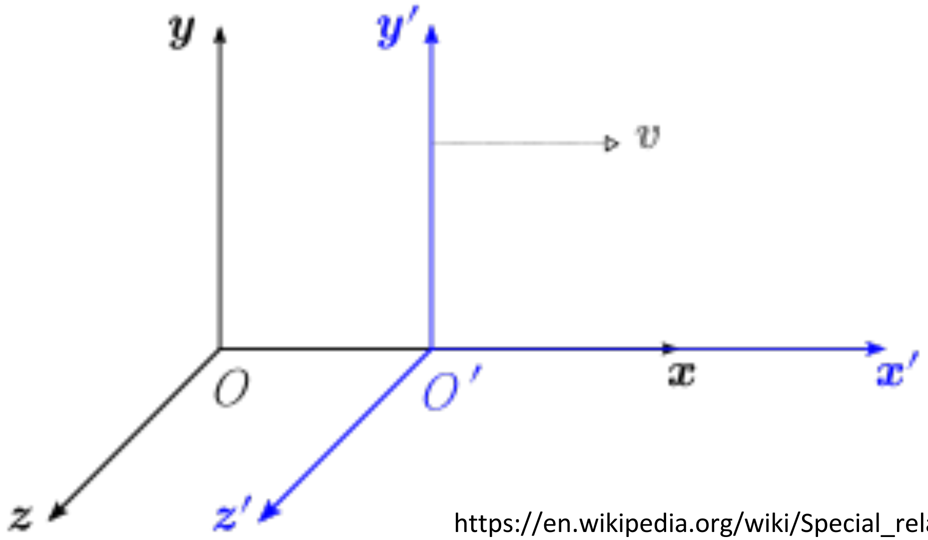
- The locations and times of events together form a 4D co-ordinate system known as **space-time**.



https://xkcd.com/1524/

Inertial frames

- Some reference frames are special
- In an **inertial frame**, a freely-moving body (no external forces) moves with constant velocity
- Inertial frames **move uniformly** with respect to each other



https://en.wikipedia.org/wiki/Special_relativity

Inertial frames

- Some examples of non-inertial frames!

https://www.thinglink.com/scene/877700809561735169
https://www.pinterest.com.au/bestonamusement/beston-frisbee-rides-for-sale-or-fairground-pendul/

Principle of relativity

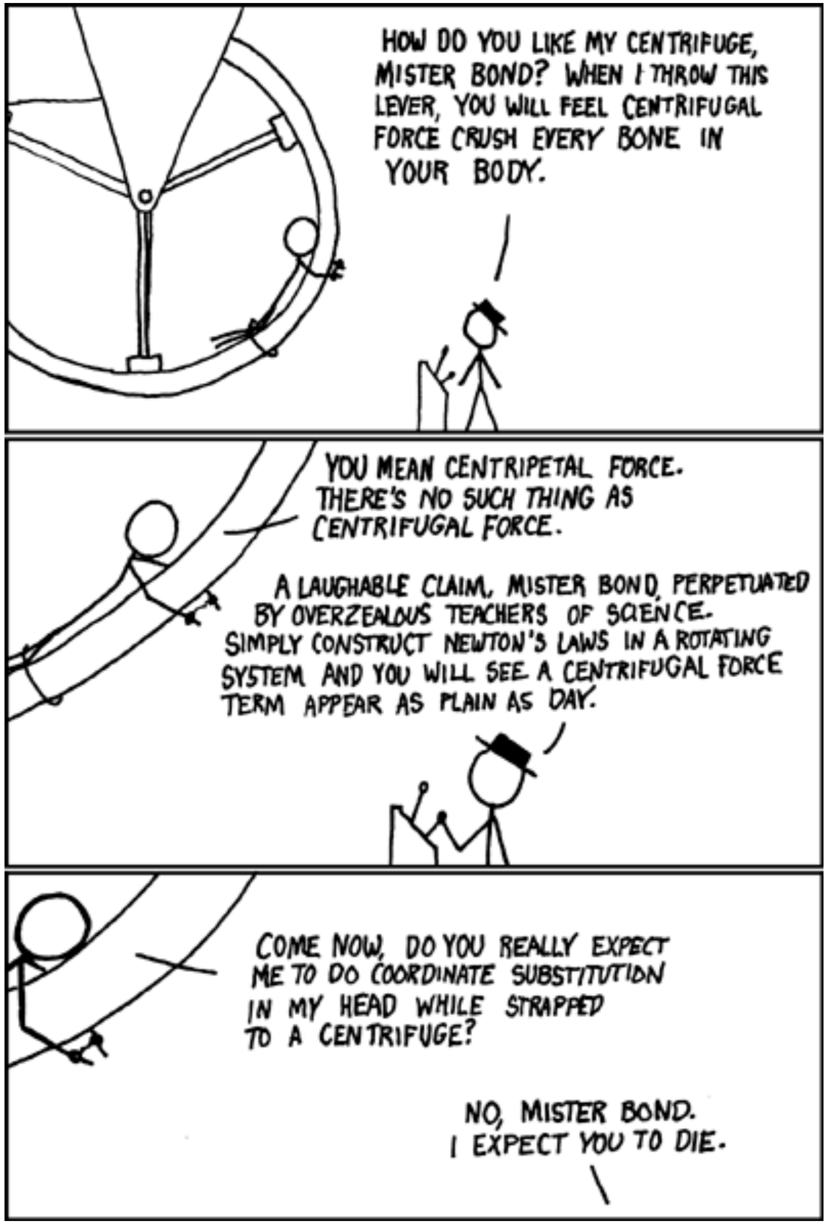
- **Laws of nature are identical in all inertial frames** (mathematically, they have the same form in each frame's co-ordinate system)
- There is no such thing as **absolute** rest/velocity

(Due to Galileo)

https://owlcation.com/stem/What-Were-Galileos-Contributions-to-Astronomy

Principle of relativity

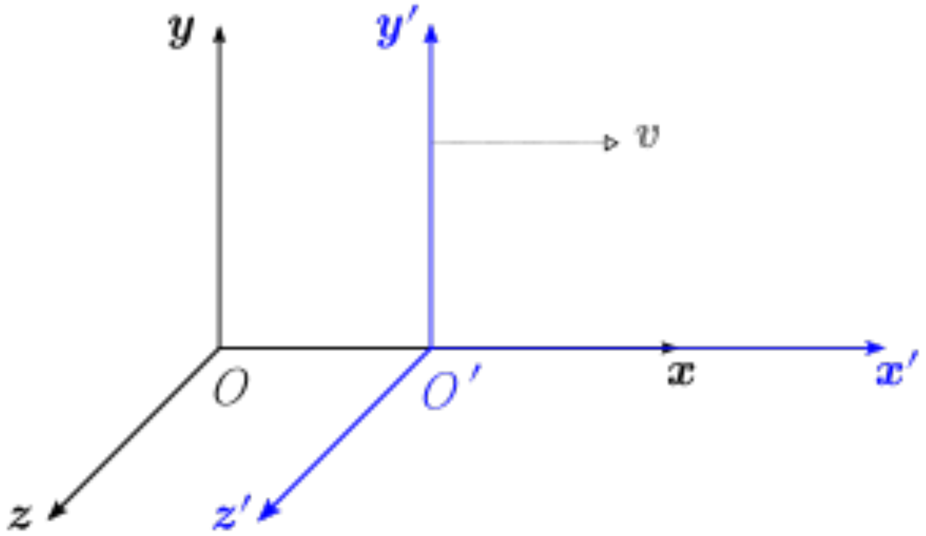
- **Example**: Newton's Laws $\vec{F} = m\vec{a}$ are invariant under the transformation $\vec{v}'(t) = \vec{v}(t) + V_{constant}$
- **Not true for non-inertial frames**, in which we experience forces we cannot account for



https://xkcd.com/123/

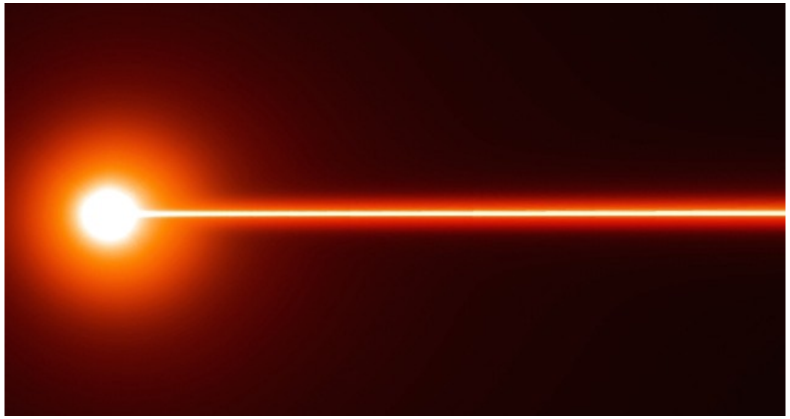
Transformations between frames

- If (t, x, y, z) are the co-ordinates of an event in frame S, what are its co-ordinates (t', x', y', z') in frame S' moving with speed v with respect to S?



- **Galilean transformation**: clocks in S and S' run at the same rate $(t' = t)$, and velocities add $(u = u' + v)$

The role of light



https://brooklyntomars.com/laser-beam-focus-352154e6acaf

- According to the Galilean transformation, the speed of light c slows down to $c - v$ in S'
- **This can be tested experimentally and is false** (e.g. Michelson-Morley experiment)
- The Galilean transformation is **inconsistent with electromagnetism**

The role of light

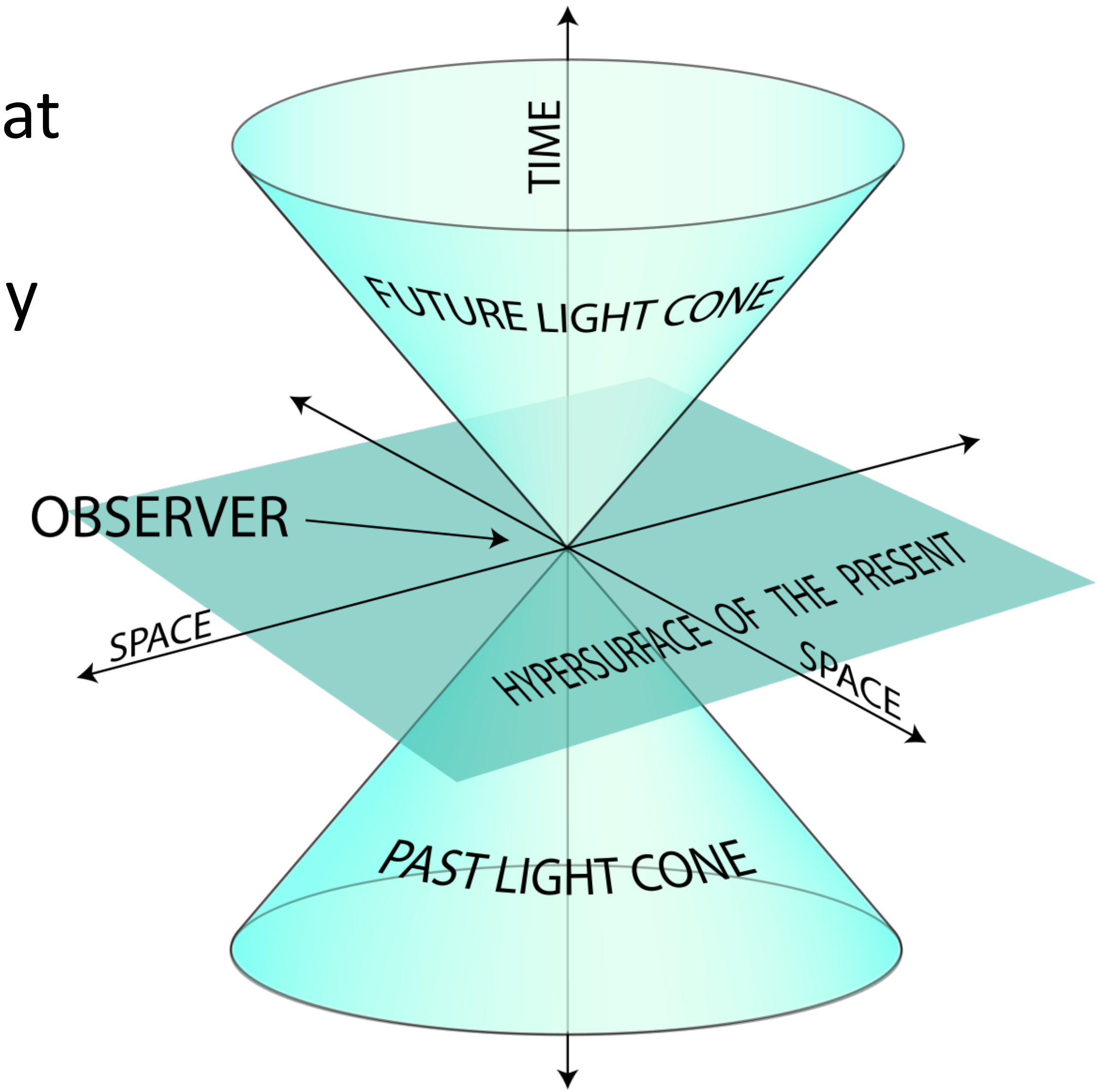
- In classical physics, **interactions are instantaneous** (change in one thing instantaneously affects another)
- **This is not true** – there is a finite maximum velocity at which interactions propagate, which turns out to be the speed of light $c = 3\times10^8$ m/s (large)

https://www.ihkplus.de/Das_Ende_der_Ketteninsolvenz.AxCMS

- **Consequences**: (1) bodies cannot move faster than c, (2) c is the same in all frames, (3) causality

The role of light

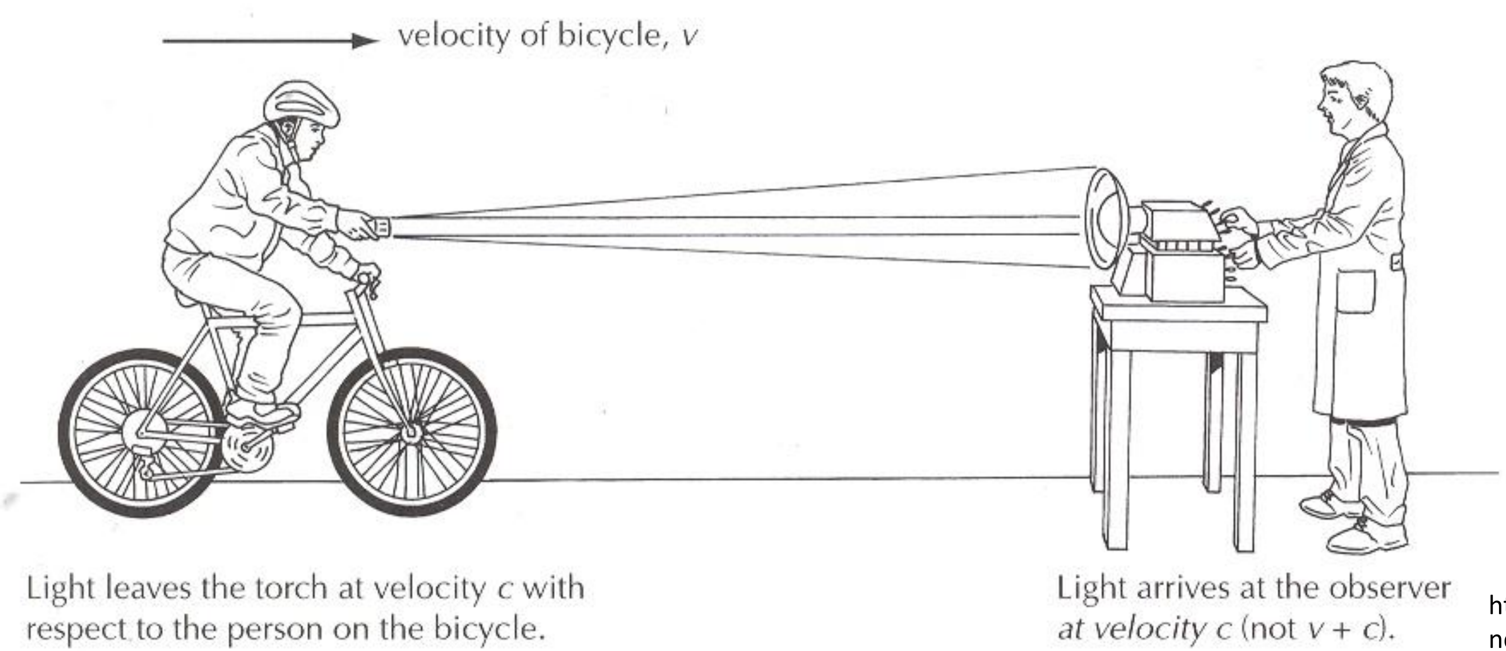
- The finite velocity at which signals can propagate is deeply connected to the idea of **causality**



https://en.wikipedia.org/wiki/Light_cone

Einstein's postulate

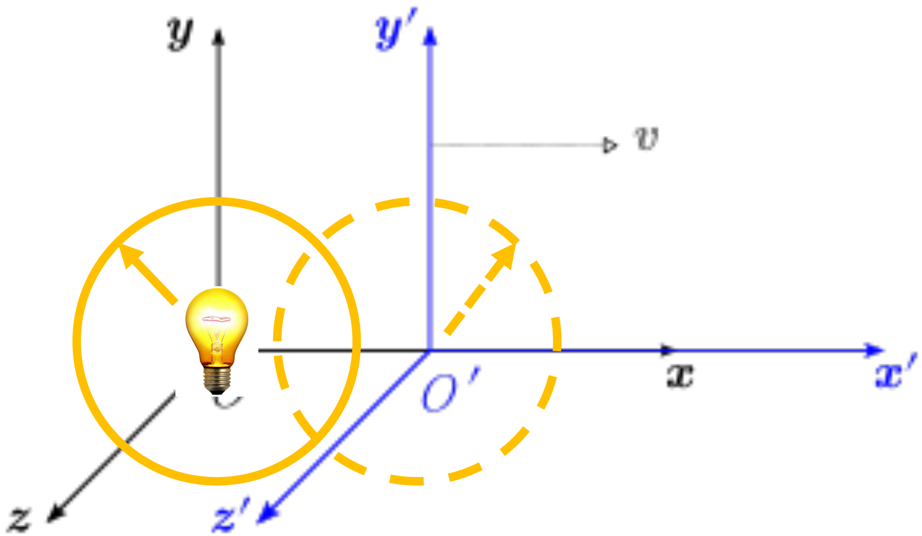
- Einstein's postulate (1905): **the speed of light is the same in all inertial frames** (regardless of the motion of the source/observer)
- The Galilean transformations are wrong!



https://www.patana.ac.th/secondary/science/anrophysics/relativity_option/commentary.html

Lorentz transformations

- How should we transform the co-ordinates of events?
- Imagine sending out a light signal from the origin:



In S: $$x^2 + y^2 + z^2 = (ct)^2$$

In S’: $$x'^2 + y'^2 + z'^2 = (ct')^2$$

Lorentz transformations

- These equations are satisfied by the **Lorentz transformations**:

$$t' = \gamma(t - vx/c^2)$$
$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$

$$t = \gamma(t' + vx'/c^2)$$
$$x = \gamma(x' + vt')$$
$$y = y'$$
$$z = z'$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

Simultaneity

- The Lorentz transformations have some remarkable consequences

- First, consider two events that are simultaneous in S $(t_1 = t_2)$. By the Lorentz transformations:

$$t_1' - t_2' = \frac{\gamma v}{c^2}(x_2 - x_1)$$

- **These events are not simultaneous in S'!**

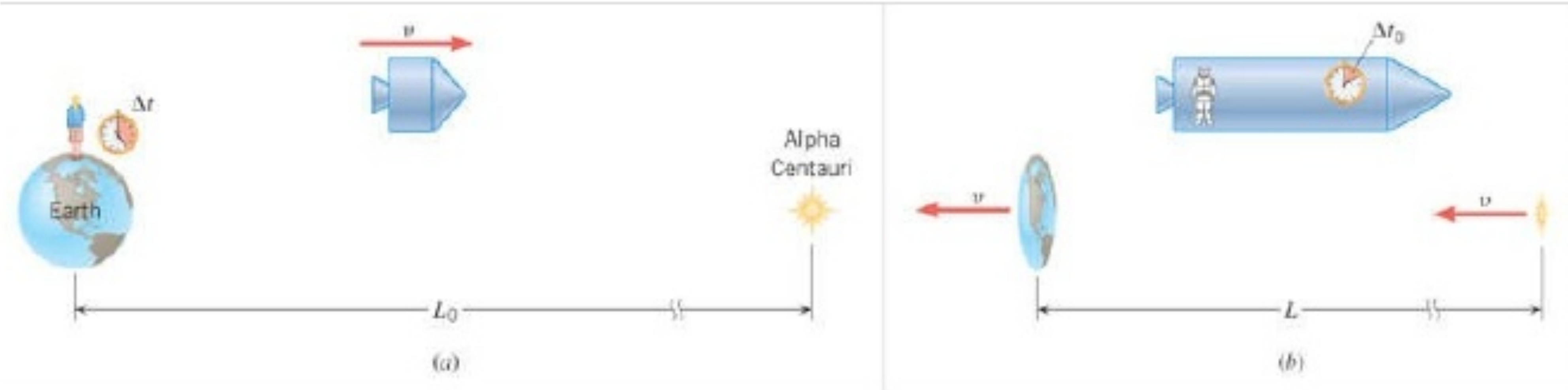
- There is no such thing as absolute time, the order of events depends on the reference frame!

Length contraction

- Consider a stick at rest in S' with ends at (x_1', x_2')
- Observers in S measure its length by recording the end positions (x_1, x_2) simultaneously in S. They find:

$$x_2 - x_1 = (x_2' - x_1')/\gamma$$

- **The length of a moving object is contracted by γ**



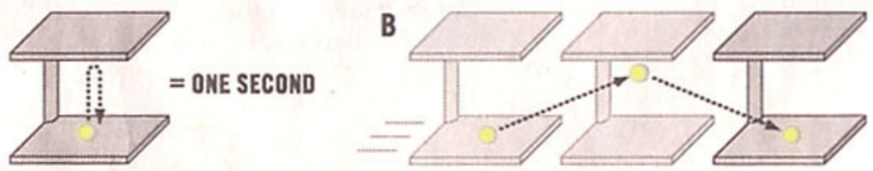
https://www.slideshare.net/cscottthomas/ch-28-special-relativity-online-6897295

Time dilation

- A clock at the origin of S' ticks at times (t_1', t_2')
- The spacing of these ticks in S is:

$$t_2 - t_1 = \gamma(t_2' - t_1')$$

- **A moving clock ticks at longer intervals**


HOW DOES MOTION SLOW THE PASSAGE OF TIME?

A hypothetical clock consists of two parallel mirrors with a photon (a ball of light) that bounces between them (A). If the mirrors are the right distance apart, the photon will complete the round-trip journey in one second. However, if the clock is moving (B), the photon travels on a diagonal path – a longer distance – and hence (since the speed of light is constant) takes longer to bounce between the mirrors. From our perspective, each second on the moving clock is thus longer than on the stationary clock.

https://patriceayme.wordpress.com/2014/06/17/time-dilation/

Space-time interval

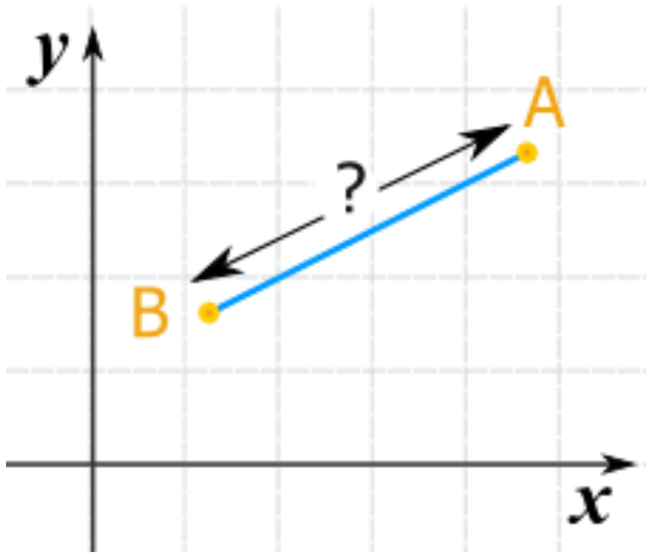
- The Lorentz transformation has a special property for how the difference in the space-time co-ordinates of two events transform between S and S':

$$-c^2\Delta t^2 + \Delta x^2 + \Delta y^2 + \Delta z^2 = -c^2\Delta t'^2 + \Delta x'^2 + \Delta y'^2 + \Delta z'^2$$

- This combination is the **space-time interval $\boldsymbol{\Delta s^2}$**
- It is an **invariant** (has the same value in all frames)

Importance of invariants

- **Invariants** – quantities which all observers agree to be the same – are useful!
- In Euclidean geometry, the distance between 2 points is the same if you rotate co-ordinates



https://www.mathsisfun.com/algebra/distance-2-points.html

The **space-time interval in relativity** (same in all inertial frames) is analogous to a **distance in Euclidean geometry** (same in all rotated frames)

Classifying intervals

- The invariant space-time interval, $\Delta s^2 = -c^2\Delta t^2 + \Delta x^2 + \Delta y^2 + \Delta z^2$, may be used to **classify pairs of events**
- Events for which $\Delta s^2 < 0$ are **timelike** (“time wins”) – an inertial observer can pass through them both. The time between the events in this frame is the **proper time** $\Delta\tau$
- Events for which $\Delta s^2 > 0$ are **spacelike** – there’s an inertial frame in which they are simultaneous. The separation between the events in this frame is the **proper separation**.
- Events for which $\Delta s^2 = 0$ are **lightlike** – they occur along the same light ray

Relativistic momentum

- Conservation of momentum can be preserved if we **modify the concept of the mass of a particle such that it depends on velocity**

$$m(v) = \gamma m_0 = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- We have introduced the **rest mass $\boldsymbol{m_0}$**

- As $v \to 0$ we recover classical momentum $p = m_0 v$

Relativistic energy

- Considering the work done on a particle $\int \frac{dp}{dt} dx$ as the energy gained, we recover a modified expression for relativistic energy

$$E = \gamma m_0 c^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- Particles have a **rest energy $\boldsymbol{m_0 c^2}$**
- As $v \rightarrow 0$ we find $E = m_0 c^2 + \frac{1}{2} m_0 v^2$
- Mathematically we find: $E^2 - p^2 c^2 = (m_0 c^2)^2$