GALAXY BULGES AND THEIR MASSIVE BLACK HOLES: A REVIEW

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ABSTRACT

With references to both key and oft-forgotten pioneering works, this article starts by presenting a review into how we came to believe in the existence of massive black holes at the centres of galaxies. It then presents the historical development of the near-linear (black hole)–(host spheroid) mass relation, before explaining why this has recently been dramatically revised. Past disagreement over the slope of the (black hole)–(velocity dispersion) relation is also explained, and the discovery of substructure within the (black hole)–(velocity dispersion) diagram is discussed. As the search for the fundamental connection between massive black holes and their host galaxies continues, the competing array of additional black hole mass scaling relations for samples of predominantly inactive galaxies are presented.
Keywords:

1. OVERVIEW

Arguably one of the most exciting aspects of galaxy bulges are the monstrous black holes which reside in their cores, sometimes lurking quietly, other times beaming out their existence to the Universe. Not only are they the dominant species on the mass spectrum of individual objects, but they play host to such a range of extremely unusual phenomenon that they appeal to people of all ages and professions.

For extragalactic astronomers, one curious aspect is the apparent coupling between the mass of the black hole, $M_{\rm bh}$, and the host galaxy bulge or spheroid, $M_{\rm sph}$, within which it resides. The importance of this is because it suggests that the growth of the two is intimately intertwined, and unravelling this connection will provide insight into their co-evolution. While the $M_{\rm bh}$–$M_{\rm sph}$ relation may arise from black hole feedback processes such that the black hole regulates the growth of the surrounding spheroid (a remarkable feat given the factor of a billion difference in physical size), correlations between both the central radial concentration of stars and the central stellar density of the spheroid with $M_{\rm sph}$ might be telling us that it is instead the spheroid mass which (indirectly) dictates the black hole mass through these relations.

This article starts by providing a background briefing to the development of ideas (since Einstein introduced his theories of relativity) which have led to our current understanding of supermassive black holes in galactic nuclei (Section 2), and the eventual observational proof which ruled out alternative astrophysical suggestions for the dark mass concentrations identified there (Section 3). Some effort has been made to reference key papers and give credit to the original developers of ideas and solutions, of whom many have been poorly cited in the literature to date.

Not surprisingly, many reviews have been written about supermassive black holes, and far more than the author was aware when approached to write this review. Enjoyable reports are provided by Kormendy & Richstone (1995) and Longair (1996), and an impressively extensive overview can be found in Ferrarese & Ford (2005), which also remains highly relevant today. In it, they too provide an historical account of active galactic nuclei (AGN), detail the many methods used to measure the masses of black holes today, and compare the demographics of black holes in distant quasars with local galaxies. It is however their section 9, pertaining to the scaling relations between the masses of black holes and the properties of their host galaxy that is the main focus of this article. For references to other aspects of massive black holes, over the past decade or so the following astrophysical reviews have focussed on: Sagittarius A* (Alexander 2005; Genzel et al. 2010); intermediate mass black holes (Miller & Colbert 2004; van der Marel 2004); massive black hole binaries (Merritt & Milosavljević 2005); AGN activity and feedback (Brandt & Hasinger 2005; Ho 2008; McNamara & Nulsen 2007), including hot accretion flows (Yuan & Narayan 2014) and cold accretion flows (Kato 2008; Abramowicz & Fragile 2013); connections with distant AGN (Shankar 2009a); redshifted fluorescent iron lines (Reynolds & Nowak 2003; Miller 2007); gravitational radiation (Berti et al. 2009, see also Amaro-Seoane et al. 2012); black hole spin (Gammie et al. 2004; Reynolds 2013); black hole seeds (Volonteri 2010, see also Koushiappas et al. 2004); and a healthy mix of various topics (e.g. Kormendy & Ho 2013; Genzel 2014) as in Ferrarese & Ford (2005).

As noted by Ferrarese & Ford (2005), in 2004 direct black hole mass measurements were known for 30 galaxies, plus another 8 galaxies for which the dynamical models might be in error. Recently, Savorgnan & Graham (2014), see also Kormendy & Ho (2013), tabulate 89 galaxies with reliably measured black hole masses. Not only has the sample size therefore tripled over the past

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decade, but new scaling relations have been uncovered and old relations have been revised — and dramatically so as we shall see in the case of the $M_{\rm bh}$–$M_{\rm sph}$ and $M_{\rm bh}$–$L_{\rm sph}$ relations (Section 4). The $M_{\rm bh}$–σ relation, involving the velocity dispersion of the galactic host, is reviewed in Section 5 and the controversial issue of its slope addressed. The apparent substructure in the $M_{\rm bh}$–σ diagram, reported in 2008 due to barred galaxies and/or pseudobulges, is additionally discussed.

Having dealt in some detail with the two most commonly cited black hole scaling relations in Sections 4 and 5, the assortment of related relations are presented. While not as popular in the literature, it may be one of these relations which provides the fundamental, or at least an important, link between the black hole mass and its host galaxy (an issue raised by Alexander & Hickox 2012). Therefore, Section 6 examines the connection between the black hole mass and the host spheroid's Sérsic index, i.e. how radially concentrated the spheroid's stellar distribution is; this dictates the radial gradient of the gravitational potential. Section 7 describes the expected association between the black hole mass and the central stellar density (prior to core depletion). Section 8 explores the link between the mass of the black hole and the missing stellar mass at the centres of giant spheroids. The connection between the black holes and the dense star clusters found in the nuclei of many galaxies — some of which may harbour intermediate mass black holes — is presented in Section 9. Section 10 discusses the black hole mass relation with the halo (baryons plus dark matter) mass, expected to exist for spheroid dominated galaxies, while Section 11 remarks on the existence of a correlation with the pitch angle of spiral arms in late-type galaxies. Finally, Section 12 considers the possibility that a third parameter may account for some of the scatter in the above bivariate distributions, leading to a more fundamental plane or hypersurface in 3-parameter space involving black hole mass and two galaxy/spheroid parameters.

2. HISTORICAL DEVELOPMENT: FROM MATHEMATICAL SPECULATION TO WIDESPREAD SUSPICION

Karl Schwarzschild (1916, 1999; see also Droste 1917 who independently derived the same solution in 1916) is widely recognised for having developed the 'Schwarzschild metric' for a spherical or point mass within Einstein's (1916) theory of general relativity[2], but it was Finkelstein (1958, see also Kruskal 1960) who realised the true nature of what has come to be called the "event horizon" bounding these gravitational prisons. Finkelstein eloquently describes this Schwarzschild surface as "a perfect unidirectional membrane: causal influences can cross it but only in one direction". Five years later, while working at the University of Texas, the New Zealand mathematician Roy Kerr (1963) formulated the metric for the more realistic[3] *rotating* black hole. Interestingly, solutions to this space-time include closed time-like curves which, in theory, allow one to travel backwards in time (a concept popularised but also questioned by Thorne 1994). Kurt Gödel (1949) was actually the first to derive such strange solutions to the equations of general relativity, although it is commonly suspected that all closed time-like curves are just a mathematical artifact, in the same way that the original singularity at the Schwarzschild radius was later explained away by a coordinate transformation (e.g. Eddington 1924; Georges Lemaître 1933), leaving just the singularity (i.e. black hole) at the centre. But even if we are to be denied our time machines[4], black holes still offer the curious and unsuspecting property of evaporating over time — radiating like a black body — before possibly then exploding (Hawkings 1974, 1975).

Evolving parallel to the above analytical developments, our acceptance of black holes as more than just a mathematical curiosity had additional connections with stellar evolution and dark stars[5]. As detailed by Yakovlev (1994), the Soviet physicist Yakov Frenkel (1928) was the first to derive equations for the energy density and pressure of super-dense stars comprised of a degenerate Fermi-gas of electrons of arbitrary relativistic extent. He is, however, not widely recognised for having done so. Also using results from Albert Einstein's (1905) theory of special relativity, Soviet physicist Wilhelm Anderson (1929) was the first to derive a maximum mass for the fermion degenerate stellar model of white dwarf stars, above which the Fermi pressure is insufficient to overcome gravity. It is however the British physicist Edmund Stoner (1929) who is somewhat better known for having presented the structure for the mass, radius and density of white dwarf stars composed of non-relativistic electrons. Using his uniformly distributed mass density model, Stoner (1930, see also Stoner 1932a,b) refined his work by formulating how the core becomes relativistic at sufficiently high densities (as had already been done by Frenkel 1928) and he too predicted a maximum stable mass (similar to Anderson 1929) for earth-sized, white dwarf stars. But it is Chandrasekhar (1931a, see also Chandrasekhar 1931b) who is well known for calculating, in a short two-page article using *polytropic density models*, that at masses above $\approx$0.91 $M_\odot$, electron-degenerate white dwarf stars are not stable. That is, there is a maximum mass (recognised today as 1.4 $M_\odot$) that white dwarf stars can have. If more massive than this limit then they must undergo further gravitational compression. Soon after, Soviet physicist Lev Landau (1932) correctly identified that the next level of resistance to their gravitational collapse would be met in the form of the denser neutron star (see also Oppenheimer & Serber 1938; Oppenheimer & Volkoff 1939). Landau (1932) and Chandrasekhar (1932, 1935)[6] predicted that the ultimate fate of an evolved massive star would be to collapse to a

[2] It is of interest to note that Einstein was not keen on the idea of singularities, and in Einstein (1939) he wrote that *"The essential result of this investigation is a clear understanding as to why the 'Schwarzschild singularities' do not exist in physical reality"*.

[3] Collapsing stars, and (accretion disc)-fed black holes, are expected to have substantial angular momentum.

[4] Time-travel enthusiasts might appreciate a nod to the hypothetical Einstein-Rosen (1935) bridge (aka "wormhole", a term introduced by John Wheeler in 1957, e.g. Misner & Wheeler 1957, and Klauder & Wheeler 1957) which are warped regions of space-time within general relativity (Morris & Thorne 1988a,b; Hawking 1988). There is additionally the cosmic string time machine of Gott (1991).

[5] John Michell (1783) was the first to calculate the existence of black holes, which he termed "dark stars", whose gravity was so strong that light would not be able to escape from their surface.

[6] Miller (2005) details the early work of Chandrasekhar on this topic.

singularity of infinite density[7]. Following further work on this idea (e.g. Baade & Zwicky 1934; Zwicky 1938; Datt 1938), Oppenheimer & Snyder (1939) carefully detailed how overly massive neutron stars are not stable and will collapse into stellar mass black holes. Quite simply, if a star is massive enough and the outward pressure from fusion is over, gravity will win over (e.g. Arnett 1967).

Wheeler (1966) wrote *"In all the physics of the post-war era it is difficult to name any situation more enveloped in paradox than the phenomenon of gravitational collapse"*. Then, in the following year (1967), more than three decades after the initial prediction of neutron stars, pulsars were discovered, finally signalling the existence of neutron stars (Hewish et al. 1968; Pilkington et al. 1968; Hewish 1970). Not surprisingly, this bolstered belief in the existence of stellar mass black holes (e.g. Vishveshwara 1970; Penrose 1969), as did (i) mathematical proof that a singularity will form if an event horizon has formed (Penrose 1965; Hawking & Penrose 1970), (ii) the X-ray pulses from Cygnus X-1 (Oda et al. 1971; Thorne & Price 1975), and likely also (iii) the pioneering searches by Weber (1969, 1970) for gravitational radiation coming from even more massive objects at the centre of our Galaxy.

As detailed by Longair (1996, 2010), Ferrarese & Ford (2005) and Collin (2006), the notion that the centres of galaxies may contain massive black holes, millions to hundreds of millions times the mass of our Sun, stems from the discovery of the great distance to, and thus luminosity of, the quasi-stellar radio source 3C 273. The optical counterpart of this radio source was cleverly discovered by Hazard, Mackey & Shimmins (1963) using the Parkes radio telescope and lunar eclipsing. Its redshift was subsequently taken with the Palomar Observatory's Hale telescope and correctly interpreted by Schmidt (1963), see also Oke (1963) regarding 3C 273 and Greenstein & Matthews (1963a,b) in the case of 3C 48 (whose redshift had remained uninterrupted over the preceding couple of years).

Baade & Minkowski (1954), Ambartsumian (1958), Woltjer (1959), Burbidge et al. (1959, 1963, 1964), Lynds & Sandage (1963) and others had already recognised active galactic nuclei (AGN) to be incredibly energetic phenomena.[8] Radio galaxy 3C 273 and other active galactic nuclei emit vasts amount of energy from a small volume of space (as indicated by quasar variability on short time scales, Smith & Hoffleit 1963)[9] and were thus thought to be powered from the gravitational potential energy[10] released as matter falls onto a compact massive object (Salpeter 1964; Zel'dovich 1964; Zel'dovich & Novikov 1964; Ne'eman 1965; Shakura & Sunyaev 1973)[11]. Based upon Eddington-limiting arguments at the time, it was immediately realised that the central object has to be massive or else the radiation pressure of the quasar would literally blow the quasar apart. Hoyle et al. (1964) acknowledged the possibility of "invisible mass" perhaps from imploded objects of very large mass[12].

Just two years after the high-redshifts were recorded for the star-like[13] radio sources, Sandage (1965) reported on the high abundance of radio-quiet quasars, referring to them as a "major new constituent of the universe". What he had revealed was that in addition to the radio-loud quasars, the Universe was teeming with many more quasars. Encapsulating the ideas of recent years, Lynden-Bell (1969) and Lynden-Bell & Ress (1971) suggested that a massive black hole resides at the cores of many galaxies (see also Wolfe & Burbidge 1970), and that the infall of orbital matter builds an accretion disc (e.g. Thorne 1974) which heats up due to friction. For a rapidly spinning black hole, this process can liberate a substantial fraction (up to 0.42 for a maximally spinning black hole) of the infalling matter's rest mass energy[14] (Bardeen & Wagoner 1969; Bardeen 1970). Further support for the presence of massive black holes were the linear radio features emanating from the nuclei of galaxies — which were likely emitted from a stable gyroscope such as a spinning black hole — and the superluminal speed of these radio jets (e.g. Cohen et al. 1971; Whitney et al. 1971).

The moniker "black hole" was used by Ann Ewing (1964) just a year after the redshift of 3C 273 was announced. She reportedly heard it at a meeting of the American Association for the Advancement of Science (AAAS), and it was later seen used in a scientific paper by John Wheeler (1968)[15]. By 1970 the label appears as a familiar term in the literature. It had, at times, previously been used to describe dusty dark patches in our own Galaxy (e.g. Barnard 1897; Campbell 1917). However it became the popular replacement for what the Soviet physicists (e.g. Zeldovich 1964) called a frozen star[16], and Western physicists called a collapsed star or a "col-

[7] In passing, it is noted that quark stars (Ivanenko & Kurdgelaidze 1965) are also expected to have a stable configuration, en route between neutron stars and black holes.

[8] While relatively low-luminosity Seyfert (1943) galaxies were of course already known in 1963, it was not yet fully appreciated that quasars are their high-energy kin, although similarities were noted by Burbidge et al. (1963) and Burbidge (1964).

[9] Reviews of AGN and their variability are given by Mushotzky et al. (1993), Ulrich et al. (1997), and Peterson (1997).

[10] As stated by Rees (1998), the black hole's gravitational well "must be deep enough to allow several percent of the rest mass of infalling material to be converted into kinetic energy, and then radiated away from a region compact enough to vary on timescales as short as an hour."

[11] Like many capable theorists, Zel'dovich and Novikov did not restrict themselves to one theory, and in Bisnovatyi-Kogan, Zel'dovich & Novikov (1967) they proposed that quasars may be billion solar mass stars burning brightly for tens of thousands of years, see also Hoyle & Fowler (1963), while Novikov (1965) additionally advocated what we now know as 'white holes'.

[12] Hoyle & Burbidge (1966) also speculated that quasars may be nearby objects and that their redshifts do not necessarily reflect the expansion of the universe, see also Hoyle et al. (2000) and Burbidge et al. (2006).

[13] Faint halos had been reported around some of these 'star-like' objects, which we now know is due to the host galaxy surrounding the bright AGN (e.g. Gehren et al. 1984; Hutchings et al. 1984, and references therein).

[14] For comparison, nuclear fusion is known to release less than 1% of the rest mass energy (0.7% in the conversion of hydrogen to helium) and thus 'super-stars' are not as efficient sources of energy as rapidly spinning accretion discs around supermassive black holes.

[15] John Wheeler is first recorded to have used the term "black hole" at his 27 December, 1967, AAAS invited lecture, a few years after Ann Ewing. However, given that he coined the term "worm hole", it seems likely that he also introduced the expression "black hole", although the author does not rule out that it may have been Fritz Zwicky.

[16] For an external observer, time appears to stop inside the Schwarzschild radius, giving rise to the term "frozen star", because the collapse of a star will appear to freeze once the star is within the event horizon.

lapsar" (e.g. Cameron 1971). The term "singularity" had also been, and still is, regularly used by the mathematicians to indicate where any quantity in the field equations becomes infinite.

Despite its strangely endearing name, the phrase "black hole" is often noted to be somewhat unfortunate in that it implies a *hole* in space through which matter may fall through. The idea of an actual singularity — a point of infinite density which arises out of classical physics after division by 0 — is also not popular and considered rather old-school. While a Planck-sized mote may be a better description, what actually exists near the centre of a black hole's event horizon is hotly debated. Mathematically-inclined readers who are interested in what a black hole may be like, might enjoy reading about the 'fuzzball' picture from string theory ('t Hooft 1980; Mathur 2005, and associated references), or descriptions of black holes in quantum gravity theories such as spin foam networks (e.g. Penrose 1971a,b; Penrose & Rindler 1986; Rovelli 1998; Domagala & Lewandowski 2004; Perez 2004) or loop quantum gravity (e.g. Ashtekar & Bojowald 2005; Hayward 2006).

3. ON FIRMER GROUND

Acceptance of the idea that supermassive black holes reside at the centres of galaxies was not as straight forward as suggested above. During the 1960s and 1970s the AGN community battled it out amongst themselves before (largely) embracing the idea that black holes must be required to power the quasar engines of galaxies. Building on Sandage (1965), Soltan (1982) reasoned that there had to be a lot of mass locked up today in massive black holes because of all the past quasar activity, and Rees (1984) advocated further for the preponderance of massive black holes in the nuclei of galaxies. Then during the 1980s and early 1990s it was primarily the inactive-galaxy community, as opposed to the AGN-community, who remained skeptical until two key papers in 1995 (discussed shortly).

Among the pioneering observational papers for the presence of a massive black hole in individual, nearby, non-AGN galaxies, Sanders & Lowinger (1972) calculated that the Milky Way houses a $0.6 \times 10^6 M_\odot$ black hole and Sargent et al. (1978) concluded that a $5 \times 10^9 M_\odot$ black hole very probably exists in M87 (see also Lynden-Bell 1969 who predicted a $30 \times 10^6 M_\odot$ black hole for the Milky Way, and a 40×10^9 black hole in M87, i.e. an order of magnitude higher). Although these works had revealed that very high masses in small volumes were required at the centres of these galaxies (see also Dressler 1984 and Tonry 1984 in the case of M31 and M32, respectively), it took some years before the observations / measurements improved and alternatives such as a dense cloud of stellar mass black holes or neutron stars could be ruled out. The three following observational works turned the tide of opinion among the remaining naysayers who demanded further proof before accepting the existence of what is indeed an extreme astrophysical object: the supermassive black hole.

1) Before an object crosses within a black hole's event horizon, any radiation it emits away from the black hole will be gravitationally redshifted, the extent of which depending on how close the object is to the event horizon. Such a tell-tale signature of redshifting was reported on 22 June 1995 by Tanaka et al. (1995) who detected the highly broadened, ionised iron Kα line (6.4 keV) from the galaxy MCG-6-30-15. This highly asymmetric, predominantly redshifted, X-ray emission line had a width corresponding to roughly one-third of the speed of light, and was thought to have been emitted at just 3 to 10 Schwarzschild radii from the black hole. Such relativistic broadening has since been shown to be commonplace (Nandra et al. 1997), thanks to the enhanced sensitivity and spectral resolution of the Japanese ASCA X-ray satellite (Tanaka et al. 1994).

2) Additional convincing evidence for the reality of massive black holes had came from the very high mass density required to explain the central object in the Seyfert galaxy NGC 4258 (M106). Using the Very Long Baseline Array in New Mexico, Miyoshi et al. (1995) showed that the H_2O maser emission from this galaxy originates from a thin, rotating nuclear gas disc/annulus displaying a clear Keplerian rotation curve and requiring a mass of $3.6 \times 10^7 M_\odot$ within a size of just 0.13 parsec[17] (see also Haschick et al. 1994, Watson & Wallin 1994, and Greenhill et al. 1995a,b). In their January 12 paper, Miyoshi et al. (1995) note that the short collisional timescale ($< 10^8$ years) for a swarm of solar mass dark stars with such density ($> 4 \times 10^9 M_\odot$ pc^{-3} inside of the inner 4.1 milliarcseconds) implies that such a hypothetical star cluster could not survive (see also Maoz 1995, 1998); a single supermassive black hole is the only viable candidate. A second example of extreme mass density ($3.2 \pm 0.9 \times 10^8 M_\odot$ pc^{-3}) has since been shown in the Circinus galaxy by Greenhill et al. (2003).

3) Several years later, high spatial resolution measurements of stellar orbits around the central object in our own Milky Way galaxy also eventually ruled out the possibility that it could be a swarm of neutron stars or stellar mass black holes, with the high density favouring the existence of a massive black hole (Schödel et al. 2002; Ghez et al. 2005, 2008; Gillessen et al. 2009). confirming earlier suspicions (Lacy et al. 1979, 1980; Eckart & Genzel 1996, 1997; Genzel et al. 1996, 1997; Ghez et al. 1998; see also Alexander 2005 and references therein).

As was appropriately emphasized by Merritt & Ferrarese (2001b), within the black hole's sphere-of-influence — whose radius is defined as $r_{\rm infl} = GM_{\rm bh}/\sigma_{\rm sph}^2$ (e.g. Peebles 1972; Frank & Rees 1976) where $\sigma_{\rm sph}$ is roughly the host spheroid's velocity dispersion immediately beyond $r_{\rm infl}$ — one expects to find Keplerian dynamics which are dominated by the black hole. The velocity dispersion of the stars (or the rotational velocity of a relatively lighter disc, as in the case of NGC 4258) inside $r_{\rm infl}$ should thus decline with the inverse square root of the radius, i.e. $\sigma(R) \propto R^{-0.5}$, just as rotational velocities of Keplerian discs or solar systems have $v_{\rm rot} \propto 1/\sqrt{R}$.

The absence of this clear detection for many galaxies has led Merritt (2013) to question their reported black hole measurements, which may be better interpreted as upper limits until we are better able to resolve the sphere-of-influence (see also Valluri, Merritt & Emsellem 2004). With this cautionary note, we proceed to the topic of black hole scaling relations, which at the very least would still be upper envelopes in the various diagrams of black

[17] Based on a galaxy distance $D = 6.4$ Mpc.

Figure 1. Artist's impression of the horror at a galactic centre. Credit: Gabriel Pérez Díaz. [To appear in published version.]

hole mass versus host spheroid properties. It may however then be unusual that all of the clear-cut examples for a definitive black hole reside on this upper envelope (but see Ford et al. 1998; Ho 1999, his section 7; and Batcheldor 2010a).

4. THE $M_{\rm BH}$–$L_{\rm SPH}$ AND $M_{\rm BH}$–$M_{\rm SPH}$ RELATIONS

Commenting on the ratio of black hole mass to spheroid mass in M31 and M32, Dressler & Richstone (1988) suspected a relation, and used it to predict billion solar mass black holes in bright elliptical galaxies, While the prediction was not new, in the sense that the authors were aware that past theoretical papers had stated that quasars in big elliptical galaxies could have 10^9 solar mass black holes (e.g. Rees 1984; Begelman, Blandford & Rees 1984, and references therein), the idea of a scaling relation with the *spheroid* does seem to be new[18]. Dressler (1989) further advocated this connection between the black hole and the host spheroid (not the disc), and from a sample of 5 galaxies he noted that there is a "rough scaling of black hole mass with the mass of the spheroidal component".

This differed slightly from Hutchings et al. (1984) who had reported that the "black hole [] mass is related to that of the galaxy, increasing 60% faster than that of the galaxy". The study by Hutchings et al. (1984) was of poorly resolved, distant quasars which prevented them from performing a bulge/disc decomposition and as such they did not report on a black hole mass relation with the host spheroid. However, there is an upper limit to the brightness of quasars which has been observed to scale with the brightness of the host galaxy (which are typically spheroid-dominated for the brightest quasars). Using real data, Yee (1992) fit a linear relation to this limit, which he called the $M_{\rm QSO}$–$M_{\rm G}$ relationship, and wrote that "it may arise due to a correlation of the mass of the central engine and the galaxy mass", such that "the brightest quasars for a given galaxy mass are the ones shining at or near the Eddington limit (which is set by the mass of the central engine), while others are at lower luminosities"[19]. As noted by McLeod (1997, see also McLeod et al. 1999), Yee (1992) had effectively discovered the linear, high-mass end of the $M_{\rm bh}$–$M_{\rm sph}$ distribution.

With three more galaxies than Dressler (1989), Kormendy & Richstone (1995, see also Kormendy 1993) wrote a review article in which they plotted this data and reiterated in mathematical form what Dressler had said, and Yee (1992) had shown for massive bulges, i.e. $M_{\rm bh} \propto M_{\rm bulge}$. While they did not fit a relation to the data, they did report a mean $M_{\rm bh}/M_{\rm bulge}$ ratio of 0.22% (including the Milky Way) and thereby effectively created a more quantitative basis for a linear $M_{\rm bh}$–$M_{\rm bulge}$ relation.

Following the prediction by Haehnelt & Rees (1993) that ≈30% of nearby galaxies likely house a central massive black hole, Kormendy & Richstone (1995) remarked that at least 20% of nearby galaxies possess such a black hole – while noting that alternatives such as massive concentrations of dark stars could not yet be ruled out. Magorrian et al. (1998) built on this and suggested that *most* nearby galaxies harbour a massive black hole (see also Sigurdsson & Rees 1997, and the reviews by Ford et al. 1998 and Richstone et al. 1998), supporting the strong suspicion held by many (e.g. Blandford 1986; Rees 1990). Moreover, this followed closely on the heels of the observation that many quiescent galaxies have weak central radio sources (e.g. Keel 1985; Sadler et al. 1989, 1995; Ho et al. 1997), likely signalling low-level accretion onto near-dead quasars.

Rather than the pure 'linear' scaling, a single power-

[18] Jarvis & Dubath (1988) appear to have also picked up on this connection when they wrote in regard to M31 and M32 that: "The likely presence of black holes in two of the closest galaxies with bulge-like components compels us to look at the nuclei of other nearby or large galaxies", which they did for the Sombrero galaxy (NGC 4594).

[19] In passing, and as noted by Alexander & Natarajan (2014), it is possible to exceed the Eddington limit to black hole growth (as noted by Begelman 1979 and Soffel 1982), due to an effective gas drag of the photons. Moreover, non-spherical accretion in the form of a disc can also result in black hole growth superseding the Eddington limit (Nayakshin et al. 2012).

law relation was introduced by Magorrian et al. (1998) to describe the distribution of 32 points in the $M_{\rm bh}$–$M_{\rm bulge}$ diagram, such that the log-linear slope was 0.96±0.12 (which is of course still consistent with a slope of 1 and thus a linear relation)[20]. In other works, using variously updated masses and samples, Ho (1999) reported a median $M_{\rm bh}/M_{\rm bulge}$ ratio of 0.2%, and Merritt & Ferrarese (2001c) and Kormendy & Gebhardt (2001) reported a ratio of 0.13%, although with notable scatter. McLure & Dunlop (2002) noticed that the scatter was considerably reduced once the disc galaxies were excluded, suggestive of poor bulge/disc decompositions used to estimate the bulge masses. Marconi & Hunt (2003) subsequently performed careful bulge/disc decompositions on near-infrared K-band images, less effected by dust and star formation. They also showed that the dynamical/virial mass of the spheroid correlated linearly with the black hole mass, and Häring & Rix (2004) provided improved dynamical masses for the derivation of their near-linear relation. For the next decade, studies of the $M_{\rm bh}$–$L_{\rm bulge}$ and $M_{\rm bh}$–$M_{\rm bulge}$ diagram remained dominated by high-mass galaxies[21] having $M_{\rm bh} >\sim 0.5 \times 10^8 M_\odot$ and, despite each paper's incremental improvements, continually recovered a single, near-linear $M_{\rm bh}$–$M_{\rm bulge}$ relation (e.g. Ferrarese & Ford 2005; Lauer et al. 2007; Graham 2007b, 2008a, his section 6; Gültekin et al. 2009; Sani et al. 2011; Beifiori et al. 2012; Erwin & Gadotti 2012; Vika et al. 2012; van den Bosch et al. 2012; McConnell & Ma 2013; Rusli et al. 2013a). A recent notable exception has been Läsker et al. (2014b) who advocate, with a near-infrared sample of 35 galaxies, that the black hole mass correlates equally well with the total (bulge plus disc) luminosity as it does with the bulge luminosity at 2.2 μm, and that one has $M_{\rm bh} \propto L_{\rm bulge}^{0.75\pm0.10}$ and $M_{\rm bh} \propto L_{\rm galaxy}^{0.92\pm0.14}$. They attribute this to the smaller bulge fluxes obtained from their decomposition of the galaxies' light and the type of linear regression performed. The inclusion of more data will however be welcome, and Savorgnan et al. (2015, in prep.) will double the sample size.

There were, however, a few early deviations from the above (near) convergence of opinion on a linear relation that should be noted. First, while the Abstract of Laor (1998) largely supports the linear relation of Magorrian et al. (1998), the main text reports that $M_{\rm bh} \propto M_{bulge}^{1.5-1.8}$ (although it suggests that this may be partly due to the fact that all their lower mass quasar hosts are disc galaxies for which they may have over-estimated the bulge mass) and Second, it also notes that the low-mass inactive galaxies from Magorrian et al. (1998) better match their steeper $M_{\rm bh}$–$M_{\rm bulge}$ relation than the linear one. Third, Wandel (1999) reported a mean $\log(M_{\rm bh}/M_{\rm bulge})$ ratio of -3.5 for a sample of Seyfert galaxies with black hole masses predominantly less than $10^8\, M_\odot$. This is 0.6 dex, i.e. a factor of 4, smaller than reported by Merritt & Ferrarese (2001c) and Kormendy & Gebhardt (2001) who used a sample with ∼80% of the galaxies having $M_{\rm bh} > 0.8 \times 10^8\, M_\odot$. Wandel (1999) argued and wrote *"It is plausible, therefore, that the Seyfert galaxies in our sample represent a larger population of galaxies with low BBRs [black hole to bulge mass ratios], which is under-represented in the Magorrian et al. sample"*[22].

Fourth, while Wandel reported $M_{\rm bh} \propto L_{\rm bulge}^{1.4}$ (which equates to $M_{\rm bh} \propto M_{\rm bulge}^{1.2}$ when using the same $M/L \propto L^{0.18}$ relation as Laor 1998 and Magorrian et al. 1998), the data in Wandel (1999, their figure 1) reveal that a relation with a slope steeper than 1.4 would be likely from a *symmetrical* regression. Fifth, using upper limits for black hole masses, Salucci et al. (2000) reported on hints that the $M_{\rm bh}$–$M_{\rm bulge}$ relation is significantly steeper in spiral galaxies than in [massive] elliptical galaxies. Finally, Laor (2001) reinforced his claim that a steeper, single power-law seems more applicable than a linear relation, finding $M_{\rm bh} \propto M_{\rm bulge}^{1.53\pm0.14}$. Related to this, Ryan et al. (2007) further reveals that the linear $M_{\rm bh}$–$M_{\rm bulge}$ relation over-estimates the masses of black holes in low-mass Seyfert galaxies.

4.1. *A bend in the road*

Before beginning this section, it is necessary to introduce some nomenclature which may be unfamiliar to some readers. The term "Sérsic galaxy" or "Sérsic spheroid" shall be used to denote galaxies or spheroids (elliptical galaxies and the bulges of disc galaxies) whose surface brightness profile is well described by the Sérsic (1963, 1968) model all the way into the centre of the galaxy. Two decades ago Caon et al. (1993) demonstrated that the Sérsic model fits the surface brightness profiles of early-type galaxies remarkably well over a large dynamic range. An historical and modern review of Sérsic's model can be found in Graham & Driver (2005). Sérsic galaxies may contain additional nuclear flux components above that of the host Sérsic spheroid. The term "core-Sérsic galaxy" or "core-Sérsic spheroid" refers to a galaxy whose main spheroidal component has a partially-depleted core (i.e. a central stellar deficit of light that is not due to dust) such that the surface brightness profile is well described by the core-Sérsic model (Graham et al. 2003b). The history of galaxy surface brightness models and the impact that the above systematically (with luminosity) varying structures (i.e. non-homology and depleted cores) have on galaxy scaling laws and the unification of bright and faint early-type Sérsic galaxies is discussed at length in Graham (2013).

Re-analysing the dynamical spheroid mass and (updated) black hole mass data for 30 galaxies studied by Häring & Rix (2004), but this time separating the galaxies depending on whether or not they have a partially depleted core, Graham (2012a) found that the two populations follow different relations in the $M_{\rm bh}$–$M_{\rm sph}$ diagram. While the dozen core-Sérsic spheroids, which are the more massive spheroids, followed the near-linear relation

[20] As suspected by Magorrian et al. (1998), and noted by van der Marel (1999) and Gebhardt et al. (2000), their use of a two-integral distribution function which ignores radial velocity-dispersion anisotropy (see Binney & Mamon 1982) caused them to over-estimate the black hole masses by an average factor of 3–4.5

[21] Studies were also biased by the inclusion of one or two rare "compact elliptical" galaxies that do not represent the population at large.

[22] McLure & Dunlop (2001) correctly noted that a better bulge/disc decomposition reduces the observed flux attributed to the bulges by Wandel (1999), however the dust corrections which were not applied can largely cancel this reduction (compare figures 1 and 7 in Graham & Worley 2008).

$M_{\rm bh} \propto M_{\rm sph}^{1.01\pm0.52}$, the Sérsic spheroids followed a much steeper power-law relation, such that $M_{\rm bh} \propto M_{\rm sph}^{2.30\pm0.47}$. Excluding the barred galaxies, the Sérsic relation was $M_{\rm bh} \propto M_{\rm sph}^{1.92\pm0.38}$. This near-quadratic relation for the low- and intermediate-mass spheroids had never been shown before and it signalled a dramatic bend in the $M_{\rm bh}$–$M_{\rm sph}$ diagram.

With an increased sample size of 72 galaxies with directly measured black hole masses, Graham & Scott (2013) confirmed this behavior using near-infrared K_s-band magnitudes. Their sample of two dozen core-Sérsic spheroids gave $M_{\rm bh} \propto L_{\rm sph}^{1.10\pm0.20}$, while the four dozen Sérsic spheroids gave the relationship $M_{\rm bh} \propto L_{\rm sph}^{2.73\pm0.55}$, which reduced to $M_{\rm bh} \propto M_{\rm sph,dyn}^{2.34\pm0.47}$ when using $M_{\rm dyn}/L_K \propto L_K^{1/6}$ (e.g., Magoulas et al. 2012; La Barbera et al. 2010). Employing the ARCHANGEL photometry pipeline (Schombert & Smith 2012) applied to Two Micron All-Sky Survey images (Skrutskie et al. 2006), which effectively corrects for missing light at large radii, Scott et al. (2013) converted the K_s-band magnitudes of the spheroids into stellar masses. They found that $M_{\rm bh} \propto M_{\rm sph,*}^{0.97\pm0.14}$ and $M_{\rm bh} \propto M_{\rm sph,*}^{2.22\pm0.58}$ for the Sérsic spheroids and core-Sérsic, respectively.

We therefore now have a situation which is dramatically different to what was believed for the past two decades. It is not simply that we no longer have a single, near-linear $M_{\rm bh}$–$M_{\rm sph}$ relation for all spheroids, but the main growth phase of black holes and bulges, involving gas rich processes, follows a near-quadratic relation, with gas-poor "dry" mergers subsequently creating the core-Sérsic galaxies which depart from the high-mass end of this near-quadratic relation[23]. That is, the growth of massive black holes has been much more rapid than that of their host spheroids. Naturally, the simple addition of galaxies and their black holes, through dry merging, will establish the observed near-linear relation for the core-Sérsic galaxies. The average $M_{\rm bh}/M_{\rm sph}$ ratio of these core-Sérsic galaxies then reflects the value obtained at the high-mass end of the near-quadratic Sérsic $M_{\rm bh}$–$M_{\rm sph}$ relation from which they peeled off. In late 2012 Graham & Scott (2013) reported this mass ratio to be 0.49%, in agreement with that already noted by Laor (2001) for massive spheroids, and as subsequently noted in the review by Kormendy & Ho (2013). This ratio is basically the calibration for the Yee (1992) relation between black hole mass and galaxy mass in massive galaxies, modulo the fact that some core-Sérsic galaxies contain large discs. Furthermore, our own galaxy, with an $M_{\rm bh}/M_{\rm sph}$ ratio of 0.05%, is no longer a low outlying point requiring explanation in the $M_{\rm bh}$–$M_{\rm sph}$ diagram. It has a mass ratio in accord with the near-quadratic scaling relation for Sérsic spheroids.

Adding AGN data from half a dozen recent papers which had observed the AGN black hole masses to reside below the original $M_{\rm bh}$–$M_{\rm sph}$ relation, Graham & Scott (2014) revealed that they depart from the near-linear $M_{\rm bh}$–$M_{\rm sph}$ relation in a systematic manner consistent with the near-quadratic $M_{\rm bh}$–$M_{\rm sph}$ mass scaling relation for Sérsic galaxies. That is, they are not randomly offset. This is shown in Figure 2. This also provides the picture with which we can now interpret the observations by Laor (1998, 2001) and Wandel (1999), who were on the right track over a decade ago.

[23] Some Sérsic galaxies may follow the near-linear $M_{\rm bh}$–$M_{\rm sph}$ relation, having experienced a major dry merger event in which the nuclear star clusters from the progenitor galaxies have been eroded away but an obvious partially depleted core is not yet formed (see Bekki & Graham 2010). These may well be the galaxies at $-19.5 > M_B > -20$ mag in Côté et al. (2007. their figure 3e).

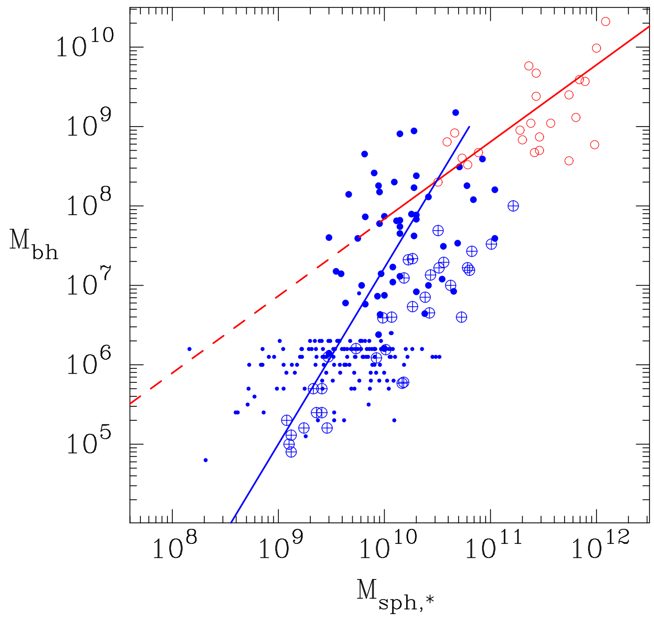

Figure 2. Black hole mass versus host spheroid's stellar mass (in units of solar mass). Core-Sérsic spheroids are shown with open red circles, while Sérsic spheroids are shown by the large blue dots. A sample of 139 low mass AGN from Jiang et al. (2011) are denoted by the small dots, while an additional 35 higher mass AGN (which may have had their host spheroid masses over-estimated by overly-high M/L_* ratios, see Busch et al. 2014) are denoted by the cross hairs. The optimal near-linear and near-quadratic scaling relations from Scott et al. (2013) are shown as the red (solid and dashed) and blue (solid) line for the core-Sérsic and Sérsic spheroids, respectively. Figure taken from Graham & Scott (2014).

If one was to separate the galaxies in Figure 2 at $M_{\rm bh} = 2 \times 10^6\, M_\odot$, one would (understandably but inappropriately) conclude that the lower mass spheroids do not follow an $M_{\rm bh}$–$M_{\rm sph,*}$ relation (Jiang et al. 2011). This had resulted in these lower mass spheroids being considered distinct by some, and sometimes labelled 'pseudobulges' as opposed to 'classical' bulges (Gadotti & Kauffmann 2009; Kormendy, Bender & Cornell 2011) with the separation said to occur at $n = 2$. This is also where the alleged divide between dwarf elliptical and ordinary elliptical galaxies was said to occur ($M_B = -18$ mag, $M_{\rm gal,*} \approx 2 \times 10^{10}\, M_\odot$ $n \approx 2$–2.5, $\sigma \approx 100$–120 km s^{-1}). However, without the fuller parameter baseline that we now have, or artificially subdividing the data at a Sérsic index of 2, or at $M_B = -18$ mag, or where the curvature in relations using 'effective' radii and surface brightnesses are a maximum (see Graham 2013 for an explanation of this), the continuity between the low- and intermediate-luminosity Sérsic galaxies can be missed, even if the data itself is accurate. This issue is discussed further in section 5.2.1.

The distribution of points in Figure 2 reveals that black holes grow faster than the stellar population of their host spheroids, for which abundant evidence is now appearing (e.g. Diamond-Stanic et al. 2012; Seymour et al. 2012; Trakhtenbrot & Netzer 2012; Agarwal et al. 2013; Alonso-Herrero et al. 2013; LaMassa et al. 2013 Lehmer et al. 2013; Drouart et al. 2014). For example, Diamond-Stanic et al. 2012 report that the black hole growth rate is proportional to the 1.67 (=1/0.6) power of the star formation rate within the inner kpc (roughly the bulge half-light radii) of their Seyfert galaxies, while the analysis from LaMassa et al. (2012) gives an exponent of 2.78 (=1/0.36) for their sample of ∼28,000 obscured active galaxies, quite different from the linear value of 1.

Figure 2 also reveals that classical bulges, pseudobulges, clump-bulges (Noguchi 1999), and mixed-bulges containing both a classical bulge and a pseudobulge, all follow the steeper scaling relation, until the onset of relatively dry mergers revealed by the scoured cores seen in the centres of (many of) the most massive spheroids.

What happens in the $M_{\rm bh}$–$M_{\rm sph}$ diagram at black hole masses less than $10^5\,M_{\odot}$ is not yet known. While the absence of a definitive black hole detection in M33 (Kormendy & McClure 1993; Gebhardt et al. 2001; Merritt et al. 2001) had reinforced the idea that black holes are associated with bulges (e.g. Dressler & Richstone 1988; Kormendy & Gebhardt 2001), bulgeless galaxies with massive black holes have since been detected (e.g. Reines et al. 2011; Secrest et al. 2012; Schramm et al. 2013; Simmons et al. 2013; Satyapal et al. 2014). Obviously these galaxies do not (yet?) participate in the observed $M_{\rm bh}$–$M_{{\rm sph},*}$ scaling relation. As noted in Graham & Scott (2013), there are however tens of galaxies known to contain AGN in bulges whose spheroid magnitudes suggest, based on this near-quadratic $M_{\rm bh}$–$M_{{\rm sph},*}$ scaling relation, that they harbour intermediate mass black holes ($10^2 < M_{\rm bh}/M_{\odot} < 10^5$). It will be interesting to see a) if this missing population of intermediate-mass black holes exists and b) where they reside in the $M_{\rm bh}$–$M_{\rm sph}$ diagram.

4.1.1. *Implications*

Of course the above represents a dramatic revision to the bulge-(black hole) connection , i.e. a completely different relation connecting supermassive black holes with their host bulges, and as such has wide-spread implications. For one, the many-merger scenario proposed by Peng (2007), and explored further by Jahnke & Macciò 2011 and Hirschmann et al. (2010), to produce a linear one-to-one scaling via the central limit theorem can be ruled out. Using a sample of galaxies with a range of initial $M_{\rm bh}/M_{{\rm gal},*}$ mass ratios, Peng (2007) noted that after many mergers it would naturally create an $M_{\rm bh}$–$M_{sph,*}$ relation with a slope of 1. Although this concept was independently ruled out by Anglés-Alcázar et al. (2013) who had emphasized that the number of actual major mergers are not frequent enough to have established such a linear relation, the quadratic slope of the $M_{\rm bh}$–$M_{\rm sph}$ relation confirms this ruling.

Some additional implications of the new relation include obvious things like (i) black hole mass predictions in other galaxies, (ii) estimates of the local black hole mass function and mass density based on local spheroid luminosity functions, and (iii) evolutionary studies of the $M_{\rm bh}/M_{\rm sph}$ ratio over different cosmic epochs (partly because $M_{\rm bh}/M_{\rm sph} \approx 0.005$, not 0.001–0.002, in nearby massive spheroids). Additionally impacted areas of research include (iv) galaxy/black hole formation theories, which extends to (v) AGN feedback models, (vi) predictions for space-based gravitational wave detections, (vii) connections with nuclear star cluster scaling relations, (viii) derivations of past quasar accretion efficiency as a function of mass (e.g. Shankar et al. 2009b), (ix) searches for the fundamental, rather than secondary, black hole scaling relation, and (x) calibrations matching inactive galaxy samples with low-mass AGN data to determine the optimal virial factor for measuring black hole masses in AGN. Given that most of these topics could generate a review in their own right, only feedback is briefly commented on here.

A large number of clever theoretical papers have tried to explain the nature of the $M_{\rm bh}$–$M_{\rm sph}$ relation in terms of feedback from the AGN (e.g. Silk & Rees 1998; Haehnelt, Natarajan & Rees 1998; Fabian 1999; Kauffmann & Haehnelt 2000; Wilman, Fabian & Nulsen 2000; Benson et al. 2003; Wyithe & Loeb 2003; Granato et al. 2004; Di Matteo et al. 2005; Springel et al. 2005; Hopkins et al. 2005, 2006; Cattaneo et al. 2006; Sijacki et al. 2007; Somerville et al. 2008; Booth & Schaye 2009, to mention just a fraction). Some papers (but not all those listed here) which have claimed success because they obtained, through gaseous processes, a linear $M_{\rm bh}$–$M_{\rm sph}$ relation over a wide range of mass, now appear in need of tweaking. Encouragingly, while not quite finding a quadratic relation with slope of 2, Hopkins & Quataert (2010) report that the black hole growth rate in their models is proportional to the 1.43 (=1/0.7) power of the star formation rate.

The so-called 'quasar' or 'cold' mode of black hole growth during gas-rich processes, as implemented in semi-analytical models, has typically assumed that black hole growth occurs via accretion which is linearly proportional to the inflowing mass of cold gas (which also produces the host spheroid), modulated by an efficiency which is lower for both unequal mass mergers (Croton et al. 2006) and less massive (more gas-rich) systems with lower virial velocities (e.g., Kauffmann & Haehnelt 2000, their eq 2; Croton et al. 2006, their eq. 8; Guo et al. 2011, their eq. 36)[24]. Graham & Scott (2013) therefore presented a new prescription for the increase in black hole mass, due to gas accretion during wet mergers, such that the black hole would grow quadratically relative to the host spheroid. The short duty (on) cycle of quasars ($\sim 10^7$–10^8 years) may then imply that the bulk of a spheroid's stars are also formed rapidly. Once the gas is largely gone, and significant galaxy/(black hole) growth is attained via major dry merger events, the low-accretion model (e.g., Blandford & Begelman 1999) presumably results in the so-called 'mechanical' or 'radio mode' feedback maintaining the spheroid-(black hole) mass ratio, as is roughly observed for the core-Sérsic galaxies.

4.2. *The $L_{\rm sph}$–σ relation*

Around the time that quasars were identified to be at large redshifts, Minkowski (1962) discovered a correlation between velocity dispersion and absolute magnitude for early-type galaxies. He refrained from fitting an equa-

[24] Note: Guo et al. (2011) excluded the square on the normalised velocity term in their eq. 36.

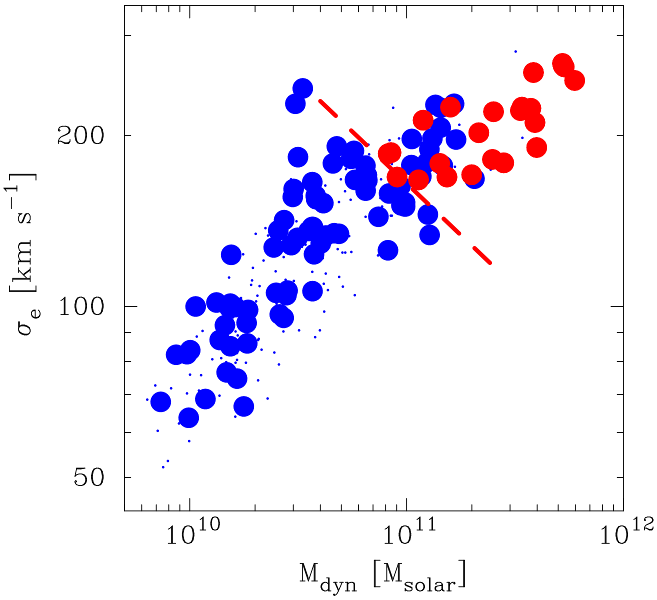

Figure 3. Dynamical galaxy mass ($M_{\rm dyn}$) — equal to twice the Jeans Anisotropic Multi-Gaussian-Expansion mass within the effective half-light radius $R_{\rm e}$ — versus the velocity dispersion $\sigma_{\rm e}$ within $R_{\rm e}$ for the ATLAS$^{\rm 3D}$ early-type galaxies (Cappellari et al. 2013, see their Fig.1). Core galaxies ($\gamma < 0.3$ according to the Nuker model (Grillmair et al. 1994; Lauer et al. 1995) as used by Krajnović 2013) are shown by the large red dots, while galaxies having steeper inner profiles ($\gamma > 0.5$) are shown by the large blue dots. Galaxies with an unknown inner surface brightness profile slope, or those with $0.3 < \gamma < 0.5$ are shown by the small dots.

tion to it, noting the need to extend the observations to low absolute magnitudes. While Morton & Chevalier (1973) achieved this, finding a continuous distribution of velocity dispersions, it was Faber & Jackson (1976) who were the first to fit an equation to Minkowski's relation. For their sample of 25 galaxies, they reported that $L \propto \sigma^4$, which has since become known as the Faber-Jackson relation. A few years later, exploring the bright end of Minkowski's relation, Schechter (1980) discovered that $L \propto \sigma^5$, a result confirmed by Malumuth and Kirshner (1981; see also von der Linden et al. 2007). Recent studies have suggested that the exponent may be 5.5 in brightest cluster galaxies (Liu et al. 2008) and as high as 6.5±1.3 in core galaxies (Lauer et al. 2007). Shortly after this, Schechter co-authored Davies et al. (1983) in which they revealed that $L \propto \sigma^2$ for low- and intermediate-luminosity early-type galaxies. Many studies have since shown that this result holds from the lowest luminosity dwarf elliptical galaxies up to $M_B \approx -20$ to -21 mag (Held et al. 1992; de Rijcke et al. 2005; Matković & Guzmán 2005; Balcells et al. 2007b; Lauer et al. 2007; Chilingarian et al. 2008; Forbes et al. 2008; Cody et al. 2009; Tortora et al. 2009; Kourkchi et al. 2012). This explained why past samples of intermediate-to-bright early-type galaxies had a slope of around 4, or 3 (Tonry et al. 1981), and confirmed the observation by Binney (1982) and Farouki et al. (1983) that a single power-law was not appropriate to describe the distribution of early-type galaxies in the L–σ diagram. Most recently, Davies has again illustrated this bend, this time in the $M_{\rm gal}$–σ diagram for early-type galaxies, through co-authorship of Cappellari et al. (2013). Their bent $M_{\rm gal}$–σ diagram is reproduced in Figure 3.

The bend in Minkowski's relation has been explained by Matković & Guzmán (2005) in terms of Sérsic galaxies (which have low- and intermediate-luminosity) following the $L \propto \sigma^2$ relation of Davies et al. (1983) while core-Sérsic galaxies (which have high-luminosity) follow the $L \propto \sigma^5$ relation of Schechter (1980). This continuity for the low- and intermediate-luminosity Sérsic galaxies, and the break-away of bright galaxies with partially depleted cores, is illustrated further in the L–μ_0 and L–n distributions seen in Graham & Guzmán (2003, their figures 9c and 10; see also Côté et al. 2007, their figure 3e). As noted in footnote 23 of this article, some galaxies may have experienced a major dry merger event but not display a partially depleted core — such as the merger remnants NGC 1316 (Fornax) and NGC 3115 (Schauer et al. 2014; Menezes et al. 2014) — which could explain why some of the high-mass galaxies in Figure 3 do not have depleted cores[25].

The bend in the $M_{\rm gal}$–σ diagram, and the $M_{\rm bh}$–$M_{\rm sph}$ diagram, is likely to have ties with the flattening that is also observed at the bright end of the colour magnitude diagram for early-type galaxies (Tremonti et al. 2004; Jiménez et al. 2011). Dry merging will increase the luminosity while preserving the colour (modulo passive evolution) among the core-Sérsic elliptical galaxies. In contrast, the Sérsic early-type galaxies display a continuous mass-metallicity relation which unites the dwarf and ordinary early-type galaxies (e.g. Caldwell 1983; Caldwell & Bothun 1987).

If the $M_{\rm bh}$–σ relation (Section 5) is roughly described by a single power-law, and given that the L–σ (and $M_{\rm gal}$–σ) relation is notably bent (Figure 3), then the $M_{\rm bh}$–L relation has to be bent, just as observed and discussed in Figure 2 and Section 4.1.

5. THE $M_{\rm BH}$–σ RELATION

While the work on the $M_{\rm bh}$–L relation from Magorrian received considerable attention, it was the $M_{\rm bh}$–σ relation (Ferrarese & Merritt 2000; Gebhardt et al. 2000) which really sparked off wide-spread global interest in black hole scaling relations. The reason may likely have been because, after having identified and removed galaxies with less secure black hole mass estimates, the $M_{\rm bh}$–σ relation was reported by both teams to be consistent with having zero intrinsic scatter (see also Kormendy & Gebhardt 2001)[26]. That is, after accounting for the measurement errors, all the scatter was accounted for, suggesting that a new law of physics had been discovered. However, the slope of this potential new law was not agreed upon. Ferrarese & Merritt (2000) had reported $M_{\rm bh} \propto \sigma^{4.8\pm0.5}$, while Gebhardt et al. (2000) reported an exponent of 3.75 ± 0.3. The former slope agreed with the energy-balancing prediction by Silk & Rees (1998, see also Haehnelt, Natarajan & Rees 1998) that $M_{\rm bh} \propto \sigma^5$, while the latter slope agreed with the momentum-balancing prediction by Fabian (1999) that $M_{\rm bh} \propto \sigma^4$. This discrepancy was to become a major

[25] It will be interesting in the future to careful apply the core-Sérsic model to see how all the points are distributed in terms of galaxies with and without partially-depleted cores.

[26] The $M_{\rm bh}$–L relation was reported to have more scatter, but this was in part because of poor bulge/disc decompositions, and the unrecognised bend in the relation.

source of controversy and uncertainty in what has become one of the most famous astronomical relations of recent years. As such, some space is dedicated to this issue here. In the following subsection, the main reason for the different slopes is presented, as this continues to be somewhat misunderstood today.

5.1. *Slippery slopes*

Ferrarese & Merritt (2000) performed a symmetrical linear regression, using the BCES routine from Akritas & Bershady (1996) which allowed for intrinsic scatter and unique measurement errors on both variables, $M_{\rm bh}$ and σ (which they took to be 13% for the velocity dispersion of external galaxies). Gebhardt et al. (2000), on the other hand, performed a non-symmetrical ordinary least squares regression by minimising the vertical offsets (i.e. in the $\log M_{\rm bh}$ direction) about their $M_{\rm bh}$–σ relation. This approach effectively assumed that the uncertainty on the velocity dispersion was zero and that the black hole masses all had the same uncertainty.

Merritt & Ferrarese (2001a) addressed the issue of the differing slopes, using four different types of linear regression, two which treated the ($M_{\rm bh}$, σ) data symmetrically and two which did not. They revealed how the slope of the $M_{\rm bh}$–σ relation increased as one assigned an increasing uncertainty to the velocity dispersion and presented a best fit slope of 4.72±0.36 for their expanded sample.

Tremaine et al. (2002) also looked at this issue of different slopes and noted that under certain conditions[27] the minimisation routine from Akritas & Bershady, which was used by Ferrarese & Merritt (2000), can be biased. As noted above, Merritt & Ferrarese (2001a) had additionally used a second symmetrical regression routine, referred to as the "Orthogonal distance regression" which had been implemented by Press et al. (1992, their Section 15.3) as FITEXY. It was such that the following quantity was minimised during the task of fitting the line $y = a + bx$

$$\chi^2 = \sum_{i=1}^{N} \frac{[y_i - (a+bx_i)]^2}{\delta {y_i}^2 + b^2 \delta {x_i}^2}, \qquad (1)$$

where N data pairs of y and x values are available in one's sample, and they have measurement errors δy and δx, respectively. Merritt & Ferrarese (2001a) pointed out that Feigelson & Babu (1992) had already noted that this routine is fine unless the distribution to be fit contains intrinsic scatter, i.e. real departures of the data from the optimal line which are not due to measurement errors. At that time, the $M_{\rm bh}$–σ relation was thought to contain no intrinsic scatter, or was at least consistent with having no intrinsic scatter.

Tremaine et al. (2002) subsequently developed their own modified version of FITEXY. I t was such that it minimised the quantity

$$\chi^2 = \sum_{i=1}^{N} \frac{[y_i - (a+bx_i)]^2}{\delta {y_i}^2 + b^2 \delta {x_i}^2 + \epsilon_y^2}, \qquad (2)$$

where the intrinsic scatter ϵ_y is solved for by repeating the fit until $\chi^2/(N-2)$ equals 1. Although Tremaine et al. (2002) claimed this expression still gave a symmetrical treatment of the data, it did not. By trying to allow for intrinsic scatter, they had inadvertently converted a symmetrical expression into a non-symmetrical expression by minimising the offsets under the assumption that all of the intrinsic scatter lay in the y-direction. They reported a slope of 4.02 ± 0.32 for their $M_{\rm bh}$–σ relation using the smaller uncertainty of 5% (compare 13%) for the velocity dispersions of the external galaxies.

Here we look at this a little more carefully, as it continues to cause confusion more than a decade later. If one was to minimise the offsets in the x-direction, about the line $y = a + bx$, or equivalently $x = (y - a)/b$, the expression would be

$$\chi^2 = \sum_{i=1}^{N} \frac{[x_i - \frac{(y_i - a)}{b}]^2}{\delta {y_i}^2/b^2 + \delta {x_i}^2 + \epsilon_x^2}, \quad \sum_{i=1}^{N} \frac{[-y_i + (a+bx_i)]^2}{\delta {y_i}^2 + b^2\delta {x_i}^2 + b^2\epsilon_x^2}, \qquad (3)$$

where ϵ_x is the intrinsic scatter, but this time implicitly assumed to reside in the x-direction. The difference between equations 2 and 3 is the final term in the denominator, which has that $\epsilon_y = b\epsilon_x$. Given this (not surprising) dependence on the slope between ϵ_y and ϵ_x, the solution reached by solving for $\chi^2/(N-2) = 1$ in equations 2 and 3 has a different value of b, i.e. a different slope. To obtain a symmetrical regression therefore requires an average of these two regressions as discussed in Novak et al. (2006)[28], which are sometimes referred to as the forward and the inverse regression.

Performing a non-symmetrical linear regression analysis and minimising the offsets in just the $\log M_{\rm bh}$ direction is preferred if one wishes to obtain a relation useful for predicting black hole masses in other galaxies, simply because this relation has the smallest offsets in the $\log M_{\rm bh}$ direction (see Feigelson & Babu 1992; Andreon & Hurn 2012). If, on the other hand, one is interested in the underlying / fundamental relation connecting $M_{\rm bh}$ and σ, then one should perform a symmetrical regression. This is discussed by Novak et al. (2006) in terms of the Observer's Question and the Theorist's Question.

Analysing the same data[29] from Tremaine et al. (2002), and assigning a 5% uncertainty to the velocity dispersion of each galaxy (including the Milky Way), Novak et al. (2006) reported a slope of 4.10±0.30 using Eq. 2 and 4.59±0.34 using Eq. 3. Had they used an uncertainty of 13%, they would have reported slopes of 4.39 and 4.59, giving an average value slope of 4.49 that was consistent with Merritt & Ferrarese (2001a) who reported an optimal slope of 4.72±0.36.

To make a point about the ongoing concerns regarding different minimisation routines, and in particular to show that the symmetrical bisector regression routine from Akritas & Bershady was not producing a biased fit in regard to the ($M_{\rm bh}, \sigma$) data, Graham & Li (2009) used

[27] The slope can be biased if (i) the uncertainty on the x values is large compared to the range of x values, or (ii) the sizes of all the x and y uncertainties are not roughly comparable to each other.

[28] An easy way to check if one has performed a symmetrical regression is to swap their x and y data around and re-feed this into their regression routine.

[29] The black hole mass for NGC 821 was updated, but this had almost no impact.

three symmetrical regression routines, one from Akritas & Bershady (1996), the expression from Tremaine et al. (2002) operating in both forward and inverse mode, and an IDL routine from Kelly (2007) based on a Bayesian estimator. All were shown to give very similar results when the same uncertainty on the velocity dispersion was consistently used, a test that was recently confirmed in Park et al. (2012) who additionally used a fourth (maximum likelihood) estimator.

5.2. *substructure and escalating slopes*

In 2007 Graham noticed that all of the barred galaxies in the $M_{\rm bh}$–σ diagram were offset, to either lower black hole masses and/or higher velocity dispersions, relative to the best-fitting line defined by the non-barred galaxies, and that excluding the barred galaxies resulted in a reduced scatter about the $M_{\rm bh}$–σ relation (Graham 2007a). At the same time, Hu (2008) had compiled a larger sample and shown the same apparent substructure within the $M_{\rm bh}$–σ diagram. Hu considered all of his offset galaxies to contain 'pseudobulges', built from the secular evolution of their surrounding disc and containing relatively under-developed black holes. They were also all barred galaxies. Graham (2008a) similarly considered the offset galaxies to have undermassive black holes, due to secular evolution over-developing the bulge, or to have elevated velocity dispersions due to the dynamics of the bar. The choice appears answered because Hartmann et al. (2013) have shown that bars are indeed capable of increasing the velocity dispersion in galaxies, and by exactly the average offset observed in the $M_{\rm bh}$–σ diagram (see also Debattista et al. 2013 and Monari et al. 2014). Furthermore, Figure 2 shows that pseudobulges and classical bulges (and clump bulges) follow the same broad distribution in the $M_{\rm bh}$–$M_{\rm sph}$ diagram; at low spheroid masses they both reside systematically below the near-linear relation defined by the massive core-Sérsic spheroids. There is not yet evidence that pseudobulges contain smaller black hole masses than classical bulges of the same mass, although more data would be welcome. In particular, removing the contribution of the bar[30], and the rotational contribution[31], from the observed central velocity dispersions of the spheroids would be helpful. It may also make more sense to use the quantity $\sqrt{3\sigma_{\rm sph}^2 + v_{\rm sph,rot}^2}$ (Busarello et al. 1992). Although, much of this may be moot in regard to pseudobulges due to the difficult task of actually identifying them, as discussed in the following subsection.

One thing that was clear from Hu (2008) and Graham (2008b) was that the growing sample size had generated an increased scatter about the $M_{\rm bh}$–σ relation[32], and the intrinsic scatter no longer appeared consistent with zero, a result shown further by Gültekin et al. (2009). The $M_{\rm bh}$–σ diagram was therefore falling from grace, and it also now presented quite a contrast to early claims which had reported that classical bulges and pseudobulges follow the same black hole scaling relations (e.g. Kormendy 2001; Kormendy & Gebhardt 2001). In Kormendy et al. (2011) the offset nature of the pseudobulges was acknowledged, and it was now claimed that black hole masses do not correlate with the properties of pseudobulges. However, the range in absolute magnitude of the pseudobulges was restricted to just 2 mag, making it challenging to identify if there is a relation present. With a fuller data set, Figure 2 reveals that all bulge types appear to follow an $M_{\rm bh}$–$M_{\rm sph}$ relation.

With a sample size of 72 galaxies, McConnell & Ma (2013) used the non-symmetrical, modified FITEXY routine, as coded by Williams et al. (2010) in MPFITEXY. They reported a slope of 5.64±0.32 for their optimal $M_{\rm bh}$–σ relation (their figure 1, which includes the alleged over-massive black hole in NGC 1277: van den Bosch et al. 2012). If they had of additionally used the inverse of this regression, in which the unknown intrinsic scatter is assigned to the log σ direction, they would have obtained a slope of 6.64, and thus an average slope of 6.14. This is steeper than previously reported, and is in part due to their inclusion of the offset barred galaxies at low masses. While McConnell & Ma (2013) do report that their 19 late-type galaxies (with both classical bulges and pseudobulges) have an $M_{\rm bh}$–σ relation with a zero point (i.e. the term 'a' in $y = a + bx$) that is 0.29 dex lower than for their 53 early-type galaxies (8.36 vs 8.07), i.e. offset by a factor of 2, they did not perform a fit to the barred and non-barred galaxies. Given that the early-type galaxies dominate at the high-mass end of the diagram, and the late-type galaxies at the low-mass end, they combine to produce the steeper relation with a slope of ≈6.

Graham et al. (2011) highlighted a potential sample selection bias such that the need to resolve (or nearly resolve) the sphere-of-influence of the black holes may be resulting in an artificial floor to the distribution of points in the $M_{\rm bh}$–σ diagram. As such, they additionally used a non-symmetrical regression, but one which minimised the offsets in the horizontal direction, i.e. they performed the 'inverse' regression as this should provide the least biased fit (see Lynden-Bell et al. 1988). Adding eight black hole masses to the compilation of 64 data pairs in Graham et al. (2011), Graham & Scott (2013) reported a slope of 6.08±0.31 using their preferred inverse regression on their sample of 72 galaxies (see Figure 4). For the 51 non-barred galaxies, their optimal slope using the inverse regression was 5.53±0.34. While this is at first glance in agreement with the preferred value of 5.64±0.32 reported by McConnell & Ma 2013, it should be realised that it is a coincidence as different things have been measured: a forward regression for all galaxy types versus an inverse regression for non-barred galaxies.

Using updated and expanded data for 57 non-barred galaxies, taken from the sample of 89 galaxies in Savorgnan & Graham (2014), the forward, inverse and average regression give a slope of 5.10, 6.48 and 5.79. Folding in the offset barred galaxies results in steeper slopes still, as seen with the McConnell & Ma (2013) data. The increase to the slope over the past few years has largely come from increased black hole masses, and new data, at the high mass end. McConnell & Ma (2013) additionally note that the flux-weighted velocity dispersion within one effective radius can be as much as 10–15% lower in their massive galaxies when excluding data within the black

[30] Graham et al. 2011, their figure 7, offer a first order approximation for this.

[31] See Kang et al. 2013, their figure 9, and Pota et al. 2013 in regard to the velocity dispersion of a globular cluster system.

[32] Potentially, this may in part be due to the inclusion of less accurate black hole mass measurements with under-estimated error bars (see Merritt 2013).

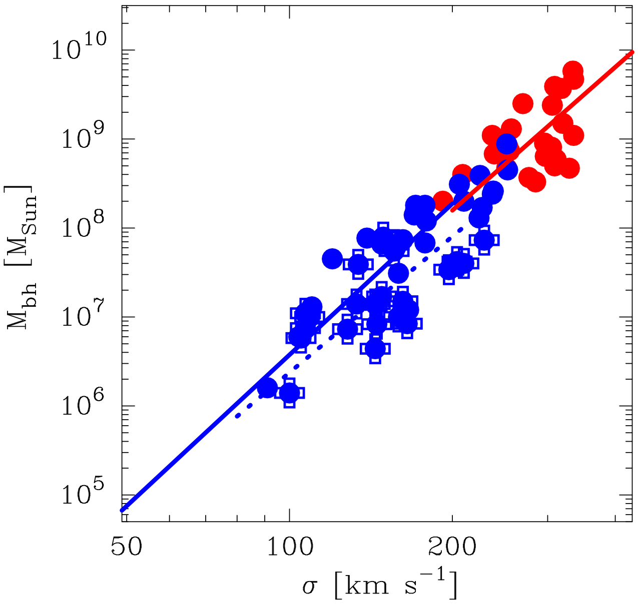

Figure 4. $M_{\rm bh}$–σ diagram taken from Graham & Scott (2013). Red points represent core-Sérsic galaxies; blue points represent Sérsic galaxies. The crosses designate barred galaxies, which tend to be offset to higher velocity dispersions. The three lines are linear regressions, in which the barred Sérsic galaxies and the non-barred Sérsic galaxies have been fit separately from the core-Sérsic galaxies (which are not barred).

hole's sphere-of-influence. This follows Graham et al. (2011) who noted that the velocity dispersion for M32's spheroid should be reduced from ∼75 km s^{-1} to ∼55 km s^{-1} (Tonry 1987) for exactly this reason. Increases to black hole masses have also come from efforts to account for dark matter halos, resulting in an average increase of ∼20% (Schulze & Gebhardt 2011; Rusli et al. 2103a), but as high as a factor of 2 in the case of M87 (Gebhardt & Thomas 2009). Incorporating a dark matter halo is akin to relaxing the past assumption/simplification that the stellar mass-to-light ratio is constant with radius[33].

This new, slightly steeper, $M_{\rm bh}$–σ relation for the non-barred galaxies suggests that if $L_{\rm sph} \propto \sigma^6$ (Lauer et al. 2007) for the core-Sérsic galaxies, then one can expect to recover $M_{\rm bh} \propto L_{\rm sph}$ for the core-Sérsic galaxies. If $L_{\rm sph} \propto \sigma^5$ (e.g. Schechter 1980) then one can expect to find $M_{\rm bh} \propto L_{\rm sph}^{6/5}$, suggestive of a second order effect on the picture of dry mergers maintaining a constant $M_{\rm bh}/L_{\rm sph}$ and $M_{\rm bh}/M_{\rm sph}$ ratio. Resolution to this minor query may simply require consistency with the regression analyses, or perhaps a careful bulge/disc separation of the galaxies involved (e.g. Laurikainen et al. 2005, 2011; Balcells et al. 2007a,b; Gadotti 2008; Läsker et al. 2014a), because core-Sérsic galaxies can contain a fast-rotating disc (e.g. Dullo & Graham 2013; Krajnović et al. 2013).

5.2.1. *Pseudobulges*

Pseudobulges are particularly hard to identify, for the multitude of reasons presented in Graham (2013, 2014). Furthermore, many galaxies contain *both* a disc-like 'pseudobulge' and a classical bulge (e.g. Erwin et al. 2003, 2014; Athanassoula 2005; Gadotti 2009; MacArthur, González & Courteau 2009; dos Anjos & da Silva 2013; Seidel et al. 2014). In addition, some may have formed from the (secular) inward migration and (classical) merging of stellar clumps (e.g. Noguchi 1999; Bournaud et al. 2007; Inoue & Saitoh 2012, and references therein). All of this makes the task of labelling galaxies as either containing a pseudobulge or a classical bulge highly problematic and untenable. In the $M_{\rm bh}$–σ analysis by Graham et al. (2011) and Graham & Scott (2013), they avoided the issue of pseudobulges and separated galaxies based on the presence (or not) of a bar and revealed that the masses of black holes in barred galaxies correlate with the velocity dispersion, despite their heightened dynamics. Given that the majority of Sérsic spheroids (i.e. those without partially depleted cores) also follow the near-quadratic $M_{\rm bh}$–L relation, it appears that the masses of black holes in pseudobulges correlate with at least one property of their host bulge, and unless pseudobulges are restricted to have a narrow range of velocity dispersion, then their black hole masses also correlate with velocity dispersion (or at least define an upper envelope in the $M_{\rm bh}$–σ diagram).

A few of the (often not properly recognised) difficulties with identifying pseudobulges are noted here, in case it is helpful to some readers. From a kinematical perspective, just as with the formation of rotating elliptical galaxies via mergers, mergers can also create bulges which rotate (e.g. Bekki 2010; Keselman & Nusser 2012) and bars can spin-up classical bulges (e.g. Saha et al. 2012), and the smaller the bulges are the easier it is. Rotation is therefore not a definitive signature of a pseudobulge. In spiral galaxies, the observable presence of the disc's inner spiral arms, which cohabit the inner region of the galaxy where the bulge also resides, are of course easier to detect in fainter bulges (which are those that have smaller Sérsic indices) due to the greater bulge/arm contrast. However the detection and presence of these underlying features does not necessitate the presence of a pseudobulge (e.g. Eliche-Moral et al. 2011; dos Anjos & da Silva 2013).

From a selection of hundreds of disc galaxies imaged in the K-band, Graham & Worley (2008) observe no bimodality in the bulge Sérsic indices, questioning the suitability of a divide at a Sérsic index of $n = 2$ which has frequently been used in the recent literature. This divide is roughly halfway between $n = 1$ (which describes the light-profiles of flattened rotating discs) and $n = 4$ (which was in the past thought to describe the majority of elliptical galaxies and large bulges). While pseudobulges are expected to have Sérsic indices $n \approx 1$ — having formed from their surrounding exponential disc (e.g. Bardeen 1975; Hohl 1975; Combes & Sanders 1981; Combes et al. 1990; Pfenniger & Friedli 1991) — the problem is that mergers do not only produce $R^{1/4}$-like light profiles. Mergers can also create bulges with $n < 2$ (e.g. Eliche-Moral et al. 2011; Scannapieco et al. 2011; Querejeta et al. 2014), just as low-luminosity elliptical galaxies (not built from the secular evolution of a disc) are well known to have $n < 2$ and even < 1 (e.g. Davies et al. 1988; Young & Currie 1994)[34].

[33] This raises another issue which is yet to be properly addressed in the literature: not only do many spheroids have radial stellar population gradients, but most Sérsic galaxies have nuclear star clusters in addition to massive black holes, and the assumption of a single stellar mass-to-light ratio when modelling the data to derive a black hole mass is therefore not appropriate.

[34] The occurrence of large-scale, rotating stellar discs and kine-

Prior to the realisation that the Sérsic index changes monotonically with spheroid luminosity and size (e.g. Caon et al. 1993; Andredakis et al. 1996) — referred to as structural nonhomology — the curved but continuous scaling relations involving the 'effective' half-light radii and 'effective' surface brightness (which have a maximum curvature around $n = 2$) had suggested that spheroids with $n < 2$ may be a distinct species rather than the low mass extension of spheroids with $n > 2$ (see Graham 2013). However we now know that this was a red-herring, and that all relations involving the 'effective' parameters are curved (e.g. Graham & Guzmán 2003; Gavazzi et al. 2005; Ferrarese et al. 2006a; Côté et al. 2006, 2007). As such, the Kormendy (1977) relation cannot be used to separate dwarf early-type galaxies from ordinary early-type galaxies, nor to separate pseudobulges from classical bulges, because at low-luminosities both types of bulge (classical and pseudo) depart from this relation, which is the tangent to the bright arm of the curved μ_e–R_e distribution.

6. THE M_{BH}–N RELATION

As noted in Graham et al. (2001), it may not be the total amount of mass in a spheroid, but rather how that mass is distributed, when it comes to the connection with the central supermassive black hole. Similarly, the velocity dispersion is but a tracer of the underlying mass distribution, and as such it can not be the fundamental parameter driving the black hole mass scaling relations.

Intriguingly, what Graham et al. (2001) revealed is that the central radial concentration of light, within the inner effective half light radii of spheroids, correlates strongly with the black hole mass. The concentration index which they used, taken from Trujillo et al. (2001), is monotonically related with the Sérsic index n, and thus an M_{bh}–n relation also exists, as shown in Graham et al. (2003a). With an expanded data set, Graham & Driver (2007) revealed that this relation is no longer well described by a single log-linear power-law, and that a log-quadratic relation performs noticeably better (see Figure 5a). Given the log-linear L–n relation observed for both elliptical galaxies (e.g. Young & Currie 1993; Jerjen & Binggeli 1997; Graham & Guzmán 2003; Ferrarese et al. 2006a) and the bulges of disc galaxies (e.g. Andredakis et al. 1995; Graham & Worley 2008, and references therein), and the bent M_{bh}–L_{sph} relation (Section 4), the M_{bh}–n relation must be bent, such that galaxies which have experienced major, relatively dry, merger events are responsible for the flattening which is seen in Figure 5 at high masses.

The existence of the M_{bh}–L_{sph} relation, coupled with existence of the L_{sph}–n relation, necessitates the existence of the M_{bh}–n relation. Although, as illustrated by Savorgnan et al. (2013), there is a need for care when measuring Sérsic indices, and studies which fail to recover the M_{bh}–n relation for the sample of galaxies with directly measured black hole masses may be dominated by poorly measured Sérsic indices, and in turn erroneous bulge magnitudes which depend on an accurate Sérsic index. Within the literature, measurements for individual galaxies have varied dramatically (e.g. Graham & Driver 2007; Laurikainen et al. 2010; Sani et al. 2011; Vika et al. 2012; Beifiori et al. 2012; Rusli et al. 2013a; Läsker et al. 2014a). Shown in Figure 5b are the average values, after the rejection of extreme outliers, plotted against black hole mass. Savorgnan et al. (2013) divided the sample into Sérsic and core-Sérsic spheroids, and fit separate linear regressions for each sub-population.

Savorgnan et al. (2015, in prep.) has nearly completed a careful multi-component analysis of all the 72 galaxies used by Graham & Scott (2013) and reconciled the differences between past attempts to measure the Sérsic index. For example, sometimes these discrepancies arise because a lenticular disc galaxy may have been modelled with either a single Sérsic component or more correctly as the sum of a Sérsic-bulge plus an exponential disc by a different author. Other times the presence of an unaccounted for nuclear disc, or a partially depleted core, has biased the luminosity-weighted fits in some studies. The M_{bh}–n relation, more so than the previous relations, is in a state of limbo until this work is completed. Despite the need for care when measuring the Sérsic index, the advantage is that one only requires uncalibrated photometric images.

Readers interested in the development of fitting bulge light profiles since de Vaucouleurs (1959) first noted departures from his $R^{1/4}$ model, may appreciate the references in section 4.1 of Graham (2013). Andredakis et al. (1995) were the first to model the bulges of disc galaxies with Sérsic's (1963) light profile model, following its application to elliptical galaxies by Davies et al. (1988) and Caon et al. (1993), and the earlier advocation of its use by Capaccioli (1985, 1987). Some of the difficulty with, and the impact of getting, the Sérsic index correct is illustrated by Gadotti & Sánchez-Janssen (2012) in the case of the Sombrero galaxy.

7. THE $M_{BH} - \mu_0$ DIAGRAM

It is not unreasonable to expect that the growth of massive black holes may be related to the growth, and subsequent space density, of stars in its immediate vicinity. Gas processes have contributed to the development of both, and the black hole mass may be more connected with the local stellar density than the total stellar mass of the host spheroid. While the de-projected stellar density, ρ_0 is ideally the quantity we would like to have (e.g. Merritt 2006b, his figure 5), and this can be derived under certain assumptions (e.g. Terzić & Graham 2005, their Eq. 4), it is of course the projected surface brightness that is observed.

Binggeli, Sandage & Tarenghi (1984) and Sandage & Binggeli (1984) provide a nice historical account of the detection of dwarf galaxies, and wrote that it was established that "the dwarf elliptical galaxies form a continuum in luminosity with the brighter E systems". Caldwell (1983; his Figure 6) and Bothun et al. (1986, their figure 7) revealed this continuum was such that fainter than $M_B \approx -20.5$ mag, there is a log-linear relation between the luminosity and the central surface brightness, μ_0. In addition to this, Binggeli et al. (1984, their figure 11) and Binggeli & Cameron (1991, their figures 9 and 18) found that, when using the inward extrapolation of King models, this L–μ_0 relation extends from

matical substructure in early-type galaxies on either side of the alleged divide at $M_B = -18$ mag ($n \approx 2$) further reveals the continuity of dwarf and ordinary early-type galaxies (e.g., Emsellem et al. 2007; Krajnović et al. 2008; Scott et al. 2014;Toloba et al. 2014).

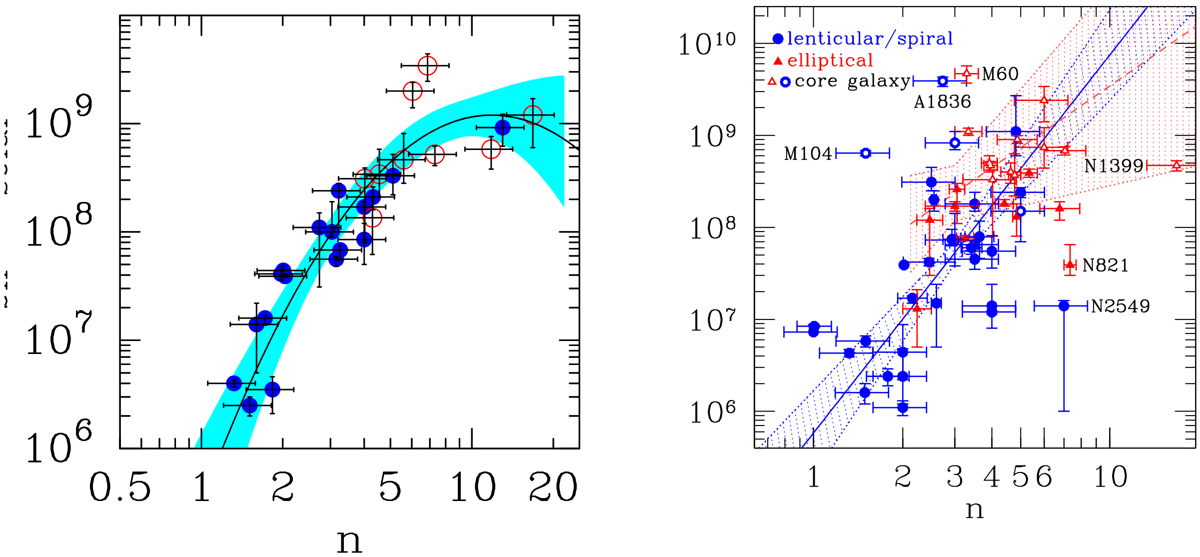

Figure 5. Left panel: $M_{\rm bh}$–n diagram taken from Graham & Driver (2007). The core-Sérsic spheroids are shown here by the red circles, while the Sérsic spheroids are shown by the blue dots. The lone Sérsic spheroid at the high-mass end is the S0 galaxy NGC 3115, identified to not have a core by Ravindranath et al. (2001). Right panel: $M_{\rm bh}$–n diagram from Savorgnan et al. (2013). Rather than a single log-quadratic relation, two log-linear relations are shown here, one for the Sérsic spheroids and one for the core-Sérsic spheroids.

$-12 > M_B > -23$ mag. This was further highlighted by Jerjen & Binggeli (1997) and Graham & Guzmán (2003) when using the inward extrapolation of the Sérsic model; extrapolated over partially depleted cores in the case of the brightest spheroids whose cores have been eroded away by coalescing supermassive black holes.

Given this log-linear L–μ_0 relation, and the bent $M_{\rm bh}$–$L_{\rm sph}$ relation (Section 4), there must be a bent $M_{\rm bh}$–μ_0 relation. It should again be emphasized that this particular value of μ_0 refers to the extrapolated / expected value prior to core depletion. Given the difficulties in routinely obtaining robust Sérsic indices for the spheroids with black hole masses (Section 6), it is perhaps not surprising that this diagram is yet to be published. Although it may be the fundamental parameter linking black holes with their bulges, to date there is only a prediction by Graham & Driver (2007) for its form. This was derived by coupling the log-quadratic $M_{\rm bh}$–n relation from Graham & Driver with the log-linear n–μ_0 relation from Graham & Guzmán (2003), and is reproduced here in Figure 6.

Given our current understanding, it makes more sense to construct the $M_{\rm bh}$–μ_0 relation using the log-linear $M_{\rm bh}$–L relations for the Sérsic and core-Sérsic spheroids given in Graham & Scott (2013, their table 3) together with the log-linear L–μ_0 relation given in Graham & Guzmán (2003, their figure 9c). Because the latter was derived in the B-band, we use the B-band $M_{\rm bh}$–L relation from Graham & Scott. For the Sérsic galaxies, this gives the relation

$$\log(M_{\rm bh}/M_\odot) = 17.24 - 0.63\mu_0, \qquad (4)$$

and for the core-Sérsic galaxies one has the relation

$$\log(M_{\rm bh}/M_\odot) = 13.62 - 0.36\mu_0. \qquad (5)$$

These predictions are shown in Figure 6. Once the careful Sérsic modelling of galaxies with directly measured black hole masses is completed by Savorgnan et al. (in prep.), it will be possible to populate this diagram and (under certain assumptions) its deprojected cousin.

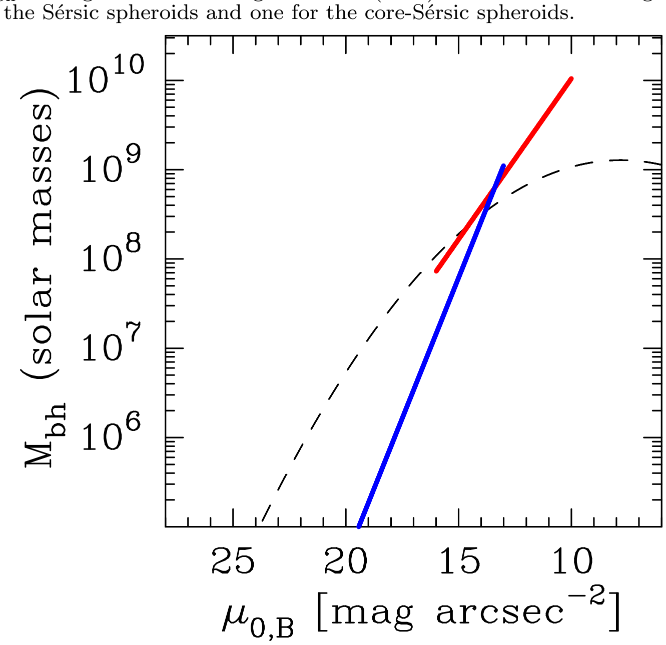

Figure 6. Predictions for the $M_{\rm bh}$–μ_0 diagram. The dashed curve is from Graham & Driver (2007), while the blue and red lines show equations 4 and 5 for the Sérsic and core-Sérsic spheroids, respectively. Clearly the uncertainty on these lines is still quite large, but a bend is nonetheless expected.

8. DEPLETED GALAXY CORES AND THE $M_{\rm BH}$–$M_{\rm DEF}$ RELATION

As noted previously, the merger of two galaxies without substantial gas, referred to as a dry merger, will result in the supermassive black holes from the progenitor galaxies sinking to the bottom of the newly wed galaxy by transferring much of their orbital angular momentum to the stars near the new galaxy's core (Begelman, Blandford & Rees 1980; Ebisuzaki et al. 1991). Such collisional construction of galaxies results in an evacuated 'loss cone' showing up as a partially depleted core in the images of nearby galaxies (e.g. King & Minkowski 1966, 1972; Kormendy 1982; Lauer 1983). Typical core sizes, as quantified by the break radius R_b of the core-Sérsic model, are tens to a few hundred parsec (e.g. Trujillo et al. 2004; Ferrarese et al. 2006a; Côté et al. 2007; Hyde et al. 2008; Richings et al. 2011; Rusli et al. 2013b; Dullo & Graham 2013, 2014; Bonfini 2014), and roughly a factor of 2 smaller than Nuker model break radii (Lauer et al. 1995). Whether or not coalescence of the black holes has already occurred in these galaxies with partially depleted cores is not clear, although see Khan et al. (2011, 2013, and references therein) in regard to the 'final parsec problem'.

Using the core-Sérsic model to quantify the central flux deficit, and in turn the stellar mass deficit, Graham (2004) discovered $M_{\rm def} \approx 2M_{\rm bh}$. Previously it was thought that $M_{\rm def}/M_{\rm bh}$ was, on average, an order of magnitude greater (e.g. Milosavljević et al. 2002; Ravindranath, Ho & Filippenko 2002), which required a troublingly large number of merger events given that the ejected mass should roughly scale with $N\,M_{\rm bh}$, where N is the cumulative number of (equivalent major) dry merger events (Milosavljević & Merritt 2001; Merritt 2006a). Using the core-Sérsic model, these new lower mass ratios were also found by Ferrarese et al. (2006a) and Hyde et al. (2008). Using the idea from Graham et al. (2003) that cores can be measured as a deficit of light relative to the inward extrapolation of the outer Sérsic profile, but fitting the Sérsic model rather than core-Sérsic model and identifying the sizes of depleted cores by eye, Kormendy & Bender (2009) reported notably larger mass ratios (typically close to 10 or higher). Hopkins & Hernquist (2010) subsequently resolved this issue in a model-independent manner and revealed that the core-Sérsic model measurements of the central mass deficits were correct. Most recently, Rusli et al. (2013b) found that ∼80% of their 23 galaxies have $1 < M_{\rm def}/M_{\rm bh} < 5$, while Dullo & Graham (2014) reported typical values for their sample of 31 galaxies to be $0.5 < M_{\rm def}/M_{\rm bh} < 4$.

Although the central mass deficit and break radius are obviously not fundamental parameters in establishing the spheroid-(black hole) connection — simply because many galaxies have black holes but not partially depleted cores — there is nonetheless an $M_{\rm bh}$–R_b relation (Lauer et al. 2007)[35] and an $M_{\rm bh}$–$M_{\rm def}$ relation (e.g. Graham 2004; Rusli et al. 2013; Dullo & Graham 2014). This relation simply exists over a restricted mass range. Dullo & Graham (2014, their Eq. 18) reported that $M_{\rm def} \propto M_{\rm bh}^{3.70\pm0.76}$ for the population ensemble (not to be confused with growth in individual galaxies). This is of interest for several reasons. One of which is that it may provide insight into the merging scenario, which currently has an unresolved problem. In general, galaxies with the greatest $M_{\rm def}/M_{\rm bh}$ ratio should have experienced the highest number of major dry mergers, and due to the increase in black hole mass but stagnation in velocity dispersion associated with such mergers (e.g. Ostriker & Hausman 1977; Hausman & Ostriker 1978; Ciotti & van Albada 2001), they should be offset to high black hole masses in the $M_{\rm bh}$–σ diagram (see Volonteri & Ciotti 2013). However, they are not (Savorgnan & Graham 2014).

Within low-luminosity early-type galaxies, the nuclear star cluster can be slightly offset (∼100 parsec) from the galaxy's photometric centre (Binggeli et al. 2000; Barazza et al. 2003). This is thought to be due to the dense star cluster's harmonic oscillation within the weak gravitational gradient of the galaxy's core. The amplitude of the nuclear cluster's rocking back and forth motion is expected to be greater in spheroids with lower Sérsic index, because they have lower central stellar densities and shallower inner density profiles, and thus less well defined gravitational centres over a greater fraction of their half-light radii (see Terzić & Graham 2005, their figure 2). Similarly, high-luminosity core-Sérsic spheroids have somewhat weakened gravitational centres (Terzić & Graham 2005, their figure 3) due to the partial depletion of stars in their cores. One may then expect to find the supermassive black holes slightly offset from the photometric centres of core-Sérsic galaxies (Miller & Smith 1992; Taga & Iye 1998). However a mechanism capable of creating more extreme (> 1 kpc) offsets is the recoil from the emission of anisotropic gravitational radiation that a newly merged black hole may receive (e.g. Bonnor & Rotenberg 1961; Peres 1962; Bekenstein 1973). The linear momentum carried away by the gravitational wave is balanced by a kick imparted to the black hole. This recoil process has the ability to evacuate a much greater loss cone, and has been proposed as an explanation for some cores having large $M_{\rm def}/M_{\rm bh}$ ratios (e.g. Boylan-Kolchin et al. 2004; Campanelli et al. 2007; Gualandris & Merritt 2008, 2012), which have been observed in NGC 1399 and NGC 5061. While only small spatial offsets are known for black holes in galaxies with directly measured black hole masses (e.g. Batcheldor et al. 2010b; Lena et al. 2014), if this process is operating one might expect to see greater displacements (e.g. Blecha et al. 2012) of black holes in galaxies with larger $M_{\rm def}/M_{\rm bh}$ ratios. However, if the damping timescale of the recoil-induced oscillation is sufficiently short, one may not find this correlation.

In passing, it might be remiss if a few words were not said about the gravitational wave signals expected from the final coalescence of massive black holes after they have scoured out the cores of massive spheroids, preferentially removing stars on plunging radial orbits (e.g. Quinlan & Hernquist 1997; Milosavljević & Merritt 2001; Thomas et al. 2014). Binary AGN, and thus massive black holes, are now known in several galaxies (e.g. Komossa et al. 2003; Liu et al. 2014, and references therein). The rapidly changing gravitational field as the black holes spiral (and thus accelerate) around each other, generates a gravitational wave-like ripple which radiates out into space (e.g. Buonanno & Damour 2000; Barack & Cutler 2004; Baker et al. 2006; Blanchet 2006; Sesana 2010; Amaro-Seoane et al. 2012). Travelling at the speed of light, the amplitude of the wave decays linearly (rather than quadratically) with distance and, also unlike light, passes unimpeded through both space and matter. Due to the large orbital size of the binary black hole, space-based interferometers at great separations are required to sample the long wavelength of the waves generated by the black hole binary. Build-

[35] Lauer et al. (2007) found that using the radius where the negative, logarithmic slope of the surface brightness profile equals 0.5 (which matches well with the core-Sérsic break radius: Dullo & Graham 2012, their section 5.2) produces a stronger relation than obtained when using the Nuker model break radii.

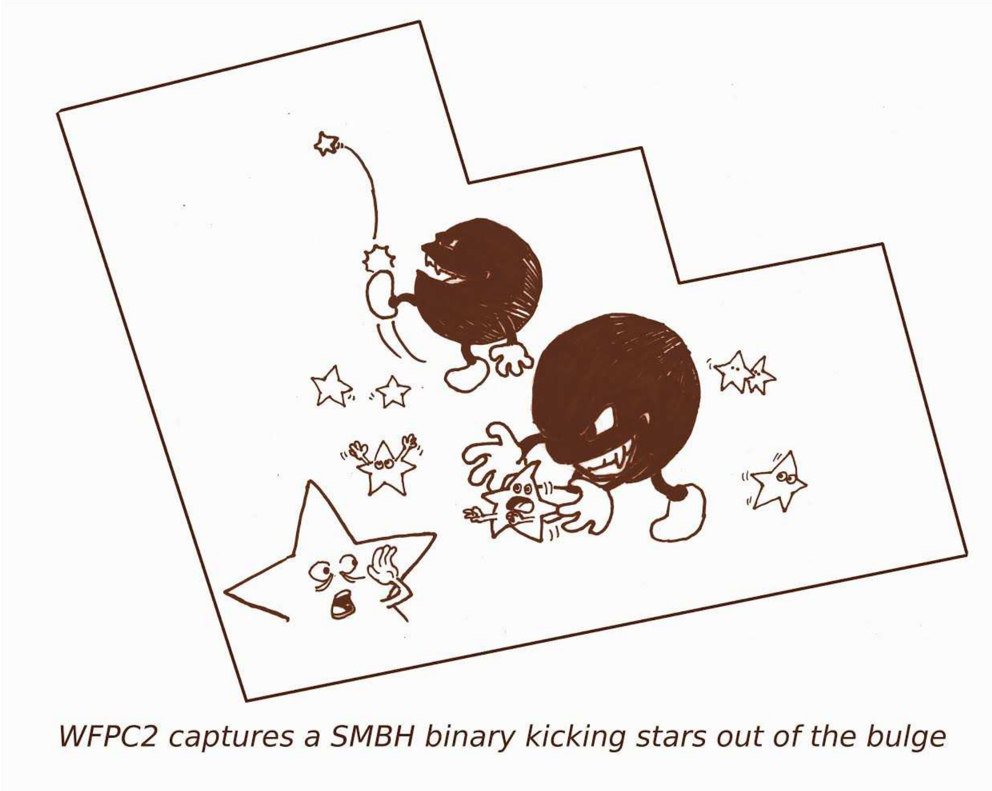

Figure 7. Cartoon showing a pair of supermassive black holes kicking stars away as they dance towards coalescence at the centre of a galaxy. Credit: Paolo Bonfini.

ing on the hopes of the Laser Interferometer Space Antenna (LISA: Danzmann & Rüdger 2003), the European LISA Pathfinder mission[36] (LPF: Anza et al. 2005; McNamara 2013), formerly known as SMART-2, offers the very exciting promise of detecting these waves predicted by Einstein's theory of relativity but not yet detected.

9. INTERMEDIATE MASS BLACK HOLES AND THE (BLACK HOLE)–(NUCLEAR CLUSTER) CONNECTION

As was noted in section 4.1, the bent $M_{\rm bh}$–$M_{\rm sph}$ relation offers hope for detecting the missing population of intermediate mass black holes. This is because the linear $M_{\rm bh}$–$M_{\rm sph}$ relation predicts $10^2 < M_{\rm bh}/M_\odot < 10^5$ black hole masses in smaller / fainter spheroids. Although we may not have the spatial resolution at optical/near-infrared wavelengths to resolve the sphere-of-influence of these black holes, and thus directly measure their masses from Keplerian kinematics, there is an independent method which can be used to predict (strengthen / reject) the likely existence of such intermediate mass black holes. It is based on the observation that the black hole mass correlates with the AGN radio and X-ray flux in such a way that they define a 2-dimensional surface in 3-parameter space, which has been dubbed the 'fundamental plane of black hole activity' (Merloni et al. 2003). Therefore, obtaining radio and X-ray data is expected to prove fruitful in the hunt for the elusive intermediate mass black holes. Preferably, this data should be obtained simultaneously because the AGN are known to vary in their flux output over timescales of days.

One of the best candidates for an intermediate mass black hole is the ultraluminous X-ray source HLX-1 in the galaxy ESO 24349 (Farrell et al. 2009; Webb et al. 2014). Interestingly, this 9,000 solar mass black hole candidate does not reside near the centre of its host galaxy but in a compact star cluster (Soria et al. 2010; Wiersema et al. 2010; Farrell et al. 2012) located at a projected distance of $\sim$3 kpc from the galaxy's nucleus, perhaps shedding insight into the formation location of intermediate mass black holes. Despite early hopes for intermediate mass black holes in globular clusters (e.g. Gerssen et al. 2003; Gebhardt et al. 2005; Noyola et al. 2010; Lützgendorf et al. 2013, and references therein), there are not yet any definite candidates (e.g. van den Bosch et al. 2006; Hurley 2007; Anderson & van der Marel 2010; Vesperini & Trenti 2010; Lanzoni et al. 2013; Lanzoni 2015). Observational research programs (e.g. Bellini et al. 2014; Lapenna et al. 2014) continue the hunt as the formation of intermediate mass black holes in dense star clusters seems probable (e.g. Miller & Hamilton 2002; Baumgardt et al. 2004; Gürkan et al. 2004; Portegies Zwart et al. 2004).

Aside from globular clusters, some of the dense star clusters found in the nuclei of many low- and intermediate-luminosity spheroids (e.g. Reaves 1983; Binggeli et al. 1985; Phillips et al. 1996; Carollo et al. 1997) are already known to house massive black holes. Ferrarese et al. (2006b) and Wehner & Harris (2006) originally suggested that these star clusters may be the low-mass extension of the supermassive black holes, in the sense that galaxies housed one type of nucleus or the other. However this idea was soon modified when it was realised that such clusters and massive black hole coexist in substantial numbers of galaxies (e.g. González

[36] http://sci.esa.int/lisa-pathfinder/

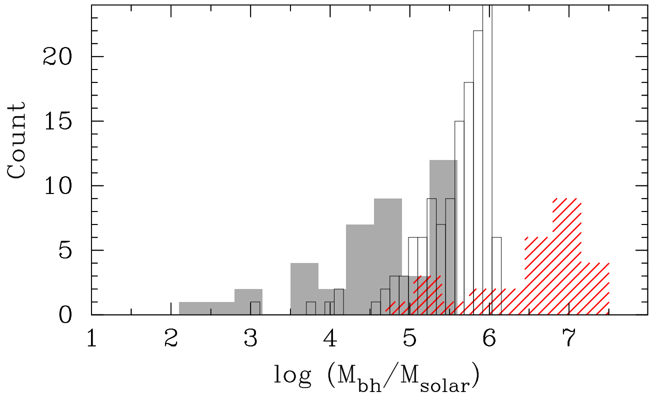

Figure 8. Predicted black hole masses. The solid histogram was obtained using the $M_{\rm bh}$–L_K relation for Sérsic spheroids applied to the K-band bulge magnitudes in Graham & Scott (2013, their table 6). The open histogram was obtained using the $M_{\rm bh}$–$M_{\rm sph}$ relation for Sérsic spheroids (shown in Figure 2) applied to the dwarf galaxy masses in Reines et al. (2013, their table 1). The shaded histogram was obtained in the same way but using the dwarf galaxy stellar masses in Moran et al. (2014, their table 1). The fainter bulges are expected to contain the least massive black holes.

Delgado et al. 2008; Seth et al. 2008; Graham & Spitler 2009). Ongoing efforts have revealed that nuclear star clusters do not follow the same mass scaling relations as supermassive black holes (Graham 2012b; Leigh et al. 2012; Neumayer & Walcher 2012; Scott & Graham 2013), and the search for intermediate mass black holes continues. Among the most promising targets are the low mass bulges of disc galaxies hosting an AGN (Graham & Scott 2012) and the low mass dwarf galaxies which also display AGN activity (e.g. Reines et al. 2013; Moran et al. 2014); see Figure 8.

Just as there is a relation between spheroid luminosity and the central surface brightness[37] of the spheroid — until the onset of partially depleted cores in massive spheroids — there is also a relationship between spheroid luminosity and the brightness of the nuclear star clusters that they host (Balcells et al. 2003; Graham & Guzmán 2003). In a somewhat similar manner to the establishment of the $M_{\rm bh}$–μ_0 relation presented in Section 7, one can predict what the $M_{\rm bh}$-$M_{\rm nc}$ relation should be like. Graham (2015) combined the relation $M_{\rm bh} \propto M_{\rm sph}^2$ for the Sérsic spheroids (Section 4.1) with the relation $M_{\rm nc} \propto M_{\rm sph}^{0.6-1.0}$ (references above) to obtain $M_{\rm bh} \propto M_{\rm nc}^{2-3.3}$. A consistent result was obtained by coupling the relation $M_{\rm bh} \propto \sigma^{5.5}$ (Section 5) with $M_{\rm nc} \propto \sigma^{1.6-2.7}$ (references above) to give $M_{\rm bh} \propto M_{\rm nc}^{2.0-3.4}$. Massive black holes therefore grow rapidly within their host star cluster, until it is evaporated (e.g. Bekki & Graham 2010) or partially devoured (e.g. Hills 1975; Frank & Rees 1976; Murphy et al. 1991; Komossa 2013; Donato et al. 2014; Vasiliev 2014). However, disentangling which came first may be an interesting pursuit, and just as there are different types of bulges, there may be different types of nuclear star clusters (e.g. Turner et al. 2012). This $M_{\rm bh}$-$M_{\rm nc}$ relation is somewhat complementary to the $M_{\rm bh}$-$M_{\rm def}$ relation, with each applicable at opposing ends of the black hole mass range currently accessible. Such co-occupancy of black holes and nuclear star clusters is a likely source of stellar tidal disruption events (Komossa et al. 2009, 2013 and references therein) and gravitational wave emission from the inspiralling of compact stellar remnants (e.g. Amaro-Seoane et al. 2007 and references therein), predictions for which are dramatically modified when using the new, near-quadratic $M_{\rm bh}$-$M_{\rm sph}$ relation (Mapelli et al. 2012). Further quantifying the coexistence of massive black holes in dense, compact, nuclear star clusters should help us to predict the occurrence of, and better understand, these exciting phenomenon.

[37] Technically it is the central surface brightness of the spheroid excluding blips from additional nuclear components such as star clusters.

10. THE $M_{\rm BH}$–$M_{\rm HALO}$ RELATION

Ferrarese et al. (2002) have revealed that there is a relationship between the black hole mass and the galaxy halo mass (baryons plus dark matter), as traced by the circular velocity at large radii (used as a proxy for the halo's virial radius). Due to the relation between this rotational velocity and the galaxy's velocity dispersion (see also Baes et al. 2003; Pizzella et al. 2005; Ferrarese & Ford 2005, their Eq. 21)[38] one can expect an $M_{\rm bh}$–$M_{\rm halo}$ relation. The extent of this relationship may be applicable only to galaxies with large bulges (or $v_{\rm circ} >\sim 100$ km s^{-1} or $\sigma >\sim 100$ km s^{-1}), because of the breakdown in the relationship between circular velocity and velocity dispersion for lower mass systems (e.g. Zasov et al. 2005; Ho 2007; Courteau et al. 2007). Nonetheless, this would make the relationship exist over a larger mass range than the $M_{\rm bh}$–$R_{\rm b}$ and $M_{\rm bh}$–$M_{\rm def}$ relations (Section 8).

For galaxies built from major dry merger events, in which the black hole mass and the galaxy stellar mass simply add together, the dark matter must also add in this linear fashion. This would then establish a linear $M_{\rm bh}$–$M_{\rm halo}$ relation — just as there is a linear $M_{\rm bh}$–$M_{\rm sph}$ relation preserving the $M_{\rm bh}/M_{\rm sph}$ ratio — at high masses ($M_{\rm bh} >\sim 10^8\,M_{\odot}$). This appears to be consistent with the data in Ferrarese et al. (2002, their figure 5). However, their linear regression to the fuller sample gives $M_{\rm bh} \propto M_{\rm halo}^{1.65-1.82}$, which is in remarkable agreement with the prediction $M_{\rm bh} \propto M_{\rm halo}^{5/3}$ by Haehnelt, Natarajan & Rees (1998). Although, with a different sample, Baes et al. (2003) reported $M_{\rm bh} \propto M_{\rm halo}^{1.27}$. Curiously, for elliptical galaxies not built from dry mergers[39], the prediction by Haehnelt et al. (1998) transforms into $M_{\rm bh} \propto L_{\rm gal}^{20/9} (= L_{\rm gal}^{2.22})$ if $M_{\rm halo}/L_{\rm gal} \propto L_{\rm gal}^{1/3}$ (Jørgensen et al. 1996; Cappellari et al. 2006). This near-quadratic relation has been seen before in Section 4.

Lending support to the $M_{\rm bh}$–$M_{\rm halo}$ relation, it is noted that another black hole mass relation exists with the halo of globular clusters that swarm around galaxies, both in

[38] It should be noted that the dynamical study by Kronawitter et al. (2000) and Gerhard et al. (2000), which led to the relationship between the circular velocity and the velocity dispersion for elliptical galaxies, was based on a sample of elliptical galaxies that had very similar absolute magnitudes. Consequently, these galaxies will have similar structural and dynamical profiles, and thus their $v_{\rm circ}$–σ relationship may not be applicable to lower- or higher-luminosity elliptical galaxies with different Sérsic indices, i.e. concentration, and dynamical profiles (e.g. Ciotti 1991).

[39] Equal mass, (major) dry mergers preserve the $M_{\rm halo}/L$ ratio and therefore galaxies built from major dry mergers follow the sequence $M_{\rm halo}/L \propto L^0$.

terms of their number (Burkert & Tremaine 2010; Harris & Harris 2011; Rhode 2012; Harris et al. 2014) and their velocity dispersion (Sadoun & Colin 2012; Pota et al. 2013). The globular cluster system (GCS) around individual galaxies are known to display a bimodality in their colour, with the red (metal rich) globular clusters thought to be associated with the galaxy's bulge while the blue (metal-poor) globular clusters are thought to be connected with the halo (Ashman & Zepf 1992; Forbes et al. 1997). Using both the observed velocity dispersion of the GCS, and the velocity dispersion with the rotational component of the system subtracted, Pota et al. (2013) report that while a correlation with black hole mass is evident, it is not yet clear if the black hole mass is better correlated with the red (bulge) or the blue (halo) globular cluster sub-population.

11. THE $M_{\rm BH}$–(SPIRAL ARM PITCH ANGLE) CONNECTION

While the applicability of the $M_{\rm bh}$–$M_{\rm halo}$ relation in lower mass spiral galaxies is unclear, there is a somewhat complementary relation which only operates in spiral galaxies. Seigar et al. (2008; see also Ringermacher & Mead 2009; Treuhadrt et al. 2012 and Berrier et al. 2013) have presented the relation between black hole mass and spiral arm pitch angle. The spiral arm pitch angle (e.g. Puerari et al. 2014, and references therein) is of course known to vary along the Hubble-Jeans sequence, as does the bulge-to-total flux ratio, or more correctly the luminosity of the bulge (e.g. Yoshizawa & Wakamatsu 1975; Ostriker 1975; Meisels & Ostriker 1984; Trujillo et al. 2002), which may explain the black hole connection with the pitch angle. As with the radial concentration of the bulge light, the pitch angle has the advantage that it can be measured from photometrically uncalibrated images and therefore offers an easy means to predict black hole masses (perhaps even when there is no bulge[40]), from which one can then do clever things like determine the black hole mass function in spiral galaxies (Davis et al. 2014).

12. FUNDAMENTAL PLANES: ADDING A THIRD PARAMETER

As noted in Section 9, stellar and supermassive black holes roughly define a plane within the 3-dimensional space of black hole mass, radio power and X-ray luminosity (Merloni et al. 2003; Heinz & Sunyaev 2003; Falcke et al. 2004; Körding et al. 2006; Li et al. 2008). While this is both interesting in its own right and highly useful, the relationship between the black hole mass, accretion disc and jet is of a different nature to the other relations presented in this article and as such is not detailed here as it is an AGN phenomenon.

One of the early attempts to introduce a third parameter into the (black hole)–(host galaxy) scaling relations was by Marconi & Hunt (2003). They used the effective half light radius ($R_{\rm e}$) of the spheroid, together with the velocity dispersion (σ), to derive a rough virial mass for the spheroid ($M_{\rm virial} \propto \sigma^2 R_{\rm e}$). They found that the total vertical scatter about their $M_{\rm bh}$–$M_{\rm virial}$ relation was slightly less than that about their $M_{\rm bh}$–σ relation (0.25 dex vs 0.30 dex). Using a sample of elliptical galaxies, Feoli & Mele (2005; see also Feoli & Mancini 2009, 2011) reported on a black hole mass relation with the kinetic energy of the host galaxy such that $M_{\rm bh} \propto (M_{\rm gal}\sigma^2)^\alpha$, where $0.87 < \alpha < 1.00$ and $M_{\rm gal}$ was derived assuming $R^{1/4}$ light profiles[41]. Given that $M_{\rm gal}$ roughly scales as $\sigma^2 R_{\rm e}$, their kinetic energy expression roughly scales with $\sigma^4 R_{\rm e}$. Additional variations of this theme, searching for a fundamental plane using combinations of σ and $R_{\rm e}$ can be found in de Francesco et al. (2006), who effectively suggested independent exponents for σ and $R_{\rm e}$, in Aller & Richstone (2007) in terms of the gravitational binding energy, in Hopkins et al. (2007) and in Soker & Meiron (2011). Given the existence of *the* Fundamental Plane (Djorgovski & Davis 1985) linking the velocity dispersion with the mean effective surface brightness ($\langle\mu\rangle_{\rm e}$) and effective half light radius, the presence of the $M_{\rm bh}$–σ relation additionally suggests that their should be an $M_{\rm bh}$–($\langle\mu\rangle_{\rm e}, R_{\rm e}$) plane (Barway & Kembhavi 2007).

With all of these attempts to define different planes, there are two issues that require attention: (i) barred galaxies, and (ii) the accuracy[42] and thus usefulness of $R_{\rm e}$.

First, the increased scatter in the $M_{\rm bh}$–σ diagram due to the inclusion of barred galaxies was reported by Graham (2008a,b) and Hu (2008). Moreover, Graham (2008a) showed that (once the barred galaxies were removed) there was no reduction in scatter when going from the $M_{\rm bh}$–σ diagram to the $M_{\rm bh}$–($\sigma, R_{\rm e}$) diagrams. If there is a more fundamental relation with some combination of σ and $R_{\rm e}$, than compared with σ alone, this should not have been observed. In other words, if the lower scatter about the hybrid relations is only achieved when including the barred galaxies, it suggests that something else is responsible for the reduction, such as barred galaxies having smaller $R_{\rm e}$ values than the elliptical galaxies which dominate at the high mass end of one's sample. It would be interesting to repeat this specific test, searching for an optimal fundamental plane without including the barred galaxies, with a larger galaxy sample. However this brings us to the second issue.

Given that there have been errors in the measurement of the Sérsic indices n (as revealed by Savorgnan et al. 2013), there are errors in the measurements of the published effective half light radii $R_{\rm e}$. Harris et al. (2014) show the large range of $R_{\rm e}$ values (for the same spheroid) reported by different authors for spheroids with directly measured black hole masses. A similar plot is shown in Figure 9 but this time restricting the data to that obtained from Sérsic $R^{1/n}$ model fits by different authors. Consequently, attempts to use $R_{\rm e}$ for measuring dynamical masses ($\propto \sigma^2 R_{\rm e}$) or as a third parameter to mop up some of the scatter about the $M_{\rm bh}$–σ relation should at this time be treated with caution.

13. CONCLUDING REMARKS

The "attraction" of black holes is vast, as evinced by a huge literature on the subject, of which but a small frac-

[40] The $M_{\rm bh}$–(pitch angle) relation is yet to be established for a sample of bulgeless galaxies.

[41] It should be noted that the assumption of $R^{1/4}$ light profiles can introduce a systematic bias with galaxy mass, Sérsic index and effective radius (e.g. Trujillo et al. 2001; Brown et al. 2003).

[42] It could be argued that a third issue is the accuracy of the black hole masses (Merritt 2013).

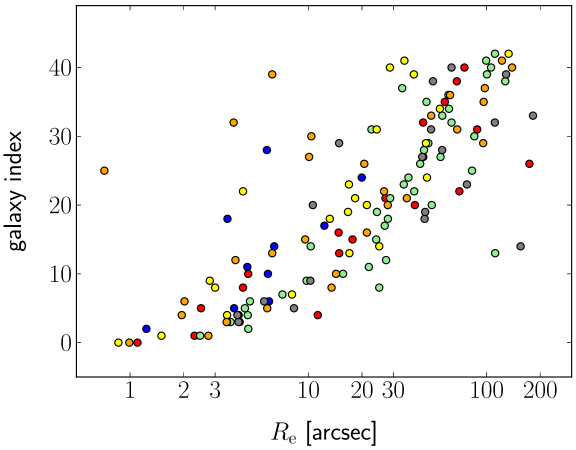

Figure 9. Major-axis effective half-light radii R_e for the spheroidal component of 43 galaxies (having directly measured black hole masses) as determined by different authors. Figure taken from Savorgnan et al. (in prep.). Legend: red = Graham & Driver (2007); blue = Laurikainen et al. (2010); green = Sani et al. (2011); yellow = Vika et al. (2012); gray = Beifiori et al. (2012); orange = Läsker et al. (2014a).

tion is noted here. The fundamental physical connection between black-hole and bulge growth still awaits discovery. While it is expected that we may narrow in on the solution as we keep plugging away at more black hole mass measurements, coupled with improving the accuracy of all quantities involved, it is reasonable to expect that something unexpected may be discovered, such is the nature and joy of our collective pursuit.

Given the role that pulsars played in convincing the community that black holes may exist in 1967–8, it is perhaps fitting that arrays of pulsar beacons are used today (e.g. Sesana et al. 2008; Hobbs et al. 2010 ; Kramer & Champion 2013) to try and detect the bob and sway of the space antennae as anticipated gravitational waves — from the inspiral of supermassive black holes at the centres of newly merged galaxies — wash by oblivious to our solar system. The future detection of such gravitational radiation would provide another strong test of Einstein's theory of general relativity (e.g. Clifford 2006), which, starting 100 years ago, led to the modern prediction of dense, dark stars and supermassive black holes.

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