Simulating Dusty Gas Using SPH

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Abstract. We briefly look at dust in astrophysical environments and its importance, with particular attention given to star forming regions and protostellar disks. Smoothed Particle Hydrodynamics (SPH) is used to simulate a two–phase flow comprising of dust and gas, coupled by gravity and friction. The SPH implementation is discussed with a brief look at the drag term. To test the code, we simulate of a sound wave in a periodic box, a collapsing stationary and rotating sphere. Our results compare well with known analytic solutions and expected results.

1 Introduction

Dust is found in practically all astrophysical environments, including the ISM, the arms of spiral galaxies, elliptical galaxies, the disks of young main sequence and pre-main sequence stars and the envelopes of AGB stars and planetary nebulae. Although dust makes up only 1% of ISM, it is observationally and dynamically important through its absorption, re–radiation and scattering of light. It is, for example, thought to be the dynamical driving mechanism of AGB star mass loss, with stellar radiation forcing the dust outwards with then drags the dust with it (see e.g. Netzer & Elitzur 1993).

Generally protostellar disk are modelled with gas alone although it is well known that dust is present (Smith & Terrile 1983, Beckwith et al. 1990, Simon et al. 1992). We want to include the dust in order to investigate its dynamical importance in protostellar environments. Noh, Vishniac & Cochran (1991) showed that dust acts to *increase* the growth rate of instabilities in protostellar disks for various azimuthal wave numbers m. We will be able to test this with our model and follow the nonlinear evolution.

2 SPH Implementation of Two–Phase Flow

Following the work of Harlow & Amsden (1975) and the SPH implementation by Monaghan & Kocharyan (1995), we have setup a two–phase dusty gas code. The applications for such a two–phase code in astrophysics extend beyond dust and gas to stars and gas (for galactic simulations) and luminous and dark matter (for cosmological simulations) by changing source terms in the momentum equation. The program could easily be extended to deal with three phases, to simulation stars, gas and dust or gas and two distinct dust populations.

In two phase flows the relative motion between the two fluids involves a momentum interchange, a heat interchange and possibly mass exchange (via a phase change). For simplicity in this test study we assume that the dust does not evaporate or coagulate and that the gas does not condense. In our SPH model the forces are due to pressure, viscosity, drag and gravity.

In this paper, the subscripts g and d will represent gas and dust particles respectively, and quantity $Q_{gd} = Q_g - Q_d$. The momentum equation of the two phase flow is given by

$$\frac{d\mathbf{v}}{dt} = -\frac{1}{\rho}\nabla P \pm \frac{K}{\hat{\rho}}(\mathbf{v}_d - \mathbf{v}_g) - \nabla \Phi \ , \tag{1}$$

where K is the drag term (see below for more details).

We define the local SPH density, $\hat{\rho}$, as a product of the global (average) density, ρ, and the void fraction, Θ, by

$$\hat{\rho} = \Theta \rho \ , \tag{2}$$

where $\Theta_d + \Theta_g = 1$. In astrophysical applications, $\Theta_g \sim 1$ and $\Theta_d \ll 1$. We also assume that the dust fluid exerts negligible pressure. The SPH particles are tagged as either *dust* or *gas* and experience the appropriate particle–particle interactions. The local densities and voids are assigned initially and updated as follows; at time t, average densities are calculated by the standard SPH summation, $\hat{\rho}_i = \sum_j m_j W_{ij}$; the dust void fractions are calculated as $\Theta_d = \hat{\rho_d}/\rho_d$; and the gas void fractions as

$$\Theta_g = 1 - \Theta_d = 1 - \sum_d \frac{m_d}{\hat{\rho}_d}\Theta_d W_{gd} = 1 - \frac{1}{\rho_d}\sum_d m_d W_{gd} \ . \tag{3}$$

The time stepping routine used is a predictor–corrector method, constrained by the force, Courant condition and viscosity, plus a drag constraint such that $\delta t_{drag} < \min(\hat{\rho}/K)$

3 Drag Force

From the SPH momentum equation, the drag force (per unit mass) is represented by

$$\frac{K}{\hat{\rho}}(\mathbf{v}_d - \mathbf{v}_g) \ , \tag{4}$$

and thus we need a representation for K. The drag force is dependent on the relative velocity, fluid viscosity and particle size. Valentine & Wohletz (1989) use

$$K = \frac{\rho_g \Theta_d c_D}{r}|\mathbf{v}_{dg}|$$

for their pyroclastic applications, assuming constant density spherical grains of radius r. The drag coefficient, c_D, which depends on the Reynolds number of the flow and the shape of the body, is uncertain in astrophysical situations. In terrestrial applications it is generally determined experimentally. For our test cases, we use K = constant, to compare our results with analytic work and other simulations.

4 Tests Cases

We set up an isothermal periodic box to model an infinite dusty–gas medium to test the code for stability and sound speed accuracy. For stability, we set the particles at rest and follow the evolution for 15 sound crossing times. The linear and angular momentum are conserved and the kinetic energy remains negligible. The choice of smoothing length can have an effect however. We found $h = 1.5$ gave the worst results. To check the accuracy of the sound speed, we set up a sine wave in the box, with an amplitude of $\sim 10\%$ of the theoretical sound speed and test with $K = 0$ and compare with the limiting case solutions to the dispersion relation of Harlow & Amsden (1975), and also a non–zero value of K to compare with the solution of the full dispersion relation. The non–zero K case gave a sound speed accurate to within 5%, and for the $k = 0$ case, to within better than 2%. The error decreases as the resolution of the wave across the box in increased.

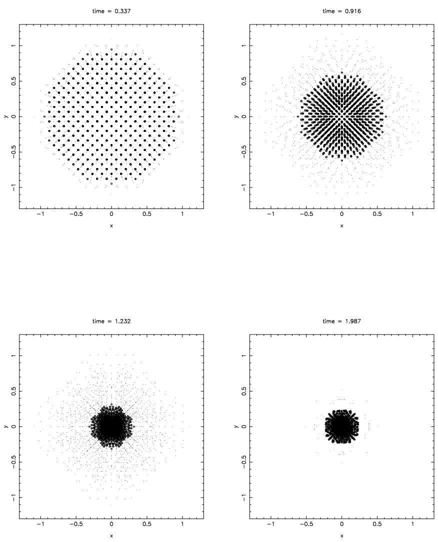

Fig. 1. Collapsing dusty–gas sphere, $K = 0$. The dust and gas are essentially decoupled and so the dust (open circles) collapses more rapidly than the gas (dots). The gravity solver is a multigrid, and thus some artificial symmetries are seen.

A collapsing initially isothermal sphere was also modelled and compared with know analytic and expected results. When $K = 0$, the gas and dust are decoupled and should collapse independently, whereas for $K \neq 0$, the fluids are coupled. We expect the gas to shock and dissipate as it is feels the effects of viscosity, which the dust does not. For a collapsing non–rotating sphere with $K = 0$ we expect the dust to falls first, followed by the gas which, unlike the dust, feels gas pressure. When $K \neq 0$, the dust will follow the gas outwards initially, then both fluids will collapse inwards. These effects were indeed observed (see Figs. 1 & 2). We also modelled a rotating sphere, which evolves to a disk. Again,

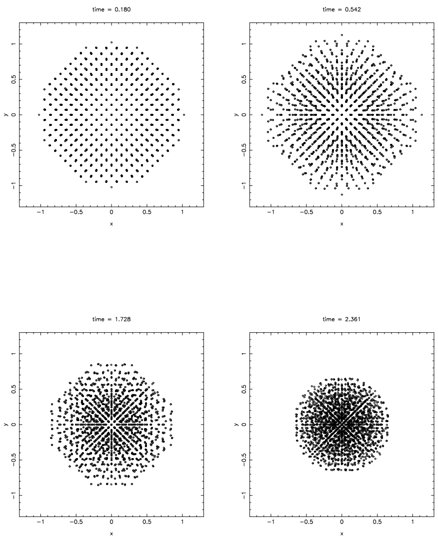

Fig. 2. Collapsing dusty–gas sphere with $K = 4$. Here the dust (open circles) and gas (dots) are tightly coupled and the dust is carried outwards initially with the gas and then they collapse inwards together. Both re-expand, the dust first as it is unaffected by viscosity. The systems eventually settles to an equilibrium configuration.

the gas shocks and dissipates energy because of the viscosity and remains in the central region whereas the dust passes through the newly formed disk and bounces around until an equilibrium is found.

5 Conclusion

We have implemented the two–phase SPH equations in our code, which has been tested for stability, sound speed accuracy and dynamical evolution. The code is now being used to study dusty–gas protostellar disks and the dynamical effects of dust on star formation.

6 References

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