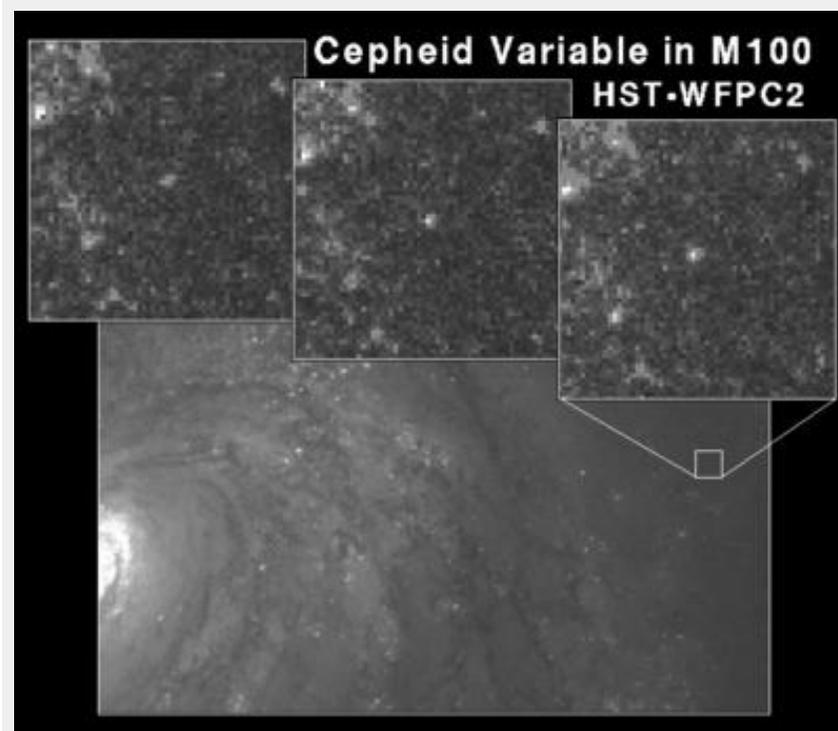


## Pulsating Stars: Stars that Breathe



**Credit:** Dr. Wendy L. Freedman, Observatories of the Carnegie Institution of Washington, and NASA

## Summary

In this Activity, we will discuss pulsating stars. In particular, we will:

- outline some early observations and theory of pulsating stars;
- describe some different classes of pulsating [star](#) and where they appear on the [HR diagram](#);
- introduce the [instability strip](#);
- develop a simple model of [stellar](#) pulsation based on sound waves and heat engines; and
- describe the  $\kappa$ -mechanism that is responsible for driving pulsations in [Cepheid](#)-type stars, and see how partial ionisation zones can explain the existence of an instability strip.

## Introduction

Pulsating stars are a type of variable star in which brightness variations are caused by changes in the [area](#) and temperature of the star's surface layers.

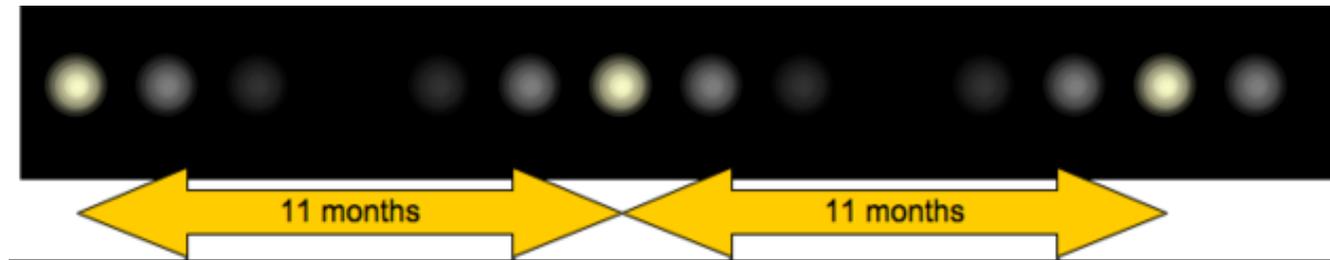
Recent evidence suggests that all stars pulsate (if we measure them carefully enough), although the presence of concentrated populations of pulsating stars on the HR diagram implies that pulsations are more important at particular stages of [stellar evolution](#).

As well as being fascinating phenomena in their own right, stellar pulsations are used to constrain theories of stellar evolution and to study the mechanisms of stellar interiors.

## The discovery of Mira

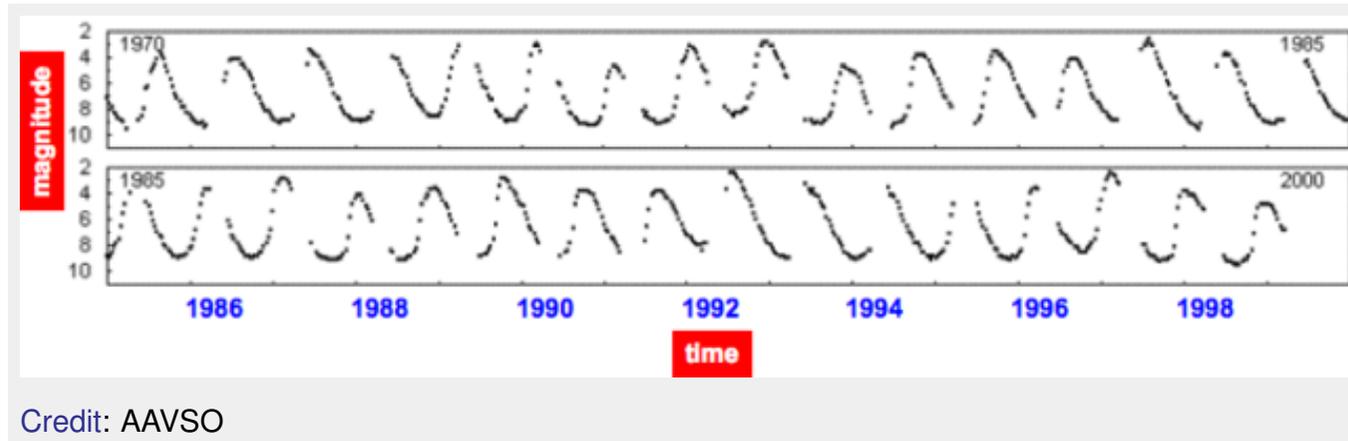
The first pulsating star,  $\alpha$  Ceti, was discovered in 1596 by David Fabricius.

Fabricius' observations showed that over a **period** of 11 months, the bright **second** magnitude star faded, disappeared, and then finally returned to its former brightness.



$\alpha$  Ceti was later called 'Mira' (meaning 'wonderful') to describe its unusual behaviour.

## Light Curve of Mira



This figure shows the light curve of Mira, compiled from observations dating back to 1970.

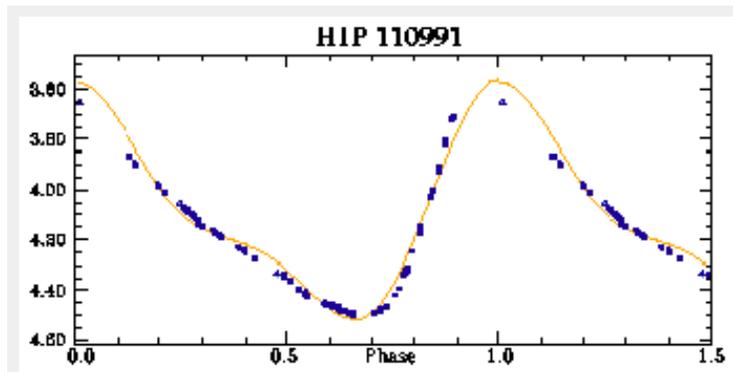
Mira's [apparent magnitude](#) varies between +3.5 and +9 over a period of  $\sim 322$  days. It is considered the prototype of long period variables, stars with pulsation periods between 100 and 400 days, and that are slightly irregular in period and [amplitude](#).

[Generate light curves for variable stars observed by the AAVSO](#)

[Try fitting light curve data to different pulsation periods](#)

## $\delta$ Cephei

The most important discovery for stellar pulsation theory, however, was the observation of periodic light variations in the yellow supergiant,  $\delta$  Cephei, in 1784.



$\delta$  Cephei light curve from HIPPARCOS.

Credit: ESTEC, ESA.

$\delta$  Cephei has a pulsation period of 5 days, 8 hours and 37 minutes and exhibits magnitude variations of  $\sim \pm 1$  mag.

It is the prototype of a kind of pulsating star called a classical Cepheid, which demonstrate a strict relationship between **luminosity** and pulsation period.

## Henrietta Swan Leavitt

Henrietta Leavitt (1868 - 1921) was employed at the Harvard College Observatory to determine the magnitude of stars from photographic plates. As part of this project, she discovered more than 2400 [variable stars](#).

Looking at the variable stars that she had discovered in the [Small Magellanic Cloud](#), Leavitt noticed that [Cepheids](#) with long pulsation periods were intrinsically more [luminous](#) than their short period counterparts.

Her discovery was the basis of the Cepheid period-luminosity relation (also called the P-L relation), which remains an important tool for measuring the [distance](#) to nearby [galaxies](#).

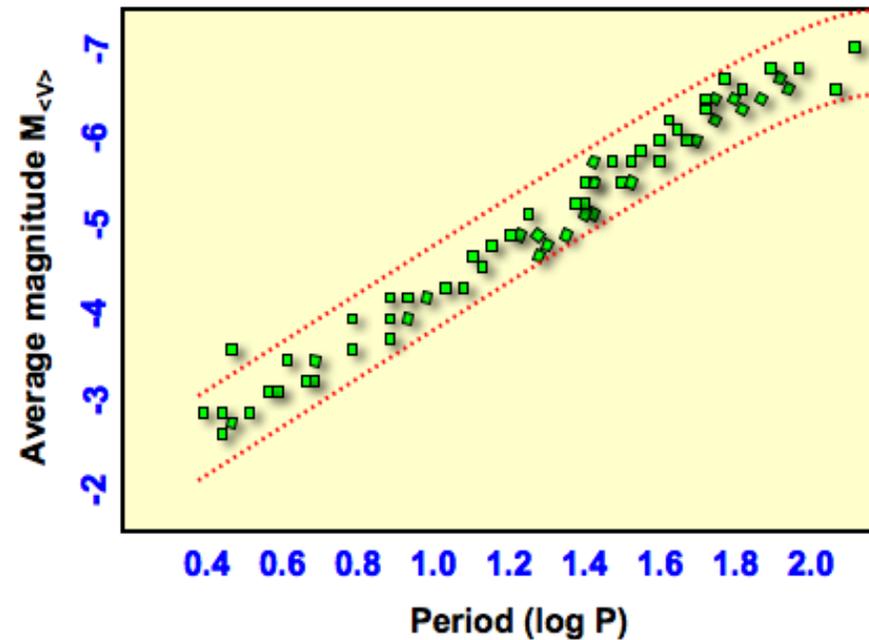
[Read Leavitt's note](#) in the Harvard College Observatory newsletter, in which she describes the relationship between brightness and period for 25 stars in the [SMC](#).



Credit: AIP Emilio Segrè Visual Archives

## Period-Luminosity Relation

This figure shows [absolute magnitude](#) as a function of period for classical Cepheids in the [Milky Way](#) and other [Local Group](#) galaxies.



From this plot (and many similar observations), **astronomers** constructed the following equations for the luminosity and absolute magnitude of a Cepheid:

$$\log_{10} \left( \frac{\langle L \rangle}{L_{\odot}} \right) = 1.15 \log_{10} \prod^d + 2.47$$

$$M_{\langle V \rangle} = -2.80 \log_{10} \prod^d - 1.43$$

where  $\langle L \rangle$  is the star's average luminosity;  $\prod^d$  is its pulsation period in days; and  $M_{\langle V \rangle}$  is the star's average absolute magnitude.

We will explore the physics behind this empirical law later in this Activity.

## A rung of the distance ladder

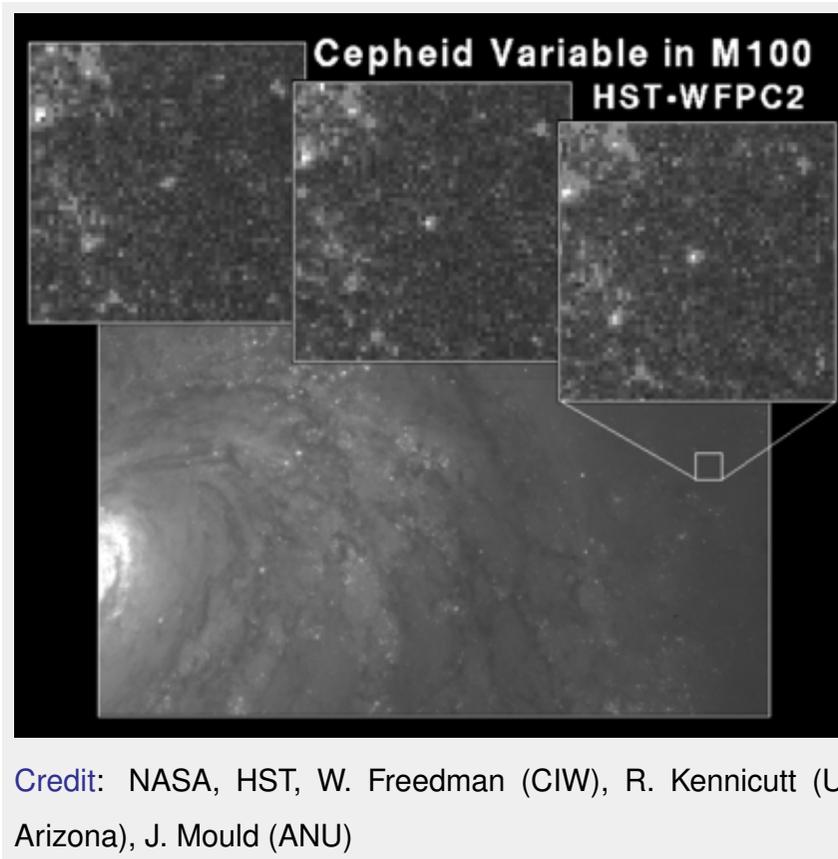
You should already appreciate the enormous power of such a relationship: it says that if we can measure the pulsation period of distant Cepheids, then we have an immediate, accurate estimate of their luminosity (and hence absolute magnitude).

We can then use the [distance modulus](#) equation that we saw in the Activity *Colour and Magnitude*, to find the distance to the Cepheid:

$$m_V - M_V = 5 \log_{10} r - 5$$

where  $r$  is the distance to the Cepheid,  $m_v$  is the star's apparent magnitude; and  $M_v$  is the absolute magnitude that we obtain from the P-L relation.

A primary scientific motivation for the [HST](#) was to detect Cepheids in the Virgo [cluster of galaxies](#), and hence derive an accurate distance estimate for the cluster.



Credit: NASA, HST, W. Freedman (CIW), R. Kennicutt (U. Arizona), J. Mould (ANU)

An HST image of a Cepheid in the outer regions of M100, a [spiral galaxy](#) belonging to the [Virgo Cluster](#). Correlating the Cepheid distance with the galaxy's [redshift](#) will help constrain the Hubble constant, and thus provide an estimate for the age and size of the [Universe](#).

You can read more about recent attempts to define the Hubble constant in the Unit HET616, *Great Debates in Astronomy*.

## Pulsating Stars on the HR Diagram

Following these discoveries, many new types of pulsating stars with a wide range of periods and luminosity variations were observed, although a reasonable theory of stellar pulsation was not developed until the 1940s.

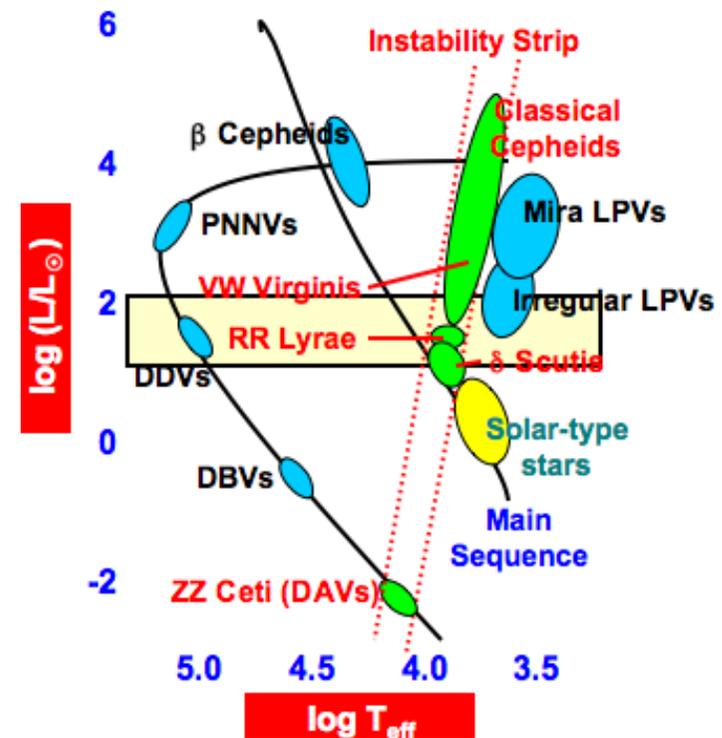
We will look at the reasons why stars pulsate later in this Activity. For the moment, let's have a look at where pulsating stars can be found on the HR diagram...

## The Instability Strip

Here we see various populations of pulsating stars plotted on the HR diagram (we will learn about their characteristics later in this Activity).

In particular you should note:

- pulsating stars are not generally found on the main sequence. Instead, most pulsating stars occupy a narrow, vertical band on the right hand side of the HR diagram. This is called the **instability strip**.
- Astronomers believe that pulsation is a transient phenomenon - as the stars follow their evolutionary tracks on the HR diagram, they will pass through the instability strip where they exhibit large brightness variations.
- Given the large number of pulsating star populations on the HR diagram, it seems reasonable to suspect that all stars pulsate. As observational techniques become more sensitive, smaller amplitude pulsations in stars on other regions of the HR diagram may be detected.



## Classes of pulsating stars

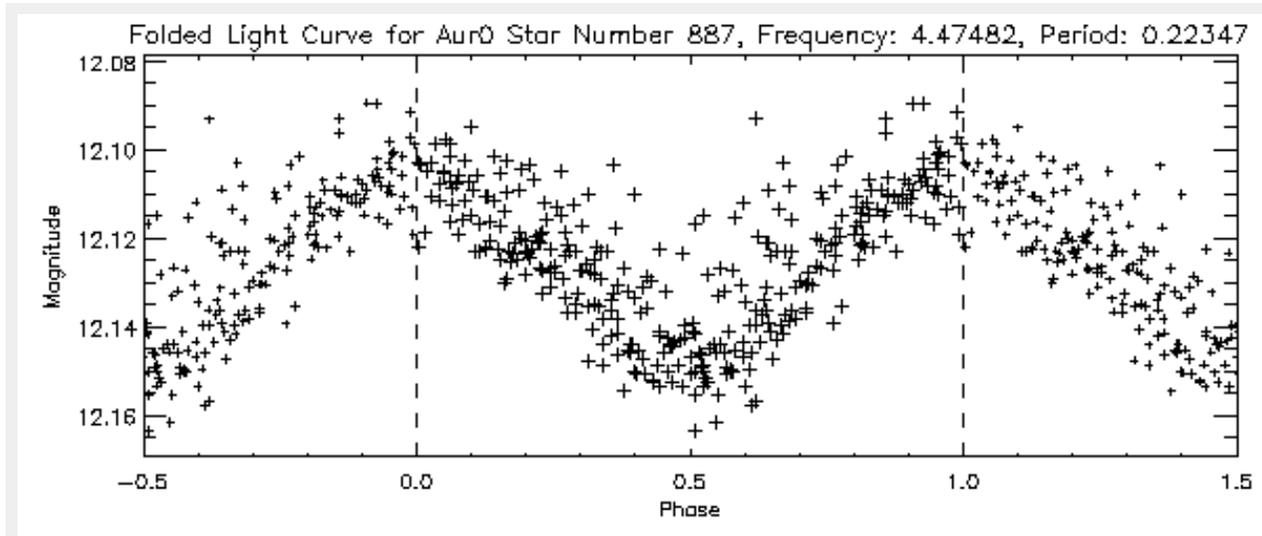
This table lists the characteristics of the major pulsating star classes that we saw illustrated on the HR diagram.

| Type                         | Period           | Population Type | Radial or Non-Radial Pulsation |
|------------------------------|------------------|-----------------|--------------------------------|
| Long period variables (LPVs) | 100-700 days     | I,II            | R                              |
| Classical Cepheids           | 1-50 days        | I               | R                              |
| W Virginis stars             | 2-45 days        | II              | R                              |
| RR Lyrae stars               | 1.5-24 hours     | II              | R                              |
| $\delta$ Scuti stars         | 1-3 hours        | I               | R, NR                          |
| $\beta$ Cephei stars         | 3-7 hours        | I               | R, NR                          |
| ZZ Ceti stars                | 100-1000 seconds | I               | NR                             |

Although in most cases, observational records for each of the different pulsating classes have been kept for over a century, astronomers had no explanation for why stars pulsate until about 1915.

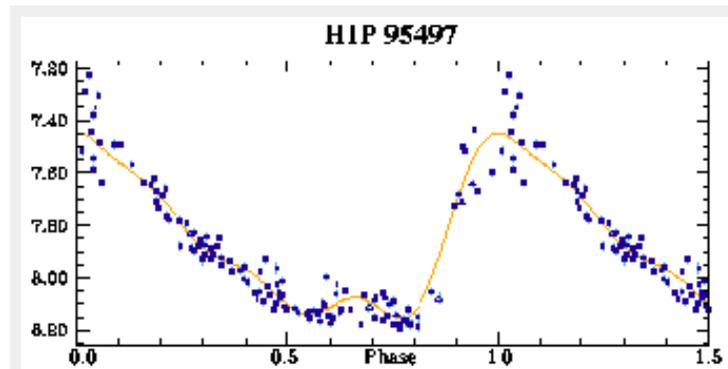
## Light curves

Here are several light curves of stars belonging to different pulsating star classes.



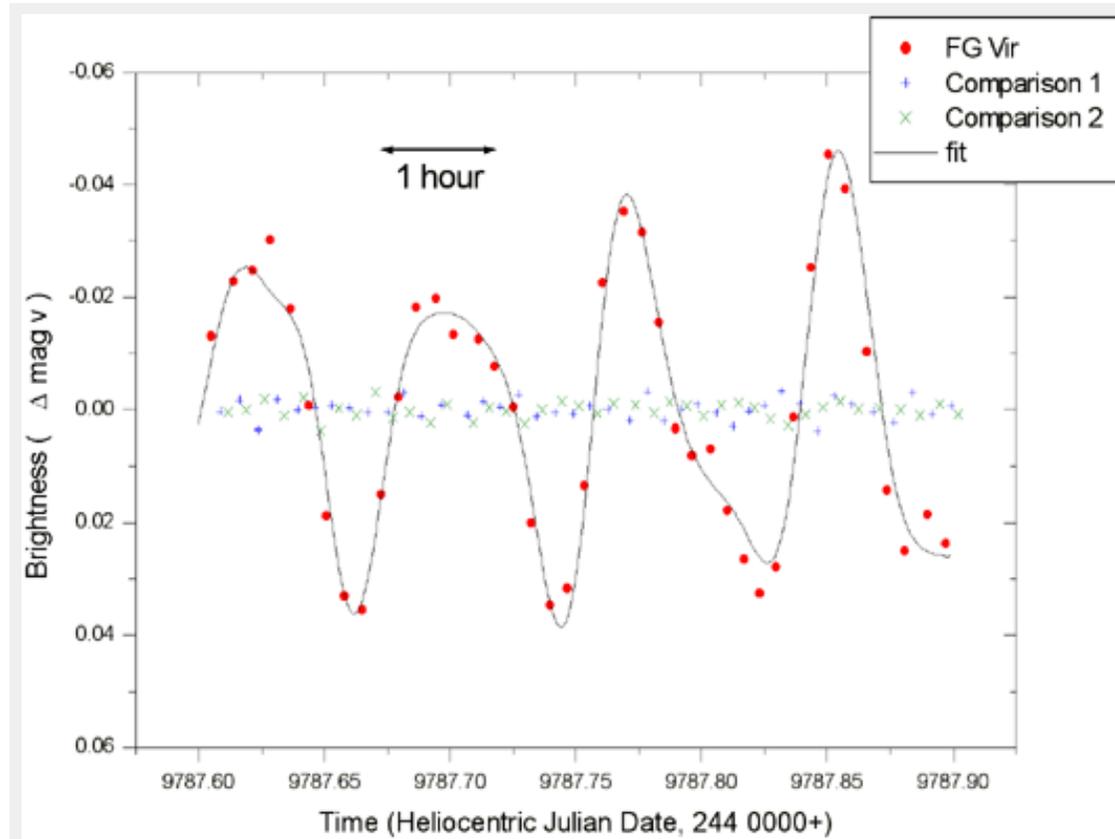
$\delta$  Cephei type variable in Auriga

Credit: STARE



RR Lyrae

Credit: ESTEC, ESA.



FG Virginis ( $\delta$  Scuti type variable)

Credit: Delta Scuti Network

## Early ideas

Early theories for the observed brightness variations of pulsating stars included dark patches on the surface of a rotating star, eclipses in a [binary system](#), and tidal effects in the atmospheres of [binary stars](#).

We now know that these ideas are wrong<sup>1</sup>. Some of the observed brightness variations could only be explained by [binary systems](#), for example, if the [orbit](#) of the (non-existent) companion star was located inside the primary star!

<sup>1</sup> In fact, [eclipsing binaries](#) do exhibit periodic variations in their light curve- you would have learnt about these systems in the Unit HET603, *Exploring Stars and the Milky Way*. In this Activity, however, we are concentrating on stars that show intrinsic variability, i.e. brightness variations that are a consequence of the physics in the stellar interior.

## 'Breathing Stars'

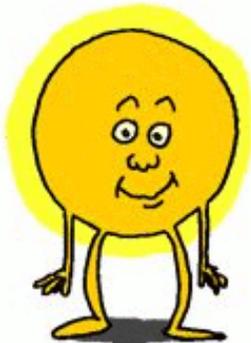
In 1914, the American astronomer, Harlow Shapley, suggested that the observed variations in temperature and brightness of Cepheid variables were caused by radial pulsation.

He argued that binary theories of stellar pulsation should be discarded and that astronomers should seek a mechanism by which single stars could rhythmically 'breathe' in and out.

Radial pulsations had been proposed by Arthur Ritter in 1879, but his ideas were overlooked until Sir Arthur Eddington attempted to provide a mathematical framework for Shapley's suggestion.

Shapley's original paper can be found on the [ADS website](#)

Look up Shapley, H., 'On the Nature and Cause of Cepheid Variation', *The Astrophysical Journal*, (1914) v. 40, pp 448-465.



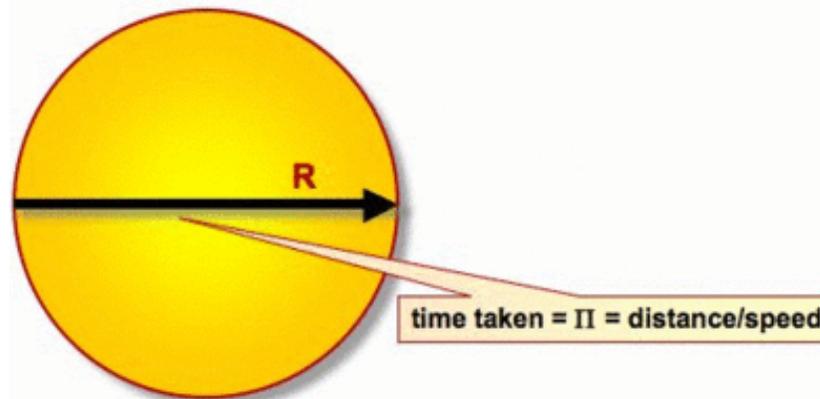
## A simple model for stellar pulsation

Eddington proposed that if we consider pulsating stars as thermodynamic heat engines, then radial oscillations may be the result of sound waves resonating in the stellar interior.

A rough estimate for the pulsation period,  $\Pi$ , is obtained by calculating the length of time it would take a sound wave to travel across the diameter of a star. That is,

$$\Pi = \frac{2R}{v_s}$$

where  $R$  is the stellar radius, and  $v_s$  is the **speed** of sound.



Arthur Eddington.

Credit: AIP Emilio  
Segrè Visual  
Archives

The speed of sound,  $\nu_s$ , can be determined from the **pressure** and **density** of the stellar interior:

$$\nu_s = \sqrt{\frac{\gamma P}{\rho}}$$

where  $P$  is the pressure,  $\rho$  is density, and  $\gamma$  is ratio of specific heats for the stellar material ( $\gamma=5/3$  for a monatomic gas).

To work out the pressure, we assume that the star has constant density and invoke the condition of **hydrostatic equilibrium**:

$$\frac{dP}{dr} = -\frac{GM_r \rho}{r^2} = -G \left( \frac{4\pi r^3}{3} \rho \right) \frac{\rho}{r^2} = -\frac{4G\pi r \rho^2}{3}$$

Integrating so that  $P=0$  at the star's surface, we can then find the pressure as a function of stellar radius:

$$P(r) = \frac{2}{3}\pi G \rho^2 (R^2 - r^2)$$

Substituting back into our equation for the pulsation period, we find,

$$\begin{aligned}\Pi &= \frac{2R}{v_s} \\ &= 2 \int_0^R \frac{dr}{\sqrt{\frac{2}{3}\gamma\pi G\rho(R^2 - r^2)}} \\ &\approx \sqrt{\frac{3\pi}{2\gamma G\rho}}\end{aligned}$$

This equation, also called the period-mean density relation, shows that the pulsation period of a star is inversely proportional to the square root of its mean density.

Although its derivation is not rigorous, substituting typical values for a classical Cepheid into the period-mean density relation produces period estimates that are in good agreement with observation.

## Eddington's Engine

What did Eddington mean when he said that a pulsating star can be characterised as a thermodynamic heat engine?

Let's consider layers inside the star as they expand and contract...

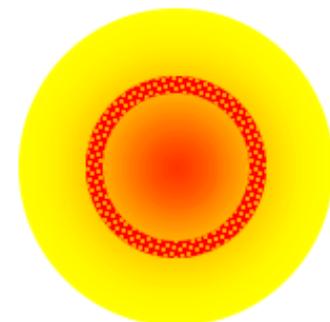
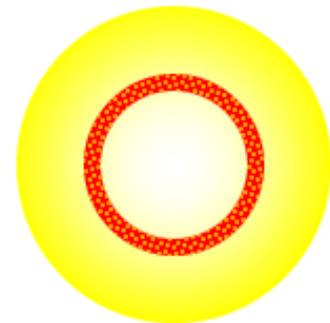
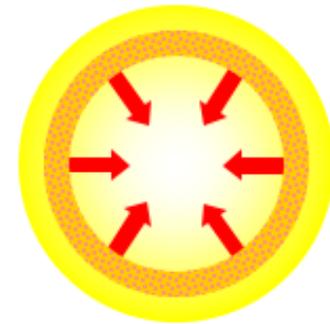


## A simple pulsation cycle

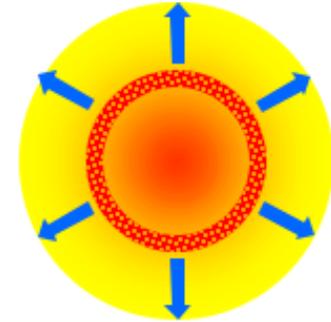
At one point in the pulsation cycle, a layer of stellar material loses support against the star's **gravity** and falls inwards.

This inward motion tends to compress the layer, which heats up and becomes more opaque to radiation.

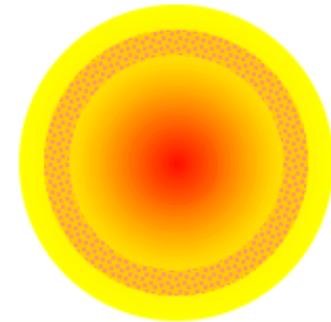
Since radiation diffuses more slowly through the layer (as a consequence of its increased opacity), heat builds up beneath it.



The pressure rises below the layer, pushing it outwards.



As it moves outwards, the layer expands, cools, and becomes more transparent to radiation.



Energy can now escape from below the layer, and pressure beneath the layer drops.



## The Cycle

The layer falls inwards and the cycle repeats.



This animation illustrates the stellar pulsation cycles.



We see that Eddington's analogy was apt: the stellar envelope *does* act like a heat engine with radiation taking the part of steam, the expanding and contracting layer acting as the piston, and the opacity of the layer acting as the valve mechanism.

Note: these diagrams are definitely not to scale!

## Kramer's Law and the $\kappa$ -mechanism

In our simple model of the stellar pulsation cycle, we require the opacity of a layer in the star to increase with compression. This is called the  $\kappa$ -mechanism.

In most regions in a star, however, opacity,  $\kappa$ , typically *decreases* with compression according to Kramer's Law:

$$\kappa \propto \frac{\rho}{T^{3.5}}$$

This law was discussed in the Activity *Energy Transport*.

## Opacity, pressure and temperature

As the layers of a star are compressed, we know that their density and temperature will increase.

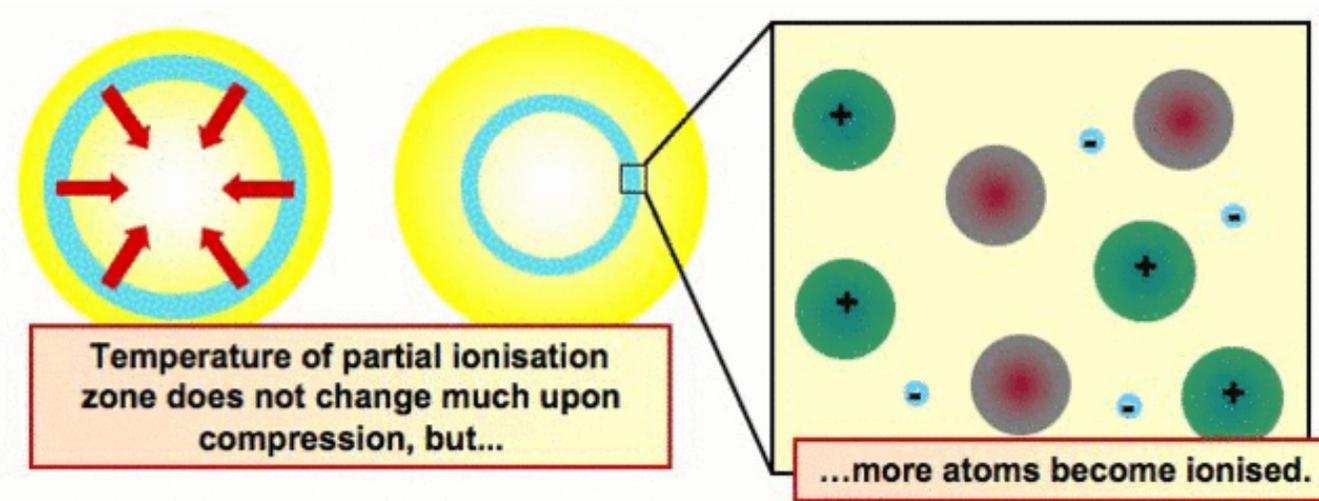
Kramer's Law, however, tells us that opacity is more sensitive to temperature than pressure, so it is very likely that the opacity of stellar material will decrease upon compression.

Evidently, for stellar pulsation to occur via Eddington's mechanism, special conditions are required.

## Zhevakin's zones

Regions of the stellar interior where increased opacity *can* provide the necessary valve mechanism to drive pulsations, were first identified by the Russian astronomer S. A. Zhevakin in the 1950s.

These were partial ionisation zones, where part of the energy released during a layer's compression can be used for further ionisation, rather than raising the temperature of the gas.



## Kramer's Law in partial ionisation zones

As the temperature of the compressed layer has not substantially increased, the increase in density produces a corresponding increase in the opacity of the layer according to Kramer's Law:

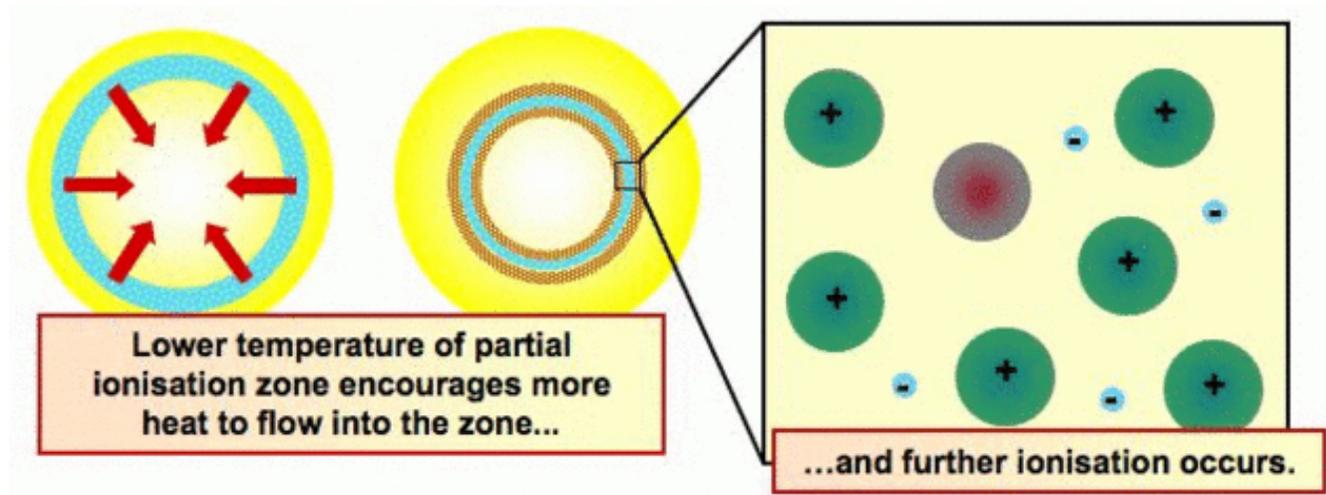
$$\kappa \propto \frac{\rho}{T^{3.5}}$$

Likewise during the expansion phase, the temperature does not decrease significantly since the **ions** release energy when they recombine with **electrons**.

## The $\gamma$ -mechanism

The  $\kappa$ -mechanism is reinforced in a partial ionisation zone because the temperature gradient between the partial ionisation zone and adjacent layers in the star encourages more heat to flow into the zone, prompting further ionisation.

This is called the  $\gamma$ -mechanism.



## Partial ionisation zones

In most stars there are two main partial ionisation zones.

The **hydrogen** partial ionisation zone is a broad region with a characteristic temperature of 1 to 1.5  $\times 10^4$  K, in which the following cyclical ionisations occur:



The **helium II** partial ionisation zone is a region deeper in the stellar interior with a characteristic temperature of  $4 \times 10^4$  K, where further ionisation of helium takes place:

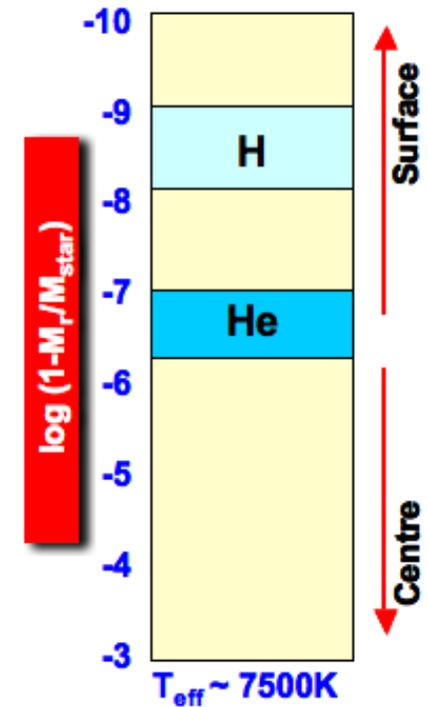


## The real-estate principle of pulsation

The pulsation properties of a star depend primarily on *where* its partial ionisation zones are found within the stellar interior.

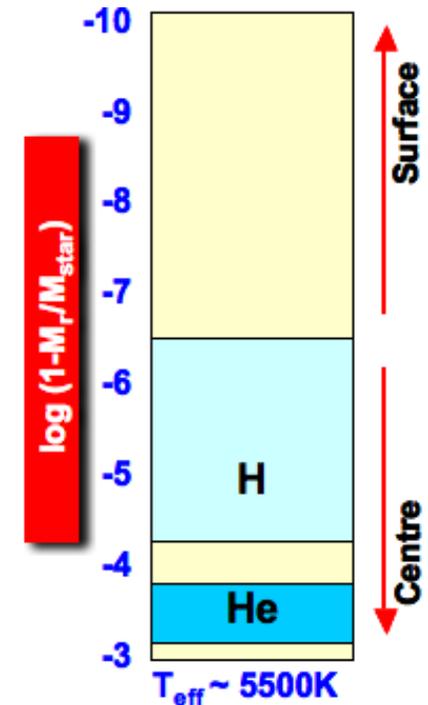
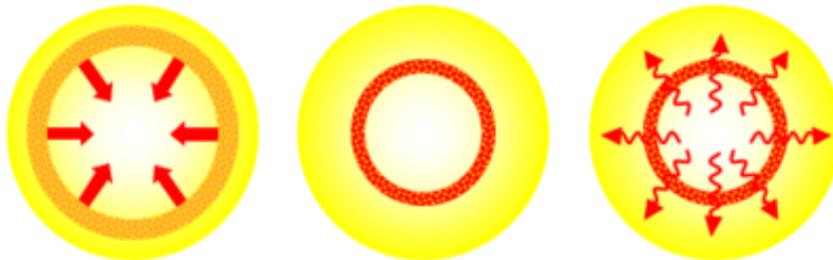
The location of the partial ionisation zones is determined by the star's temperature.

For stars hotter than  $T_{eff} \sim 7500K$ , the partial ionisation zones are located too close to the star's surface, where there is not enough mass to drive the oscillations effectively.



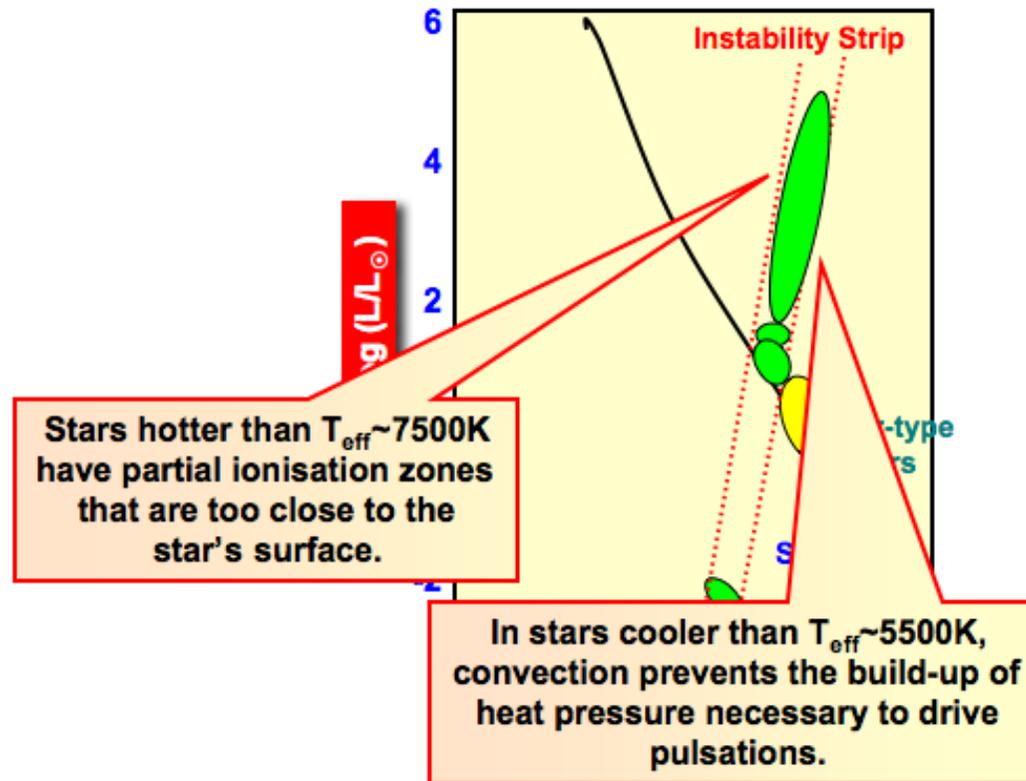
For stars cooler than  $T_{eff} \sim 5500K$ , on the other hand, the partial ionisation zones are deep in the stellar interior.

However at low temperatures, energy transport via convection becomes quite efficient in the stellar interior, preventing the build-up of heat and pressure beneath the driving pulsation layer.



## The $\kappa$ -mechanism and the instability strip

We are now in a position to understand the location of the instability strip on the HR diagram.



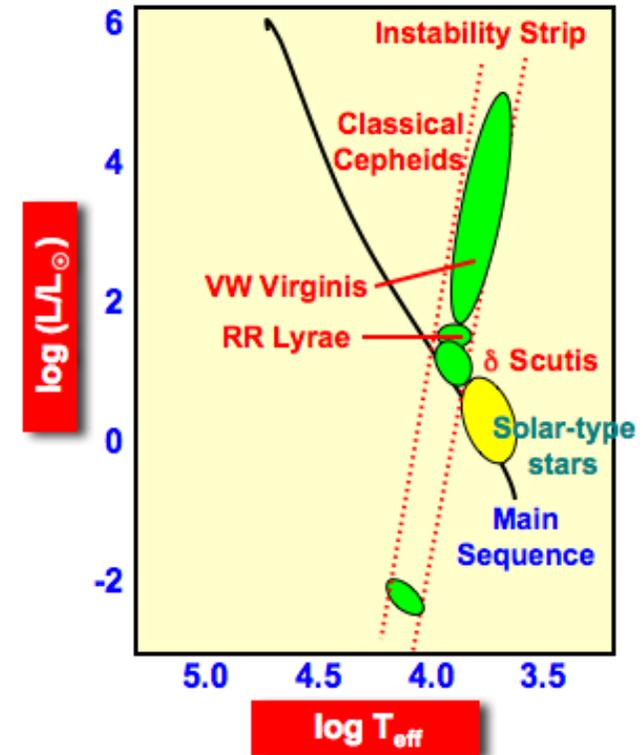
The narrow temperature range of the instability strip corresponds to the stellar temperatures that can sustain partial ionisation zones capable of maintaining stellar oscillations.

## Stars on the instability strip

$\delta$  Scuti stars are main-sequence stars with the appropriate surface temperatures.

Classical Cepheids, WW Virginis and RR Lyrae stars are giant stars of different masses that are evolving through the appropriate temperature range.

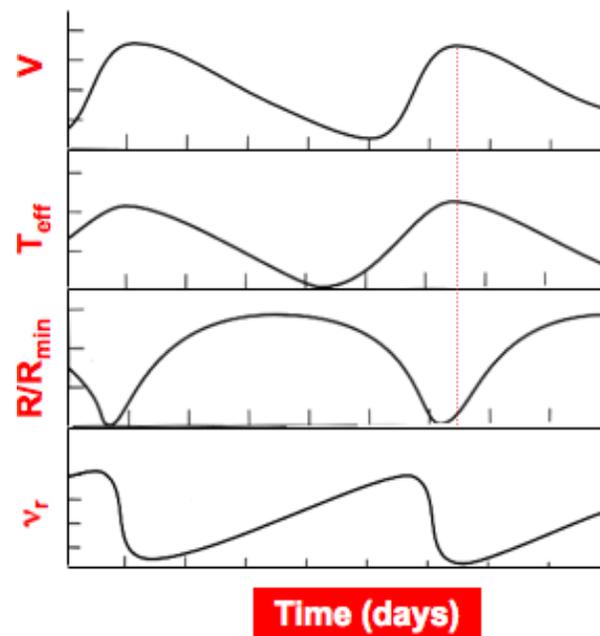
Investigating why *all* the stars on the instability strip do not pulsate remains an active area of stellar research.



## Modelling Pulsations

Computer modelling of stellar pulsation suggests that it is primarily the helium II ionisation zone which is responsible for the observed oscillations of stars on the instability strip.

The hydrogen ionisation zone, however, still plays an important role, producing an observable phase lag between the star's maximum brightness and its minimum radius.



Observed properties of a classical Cepheid. Note the phase lag between the star's maximum brightness and its minimum radius.

## Luminosity and the hydrogen ionisation zone

The maximum brightness of a pulsating star occurs when the least mass between the hydrogen ionisation zone and the star's surface is at a minimum.

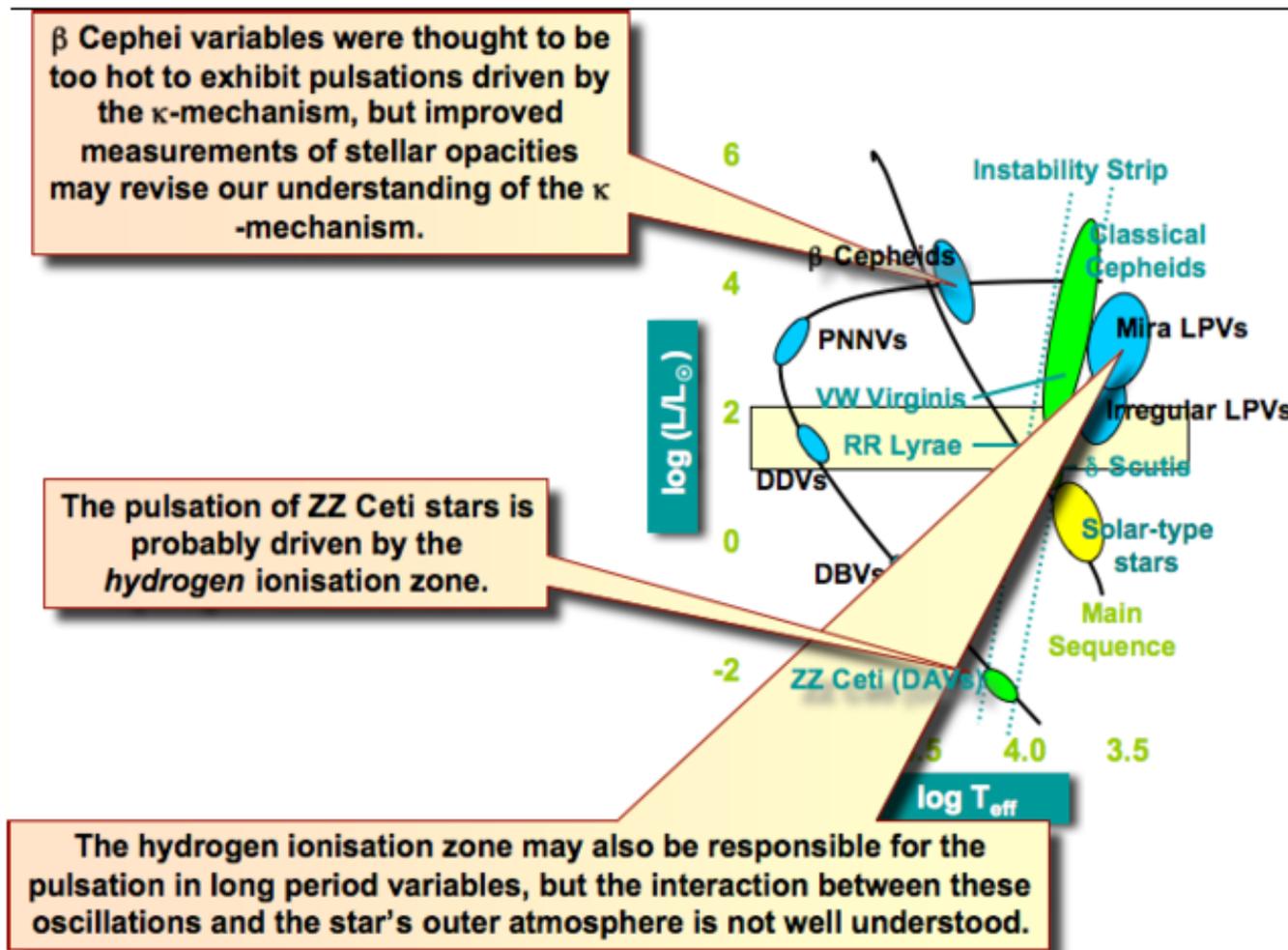
Although the energy beneath the hydrogen ionisation zone peaks when the star's radius is at a minimum, this energy propels the hydrogen ionisation zone towards the star's surface.

The observed luminosity of the pulsating star is thus at a maximum slightly after the star has been compressed to its smallest radius.

## Stars beyond the instability strip

Astronomers believe that  $\kappa$ -mechanism is almost certainly responsible for the pulsations observed in stars that are located on the instability strip: classical Cepheids, WW Virginis, RR Lyrae and  $\delta$  Scuti stars.

The pulsation mechanism for other stars, however, is not so well understood.



## Summary

In this Activity, we have looked at some of the basic properties and internal physics of pulsating stars. We discussed the history of pulsating star observations, from the first detection of Mira's pulsations to Henrietta Swan Leavitt's discovery of the period-luminosity relationship for classical Cepheids in the [Small Magellanic](#) cloud.

We saw that pulsating stars are located in many different regions of the HR diagram. In particular, there are several classes of pulsating star that are found on the instability strip. The narrow temperature range of the instability strip corresponds to the temperatures at which partial ionisation zones in the star's interior can sustain stellar oscillations.

We then developed a simple model of stellar pulsations based on the idea of sound waves and thermodynamic heat engines. These ideas were initially proposed by Arthur Eddington in the early twentieth century. Eddington's model requires that opacity acts like a valve mechanism for the stellar pulsations. Kramer's Law, however, implies that opacity tends to *decrease* rather than increase upon compression.

The special circumstances in which opacity will *increase* upon compression were identified by S. A. Zhevakin in the 1950s. Zhevakin found that partial ionisation zones absorb the energy released during a layer's compression for further ionisation of a gas rather than raising its temperature, thus increasing the layer's opacity.

Two principal partial ionisation zones were identified: the hydrogen ionisation zone and the helium II ionisation zone. Computer models show that the latter is primarily responsible for the pulsation of stars on the instability strip. The hydrogen ionisation zone is important, however, for explaining the observed phase lag between the interval of the star's maximum luminosity and minimum radius.