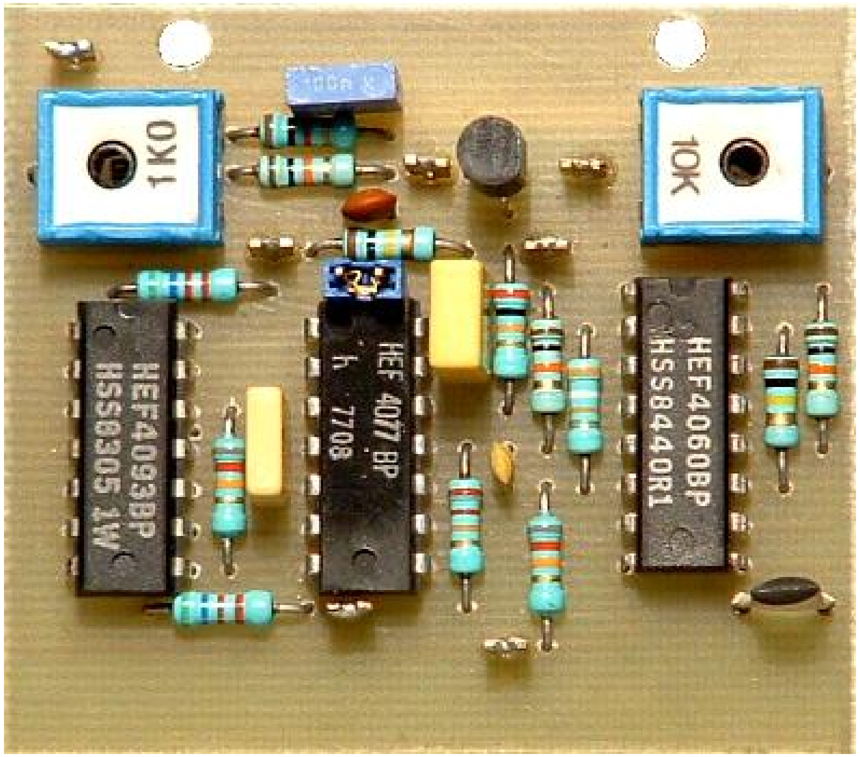
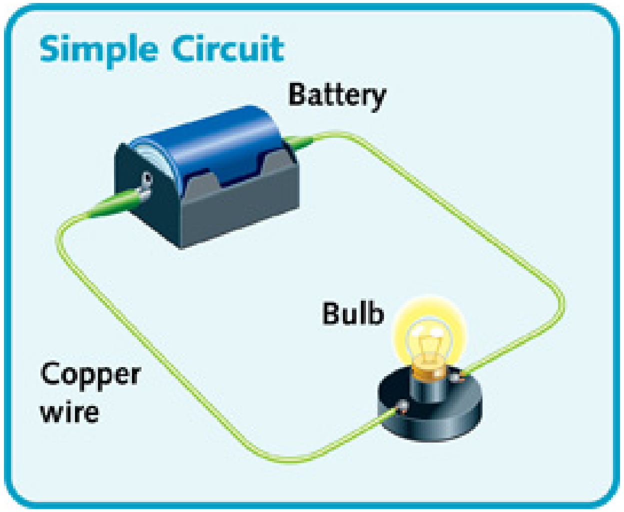
Chapter 25 : Electric circuits

- Voltage and current
- Series and parallel circuits
- Resistors and capacitors
- Kirchoff's rules for analysing circuits



Electric circuits

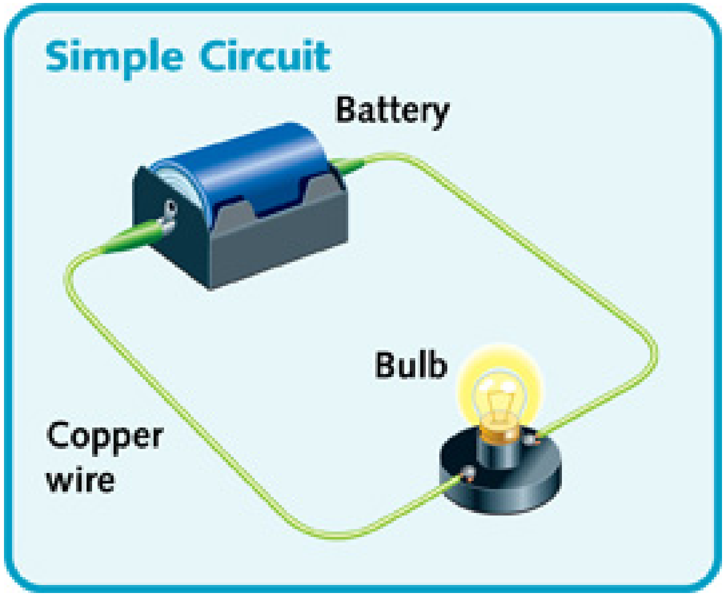
- Closed loop of electrical components around which current can flow, driven by a potential difference

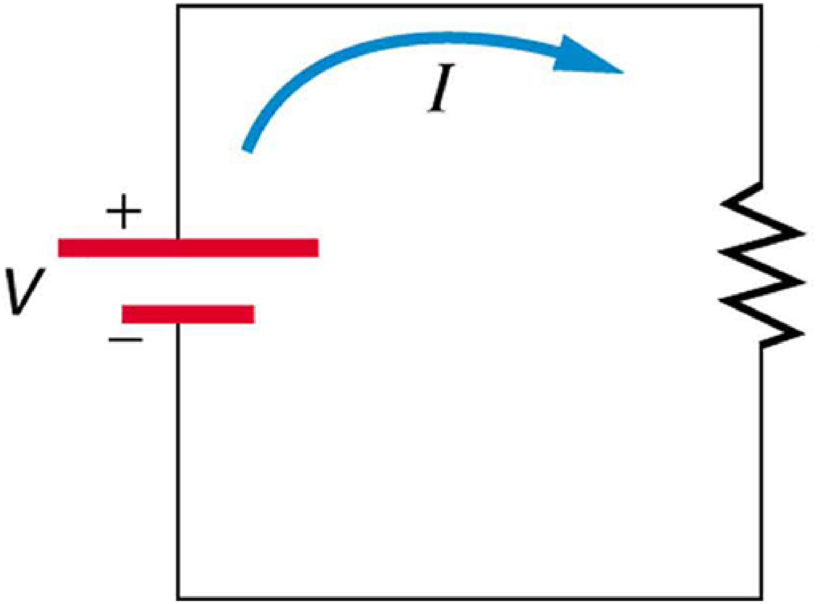


- Current (in Amperes A) is the rate of flow of charge
- Potential difference (in volts V) is the work done on charge

Electric circuits

- May be represented by a circuit diagram. Here is a simple case:



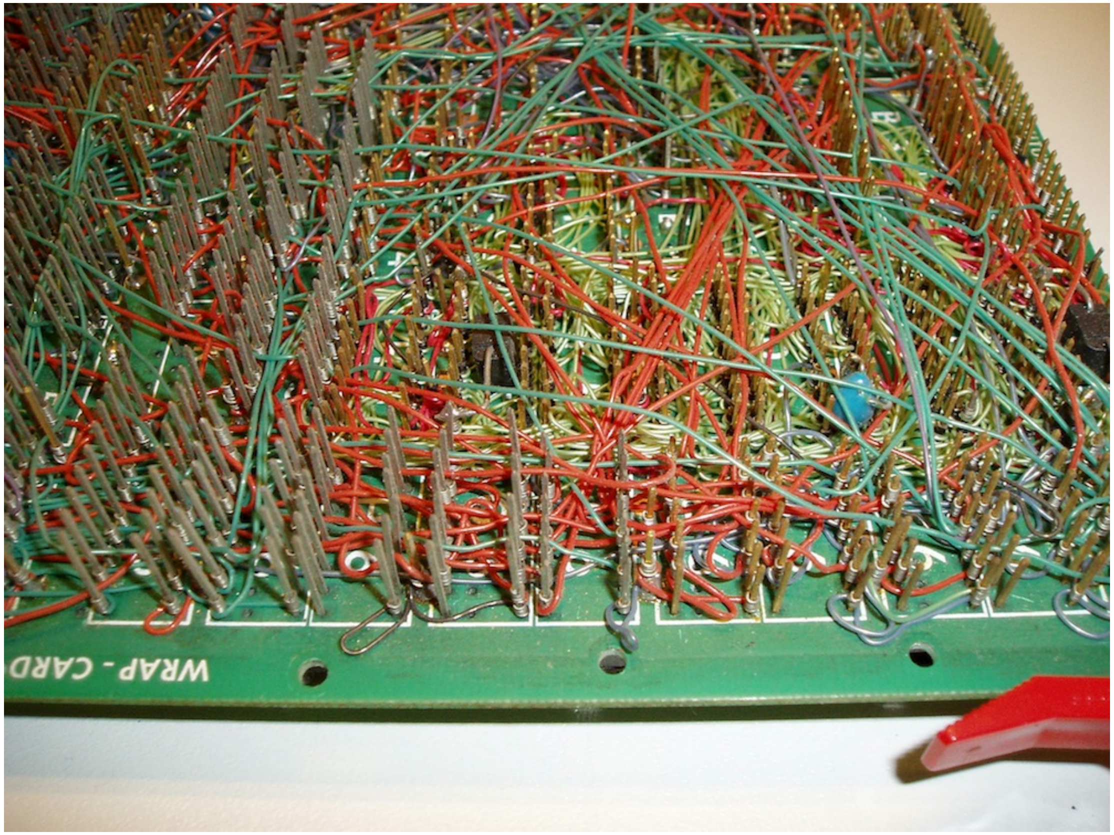


$$I = \frac{V}{R}$$

- R is the resistance (in Ohms Ω) to current flow

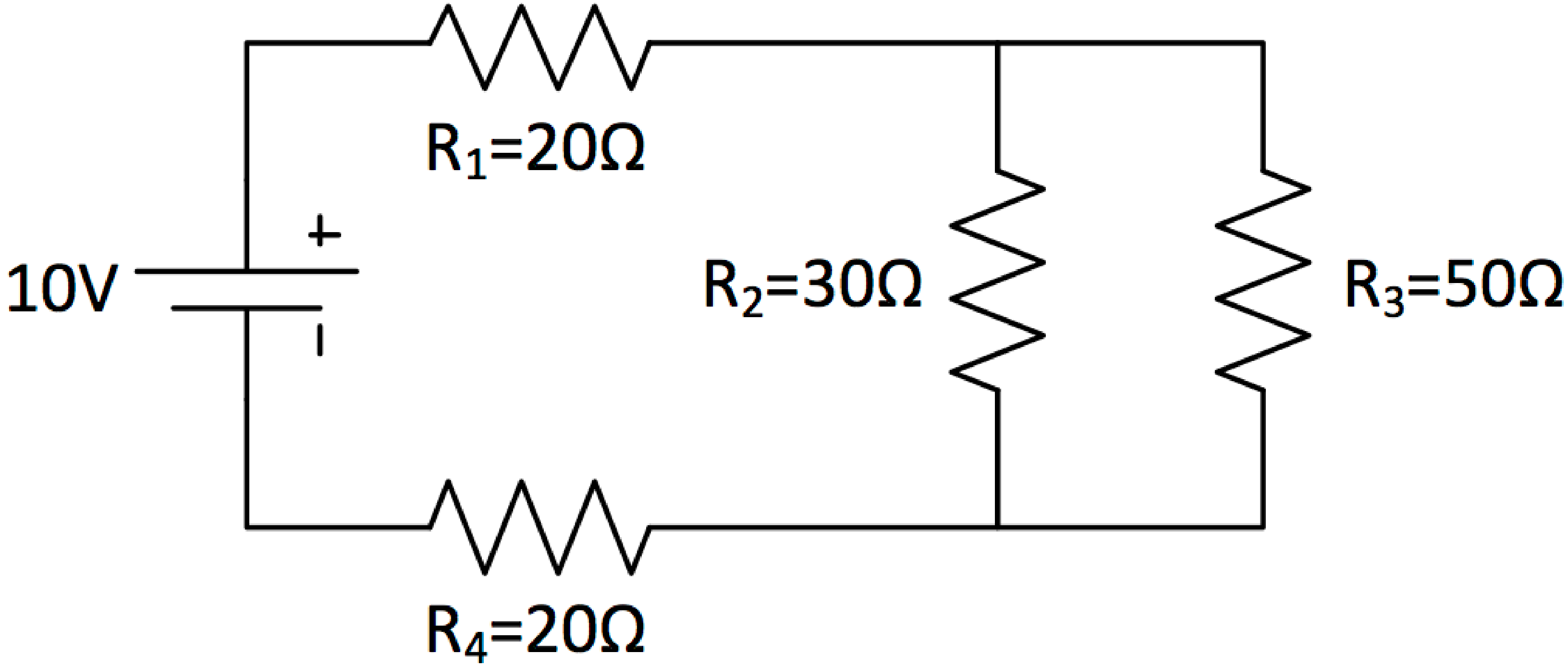
Electric circuits

- Same principles apply in more complicated cases!



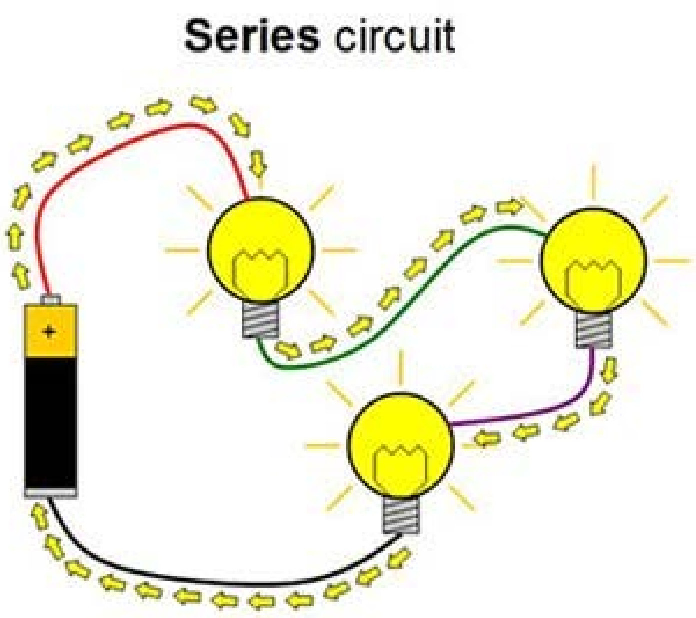
Electric circuits

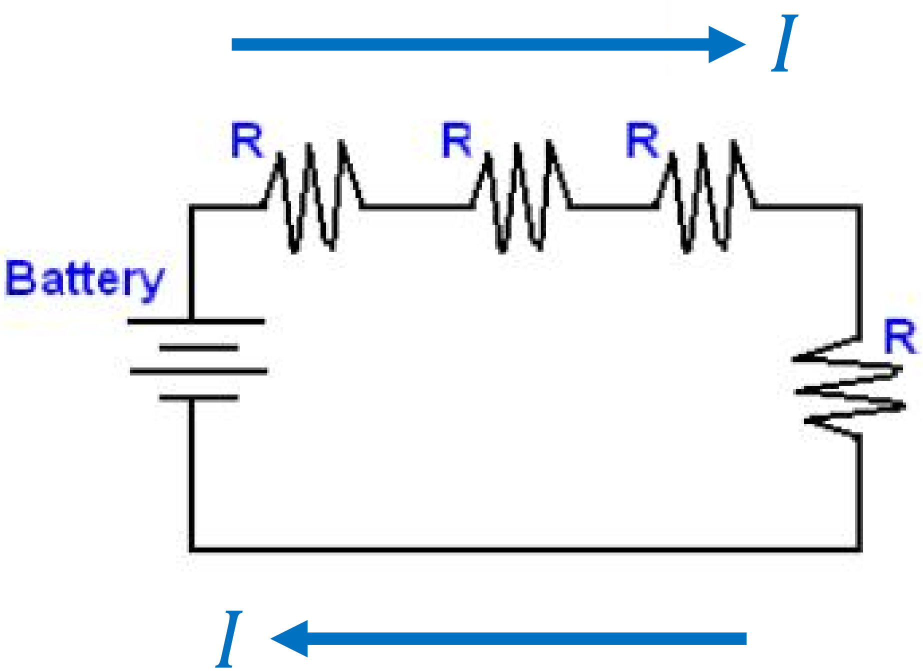
- How do we deal with a more complicated case?
  What is the current flowing from the battery?



Electric circuits

- When components are connected in series, the same electric current flows through them

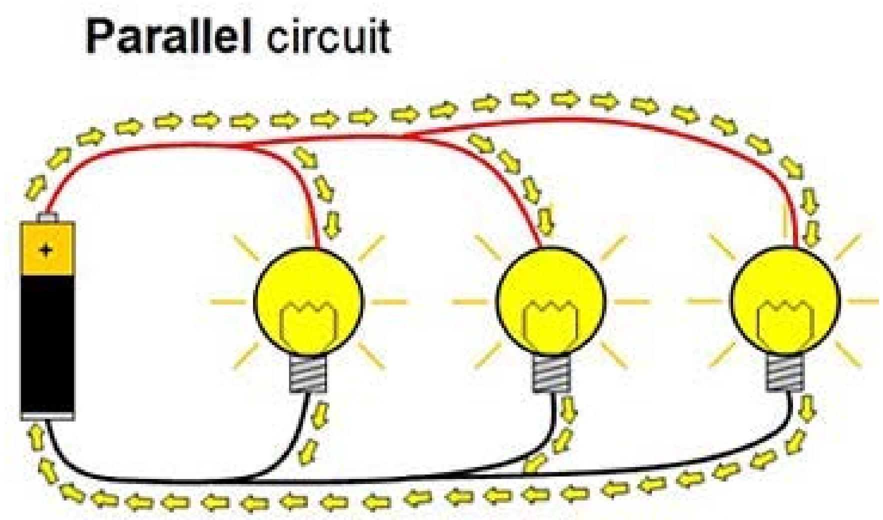


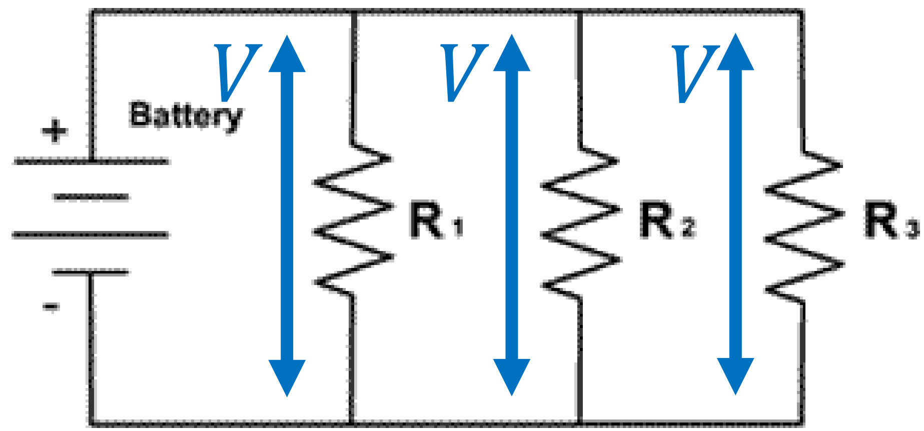


- Charge conservation : current cannot disappear!

Electric circuits

- When components are connected in parallel, the same potential difference drops across them

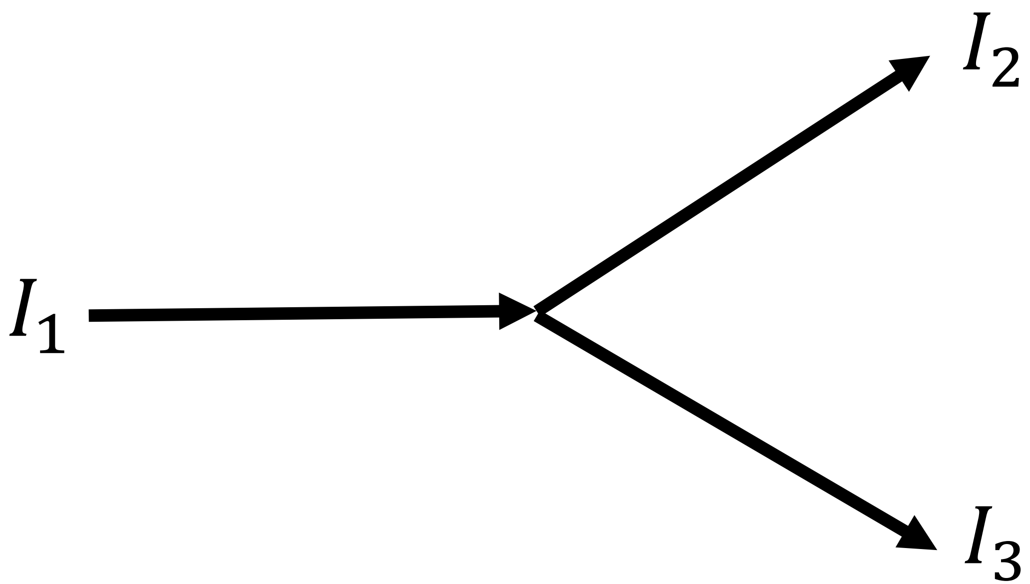




- Points connected by a wire are at the same voltage!

Electric circuits

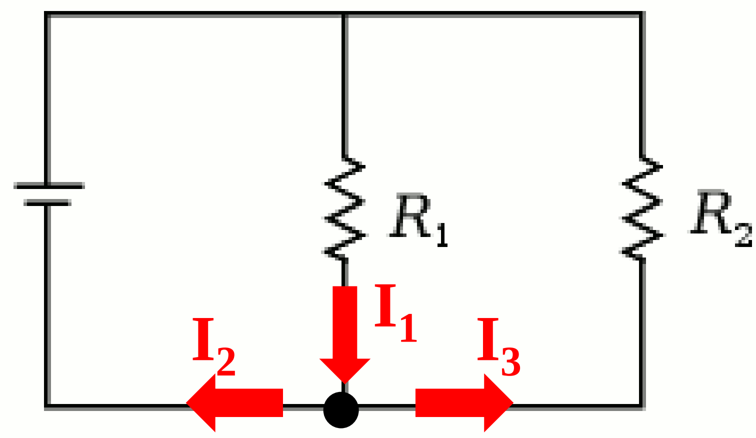
- When there is a junction in the circuit, the inward and outward currents to the junction are the same



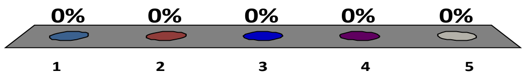
$$I_1 = I_2 + I_3$$

- Charge conservation : current cannot disappear!

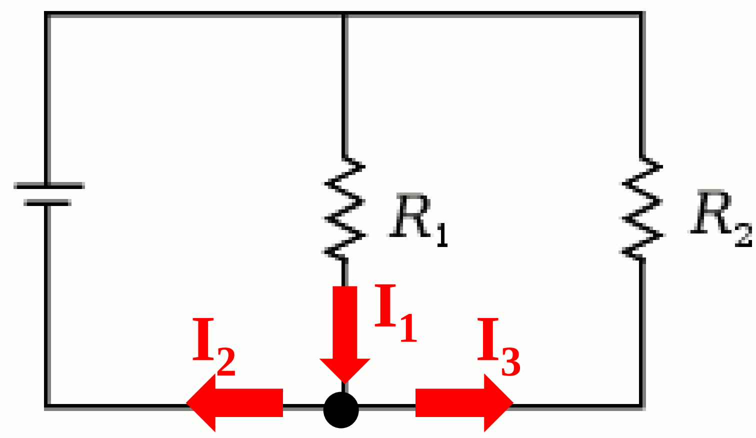
Consider the currents I_1, I_2 and I_3 as indicated on the circuit diagram. If $I_1 = 2.5$ A and $I_2 = 4$ A, what is the value of I_3?



1. **6.5 A**
2. **1.5 A**
3. **–1.5 A**
4. **0 A**
5. **The situation is not possible**



Consider the currents I_1, I_2 and I_3 as indicated on the circuit diagram. If I_1 = 2.5 A and I_2 = 4 A, what is the value of I_3?



current in = current out

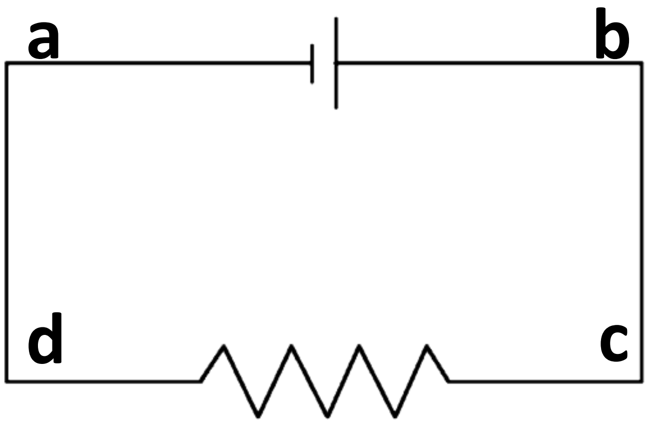
$$I_1 = I_2 + I_3$$

$$I_3 = I_1 - I_2$$

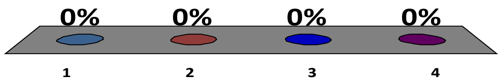
$$I_3 = 2.5 - 4 = -1.5\ A$$

(Negative sign means opposite direction to arrow.)

A 9.0 V battery is connected to a 3 Ω resistor. Which is the **incorrect** statement about potential differences (voltages)?

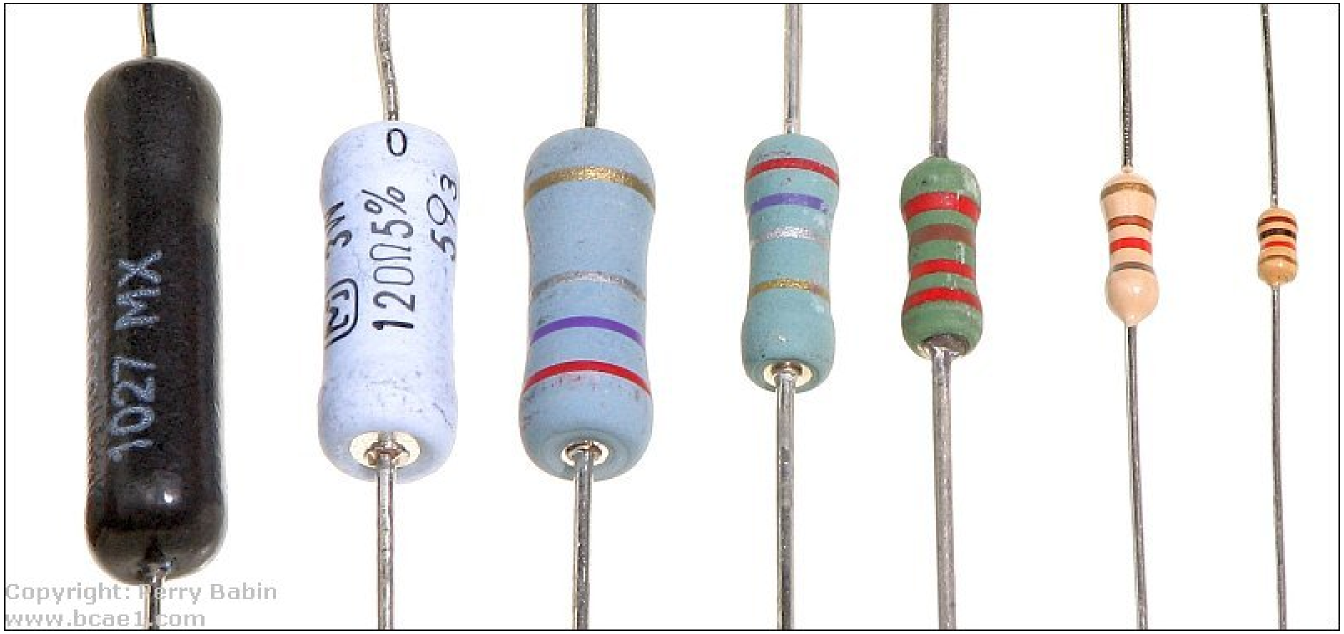


1. $V_b - V_a = 9.0$ V
2. $V_b - V_c = 0$ V
3. $V_c - V_d = 9.0$ V
4. $V_d - V_a = 9.0$ V



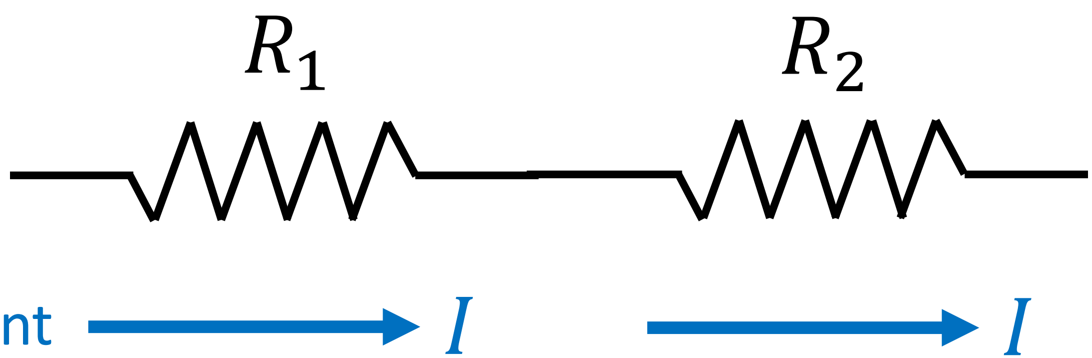
Resistors in circuits

- Resistors are the basic components of a circuit that determine current flow : Ohm's law $I = V/R$



Resistors in series/parallel

- If two resistors are connected in series, what is the total resistance?



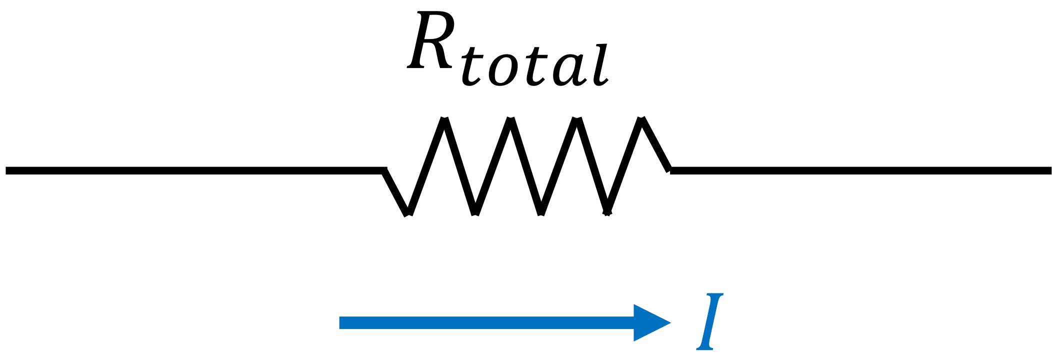
same current

Potential drop $V_1 = I\,R_1$ Potential drop $V_2 = I\,R_2$

Total potential drop $V = V_1 + V_2 = I\,R_1 + I\,R_2 = I\,(R_1 + R_2)$

Resistors in series/parallel

- If two resistors are connected in series, what is the total resistance?



Potential drop $V = I\,R_{total} = I\,(R_1 + R_2)$

$$R_{total} = R_1 + R_2$$

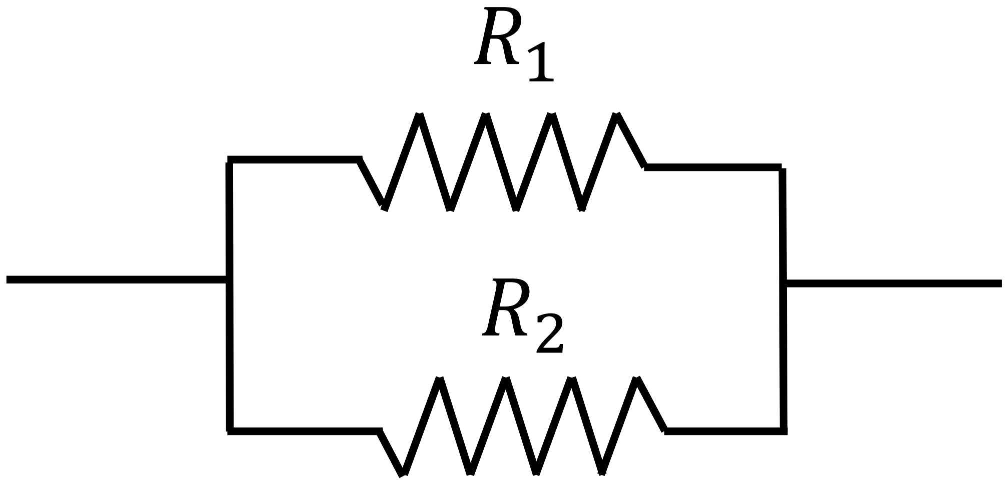
- Total resistance increases in series!

Resistors in series/parallel

- Total resistance increases in series!

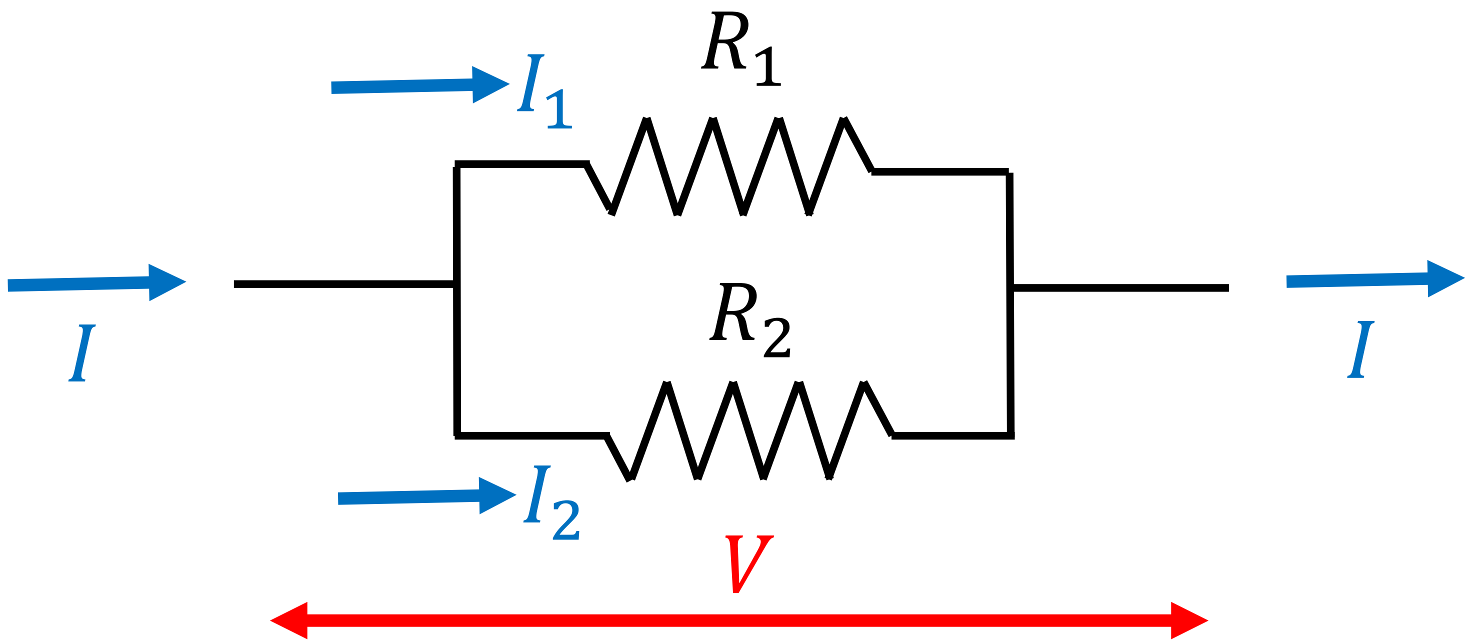
Resistors in series/parallel

- If two resistors are connected in parallel, what is the total resistance?



Resistors in series/parallel

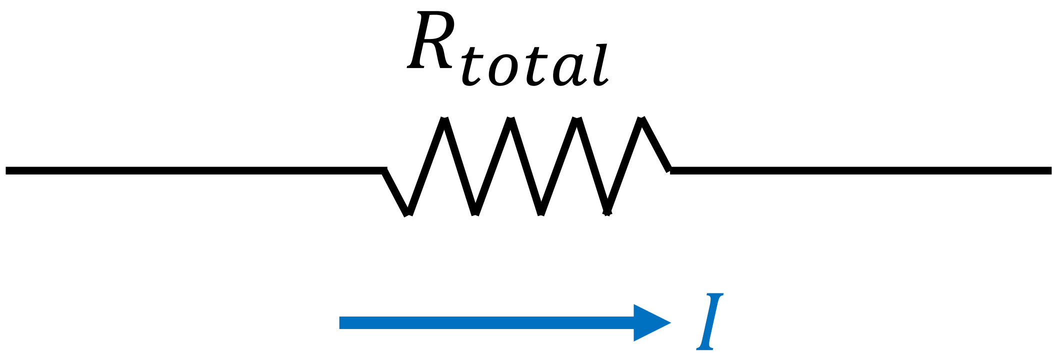
- If two resistors are connected in parallel, what is the total resistance?



Total current $I = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2} = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

Resistors in series/parallel

- If two resistors are connected in parallel, what is the total resistance?



Current I $= \frac{V}{R_{total}} = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2}$$

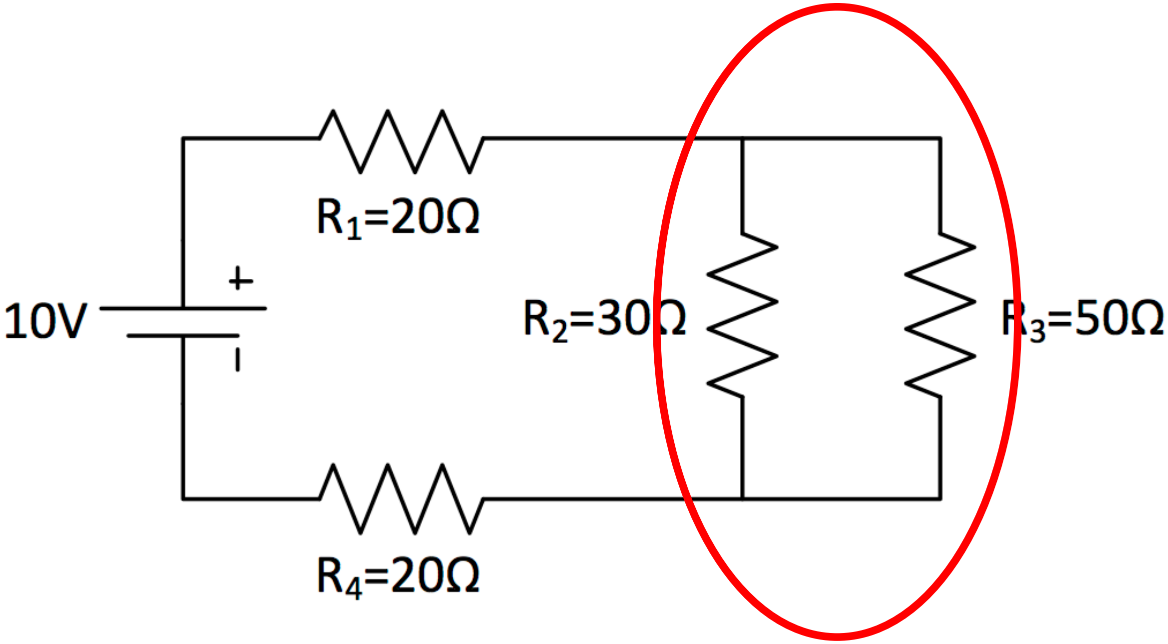
- Total resistance decreases in parallel!

Resistors in series/parallel

- Total resistance decreases in parallel!

Resistors in series/parallel

- What's the current flowing?



(1) Combine these 2 resistors in parallel:

$$\frac{1}{R_{pair}} = \frac{1}{30} + \frac{1}{50}$$

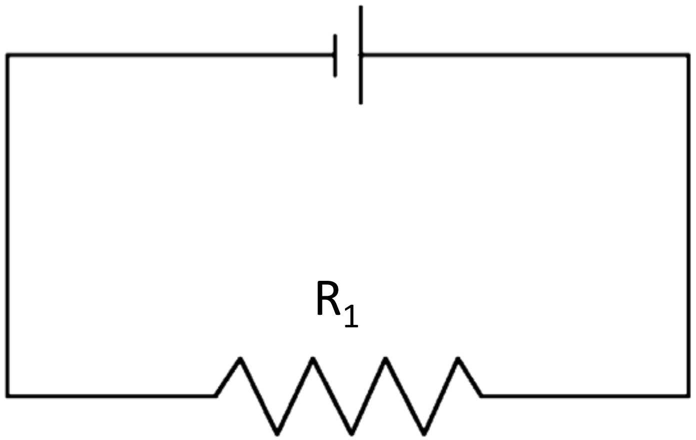
$$R_{pair} = 18.75\ \Omega$$

(2) Combine all the resistors in series:

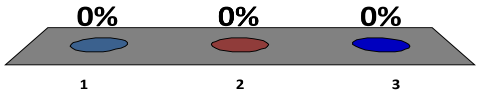
$$R_{total} = 20 + 18.75 + 20 = 58.75\ \Omega$$

(3) Current $I = \frac{V}{R_{total}} = \frac{10}{58.75} = 0.17\ A$

If an additional resistor, R_2, is added in series to the circuit, what happens to the **power dissipated** by R_1?

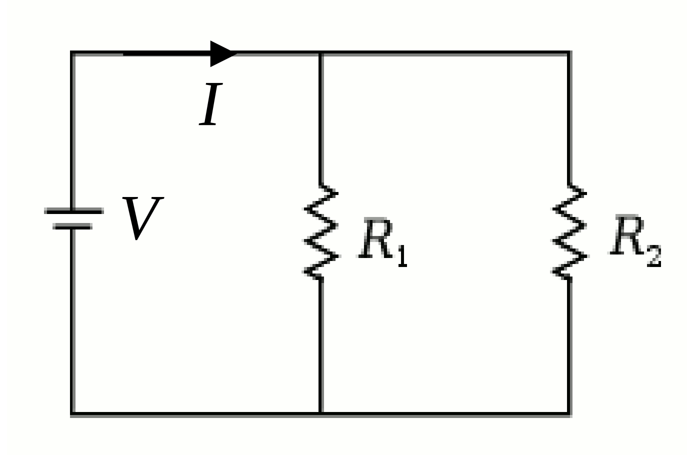


1. Increases
2. Decreases
3. Stays the same

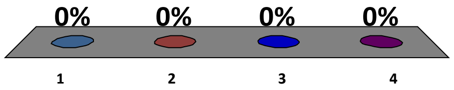


$$V = IR \qquad P = VI = I^2R = \frac{V^2}{R}$$

If an additional resistor, R_3, is added in parallel to the circuit, what happens to the **total current,** I?



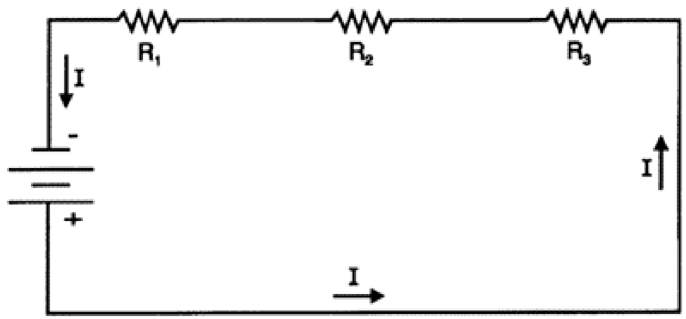
1. Increases
2. Decreases
3. Stays the same
4. Depends on R values

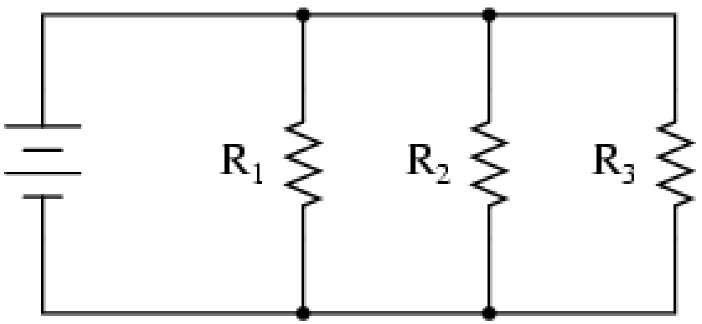


Parallel resistors: reciprocal effective resistance is sum of reciprocal resistances

$$\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

Series vs. Parallel



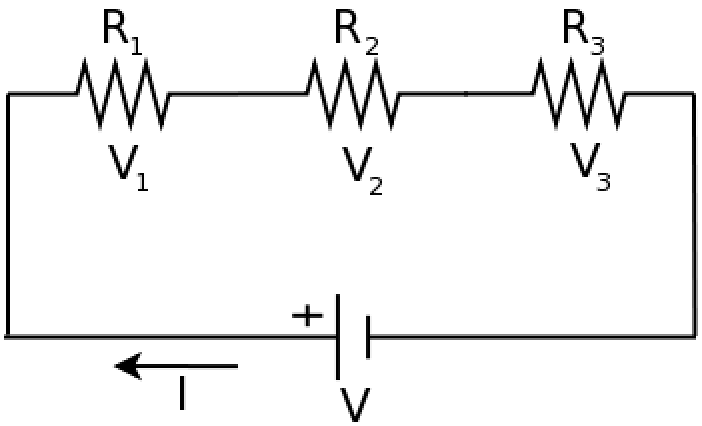


	Series	Parallel
CURRENT	Same current through all series elements	Current "splits up" through parallel branches
VOLTAGE	Voltages add to total circuit voltage	Same voltage across all parallel branches
RESISTANCE	Adding resistance increases total R	Adding resistance reduces total R

String of Christmas lights – connected in *series*
Power outlets in house – connected in *parallel*

Voltage divider

Consider a circuit with several resistors in series with a battery.



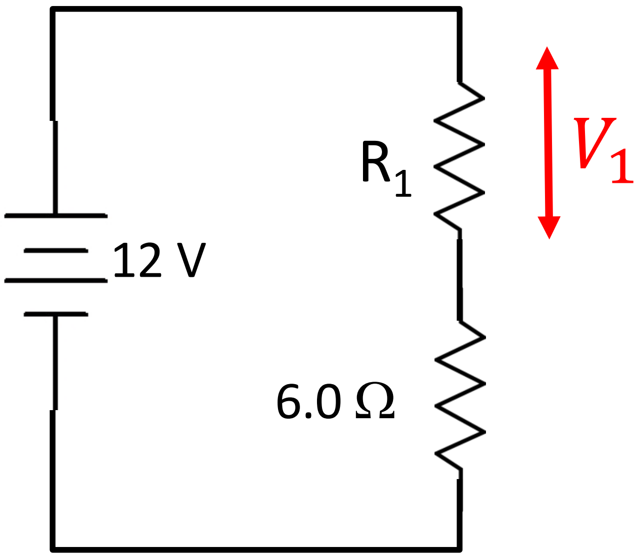
Current in circuit: $I = \frac{V}{R_{total}} = \frac{V}{R_1 + R_2 + R_3}$

The potential difference across one of the resistors (e.g. R_1)

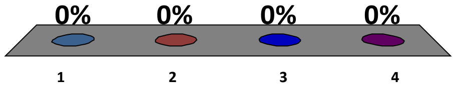
$$V_1 = IR_1 = V\frac{R_1}{R_1 + R_2 + R_3}$$

The fraction of the total voltage that appears across a resistor in series is the ratio of the given resistance to the total resistance.

What must be the resistance R_1 so that $V_1 = 2.0$ V?



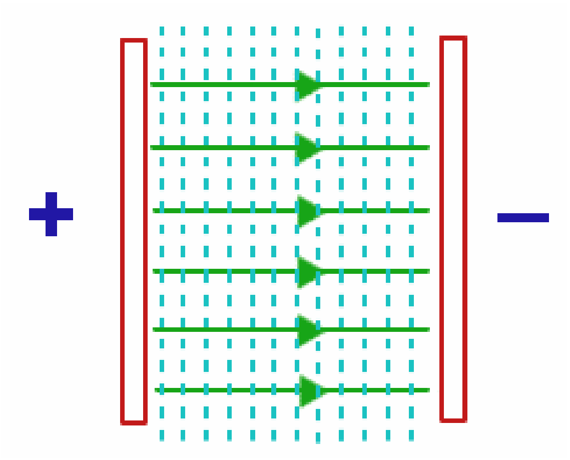
1. 0.80 Ω
2. 1.2 Ω
3. 6.0 Ω
4. 30 Ω



Capacitors

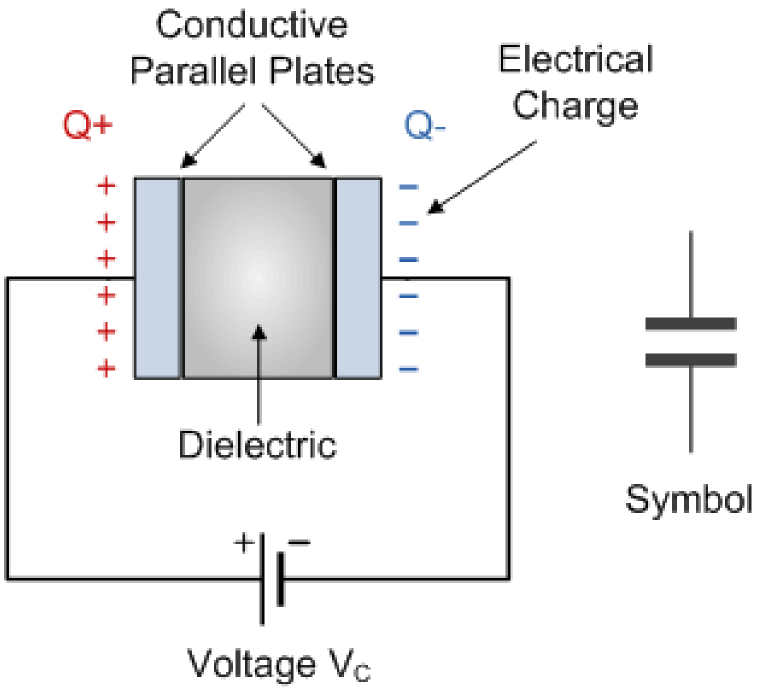
- A capacitor is a device in a circuit which can be used to store charge

A capacitor consists of two charged plates …



Electric field E

It's charged by connecting it to a battery …



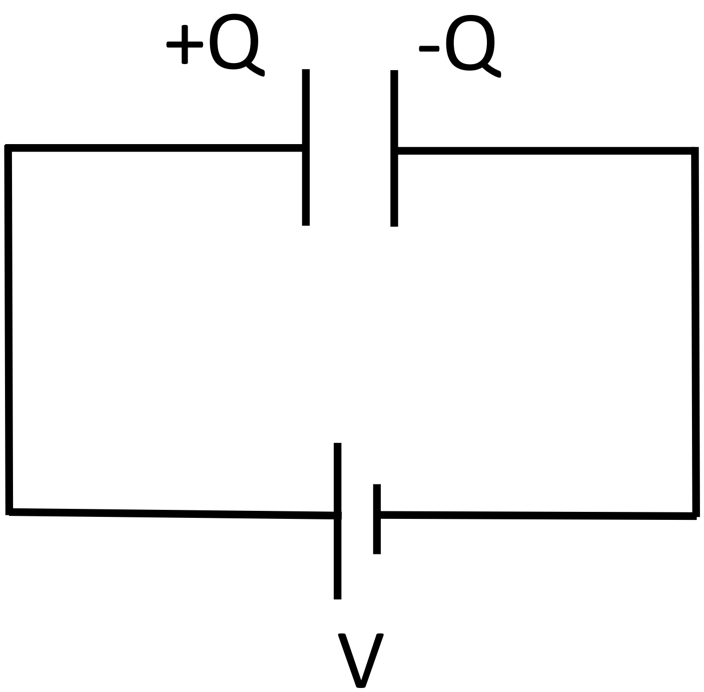
Capacitors

- A capacitor is a device in a circuit which can be used to store charge

Example : store and release energy …

Capacitors

- The capacitance C measures the amount of charge Q which can be stored for given potential difference V



$$C = \frac{Q}{V}$$

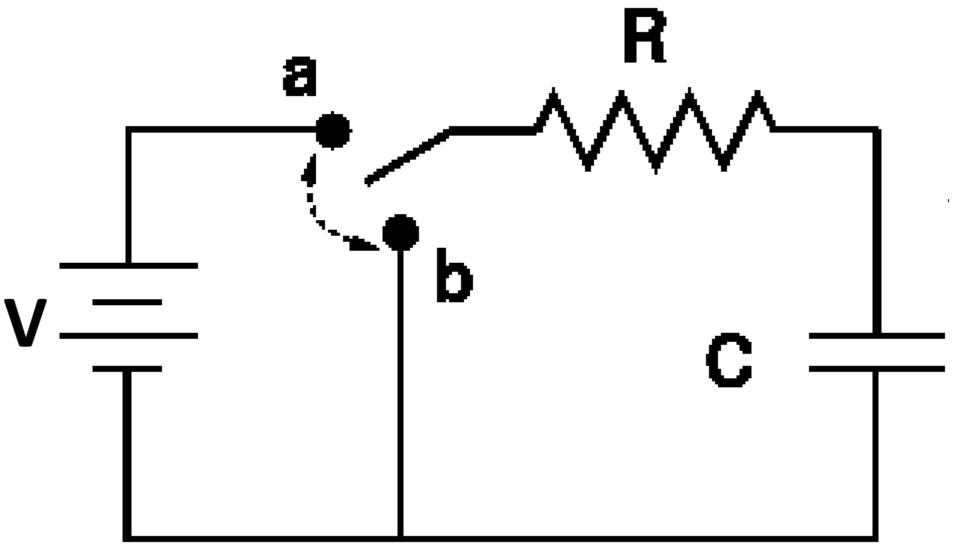
$$Q = C\,V$$

(Value of C depends on geometry…)

- Unit of capacitance is Farads [F]

Resistor-capacitor circuit

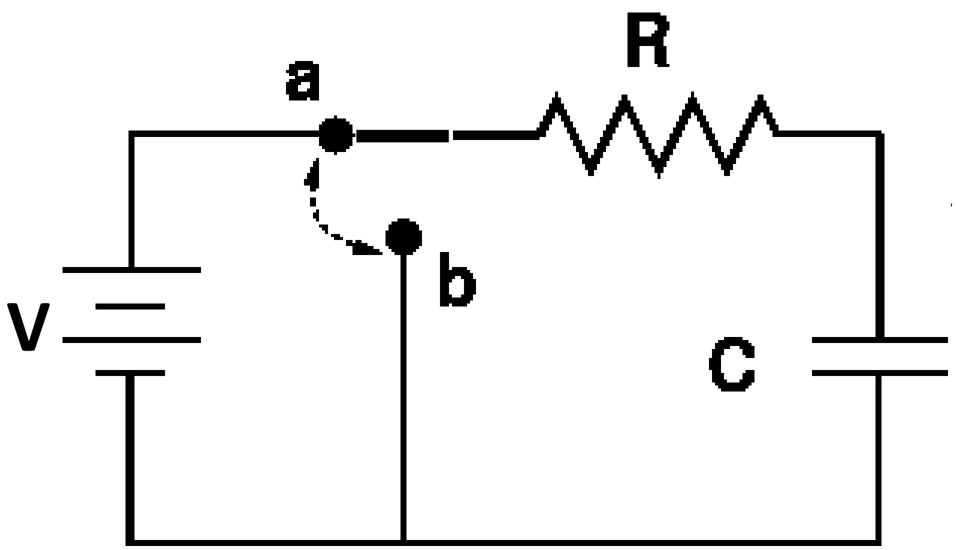
- Consider the following circuit with a resistor and a capacitor in series



What happens when we connect the circuit?

Resistor-capacitor circuit

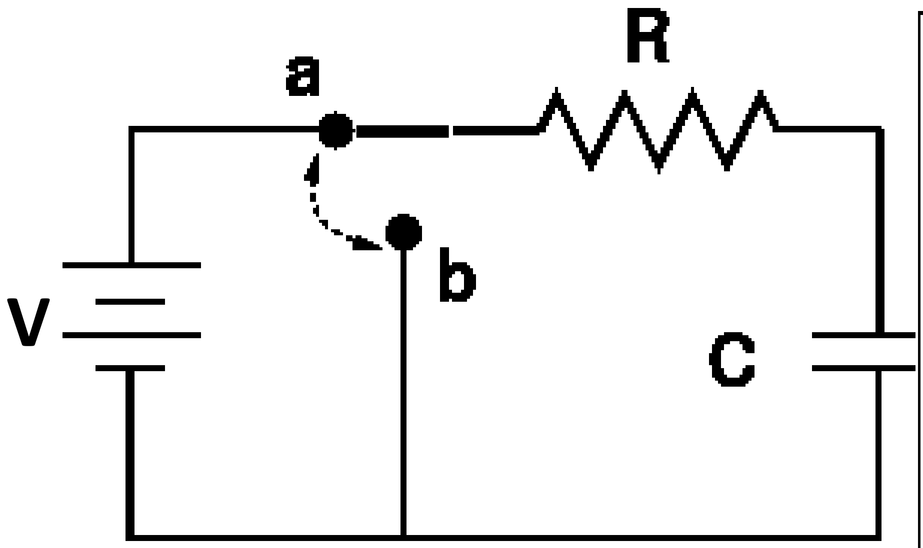
- When the switch is connected, the battery charges up the capacitor

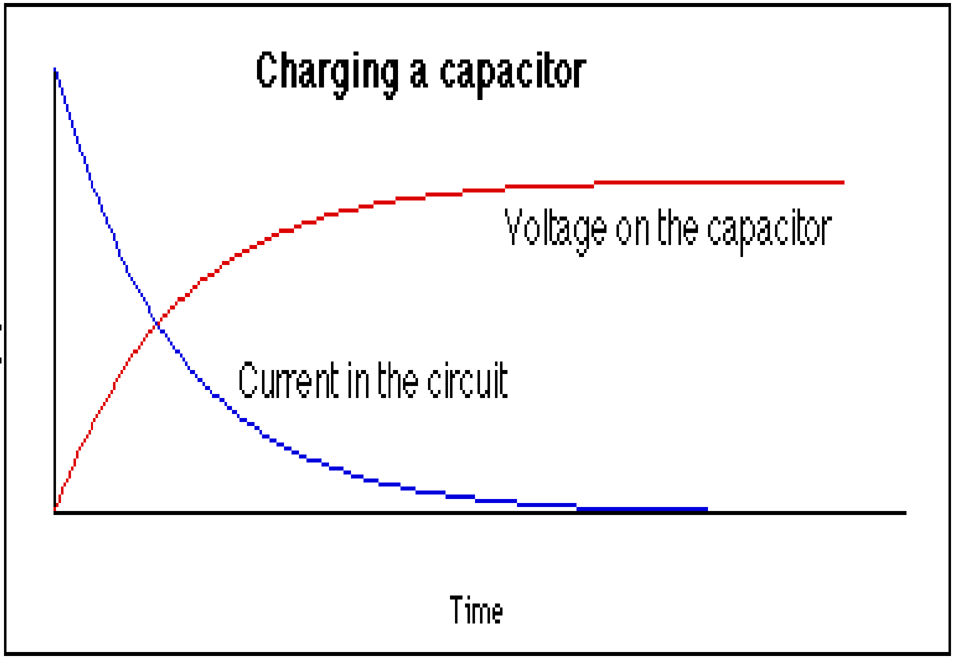


- Move the switch to point a
- Initial current flow I=V/R
- Charge Q flows from battery onto the capacitor
- Potential across the capacitor V_C=Q/C increases
- Potential across the resistor V_R decreases
- Current decreases to zero

Resistor-capacitor circuit

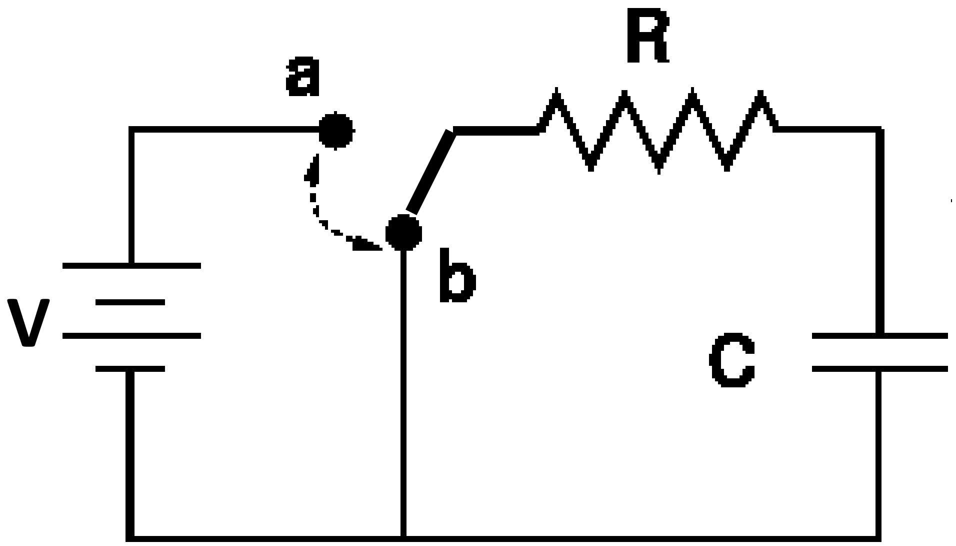
- When the switch is connected, the battery charges up the capacitor





Resistor-capacitor circuit

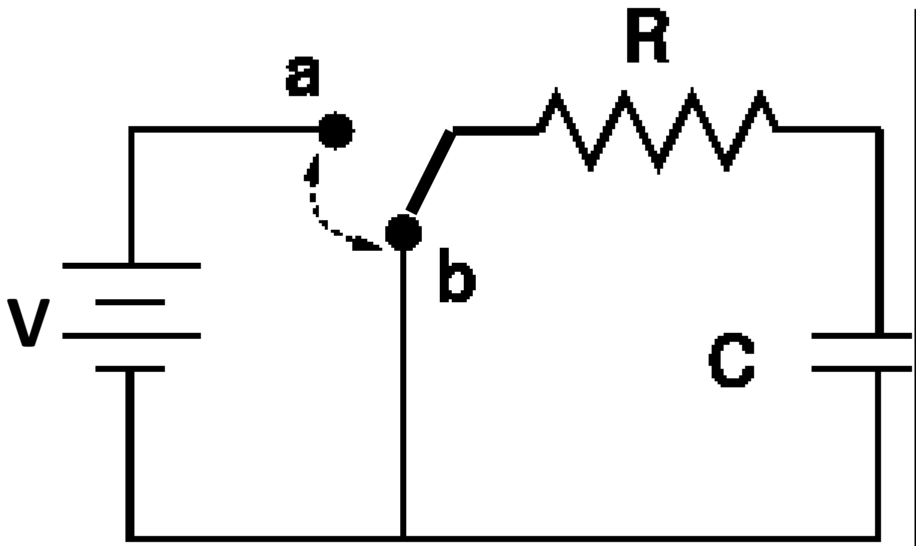
- When the battery is disconnected, the capacitor pushes charge around the circuit

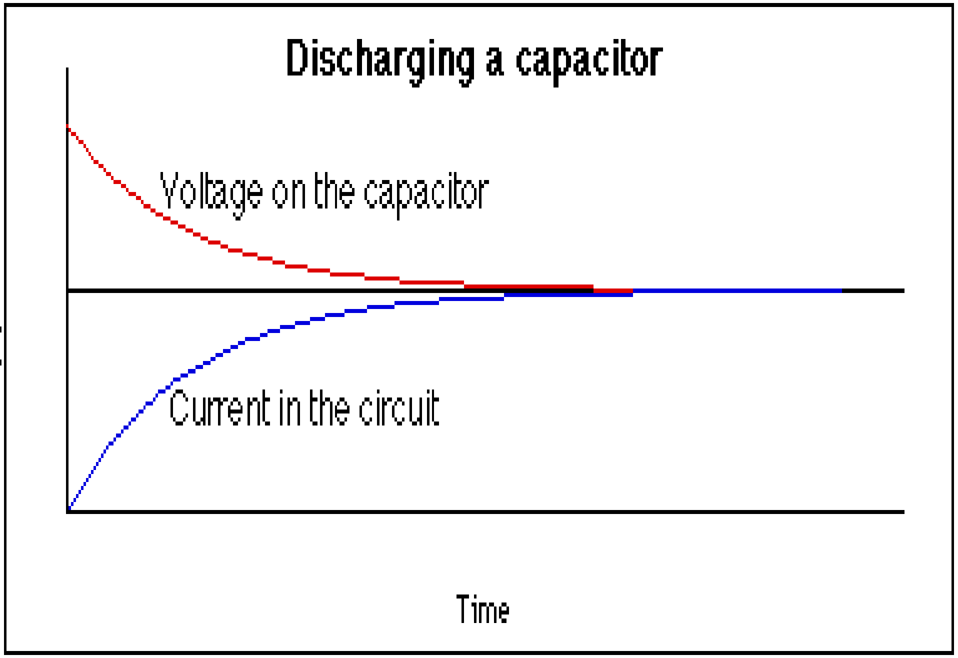


- Move the switch to point b
- Initial current flow $I=V_C/R$
- Charge flows from one plate of capacitor to other
- Potential across the capacitor $V_C=Q/C$ decreases
- Current decreases to zero

Resistor-capacitor circuit

- When the battery is disconnected, the capacitor pushes charge around the circuit

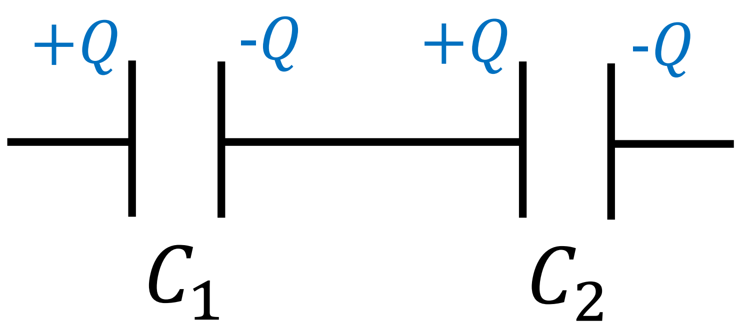




Capacitors in series/parallel

- If two capacitors are connected in series, what is the total capacitance?

Same charge must be on every plate!



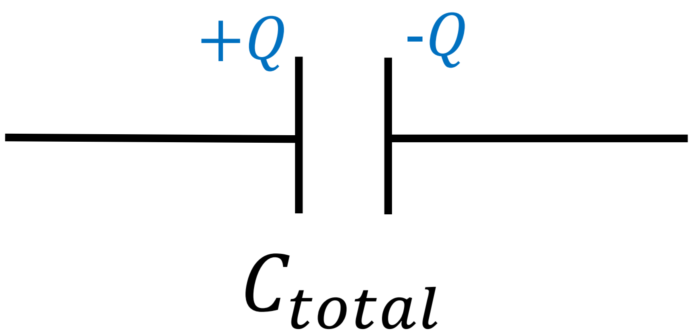
$$Q = C\,V$$

Potential drop $V_1 = Q/C_1$ Potential drop $V_2 = Q/C_2$

Total potential drop $V = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2} = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$

Capacitors in series/parallel

- If two capacitors are connected in series, what is the total capacitance?



$$Q = C\,V$$

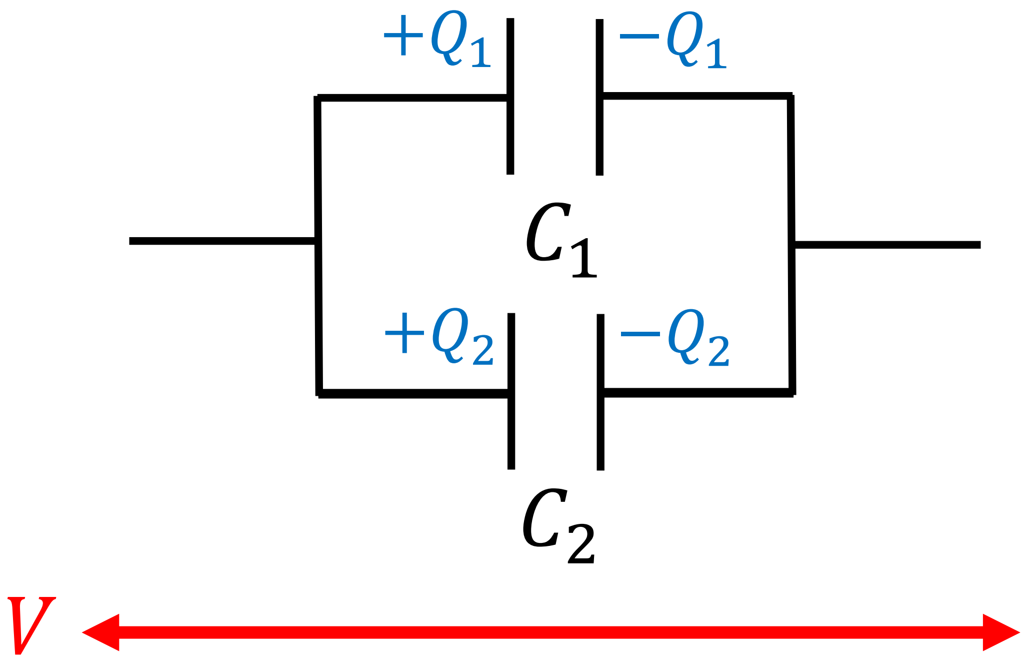
Potential drop $V = \frac{Q}{C_{total}} = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$

$$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2}$$

- Total capacitance decreases in series!

Capacitors in series/parallel

- If two capacitors are connected in parallel, what is the total capacitance?

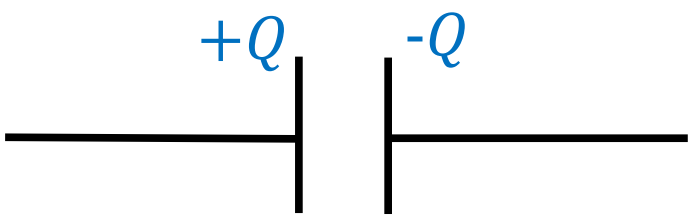


$$Q = C\,V$$

Total charge $Q = Q_1 + Q_2 = C_1 V + C_2 V = (C_1 + C_2)\,V$

Capacitors in series/parallel

- If two capacitors are connected in parallel, what is the total capacitance?



C_{total}

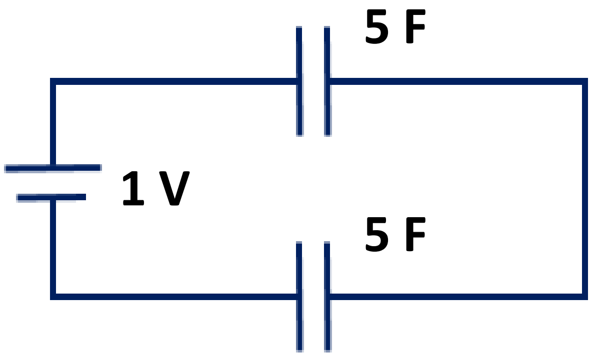
$$Q = C\,V$$

Total charge $Q = C_{total}V = (C_1 + C_2)V$

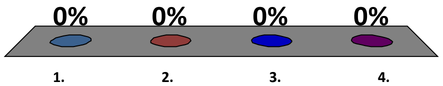
$$C_{total} = C_1 + C_2$$

- Total capacitance increases in parallel!

Two 5.0 F capacitors are in series with each other and a 1.0 V battery. Calculate the charge on each capacitor (Q) and the total charge drawn from the battery (Q_{total}).



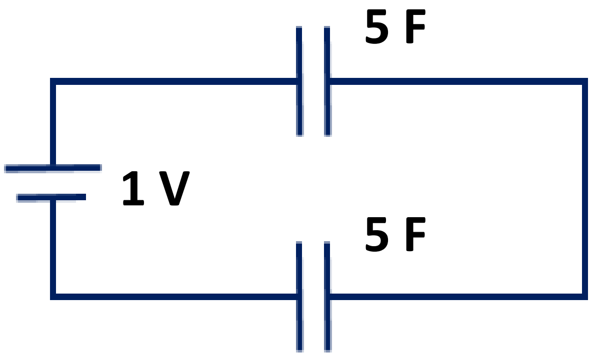
1. $Q = 5.0$ C, $Q_{total} = 5.0$ C
2. $Q = 0.25$ C, $Q_{total} = 0.50$ C
3. $Q = 2.5$ C, $Q_{total} = 2.5$ C
4. $Q = 2.5$ C, $Q_{total} = 5.0$ C



$$Q = CV$$

$$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$

Two 5.0 F capacitors are in series with each other and a 1.0 V battery. Calculate the charge on each capacitor (Q) and the total charge drawn from the battery (Q_{total}).



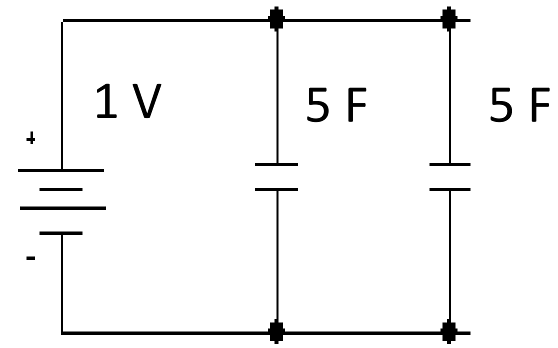
Potential difference across each capacitor = 0.5 V

Charge on each capacitor $Q = CV = 5 \times 0.5 = 2.5\ C$

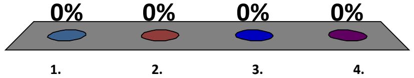
$$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} \rightarrow \frac{1}{C_{total}} = \frac{1}{5} + \frac{1}{5} = \frac{2}{5} \rightarrow C_{total} = 2.5\ F$$

$$Q_{total} = C_{total}\, V = 2.5 \times 1 = 2.5\ C$$

Two 5.0 F capacitors are in parallel with each other and a 1.0 V battery. Calculate the charge on each capacitor (Q) and the total charge drawn from the battery (Q_{total}).



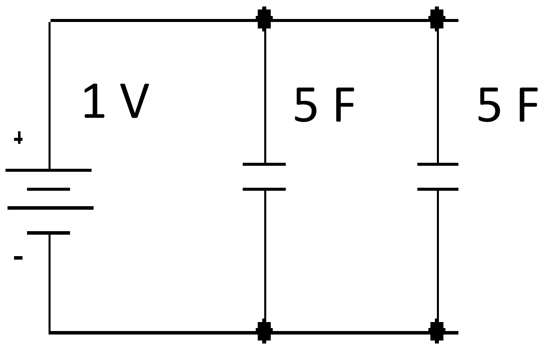
1. $Q = 5.0\ C$, $Q_{total} = 5.0\ C$
2. $Q = 0.2\ C$, $Q_{total} = 0.4\ C$
3. $Q = 5.0\ C$, $Q_{total} = 10\ C$
4. $Q = 2.5\ C$, $Q_{total} = 2.5\ C$



$$Q = CV$$

$$C_{total} = C_1 + C_2 + C_3 + \cdots$$

Two 5.0 F capacitors are in parallel with each other and a 1.0 V battery. Calculate the charge on each capacitor (Q) and the total charge drawn from the battery (Q_{total}).

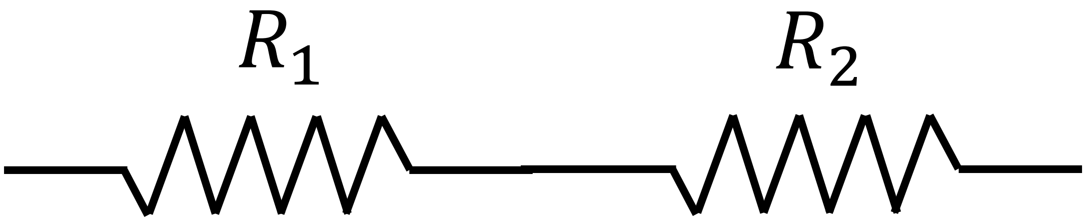


Potential difference across each capacitor = 1 V

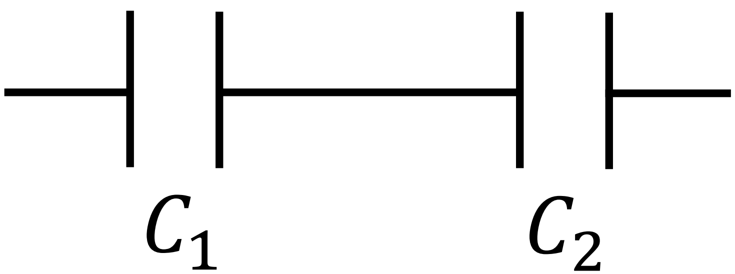
Charge on each capacitor $Q = CV = 5 \times 1 = 5\ C$

$$Q_{total} = 10\ C$$

Resistors vs. Capacitors

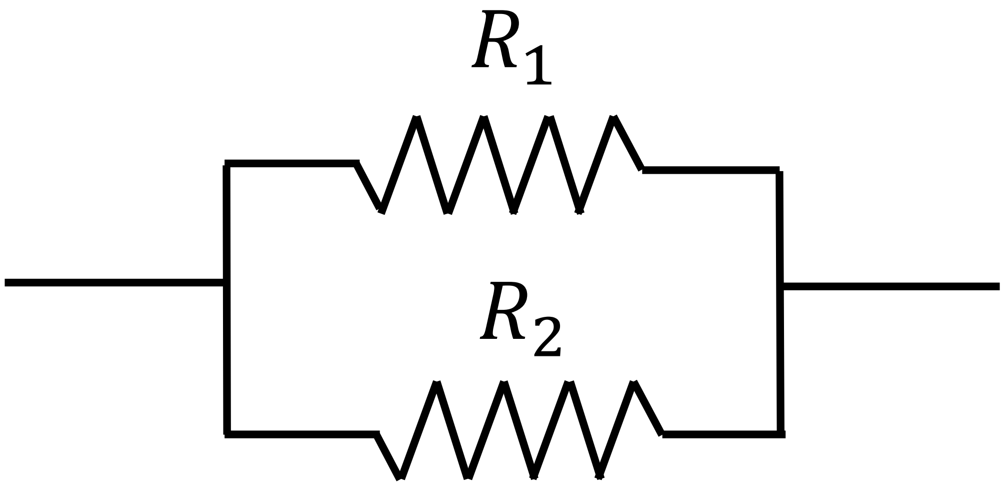


$$R_{total} = R_1 + R_2$$

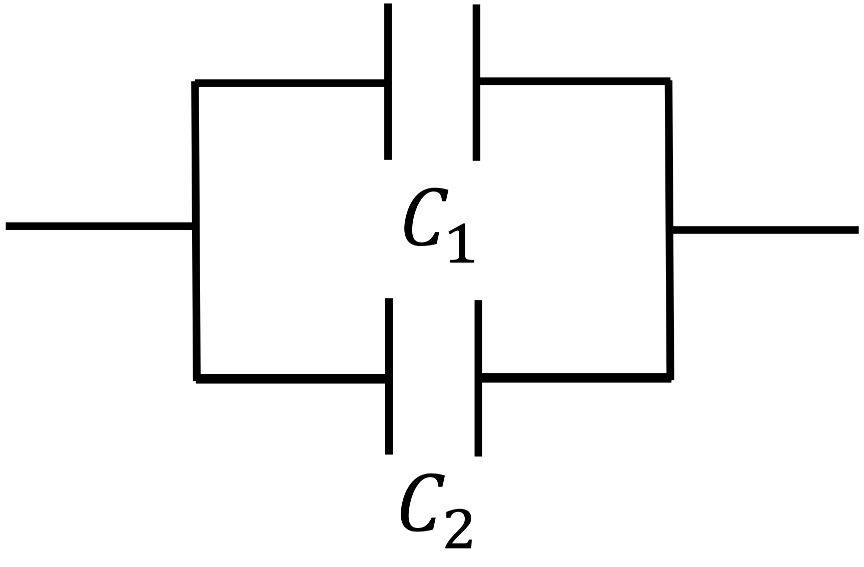


$$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Resistors vs. Capacitors



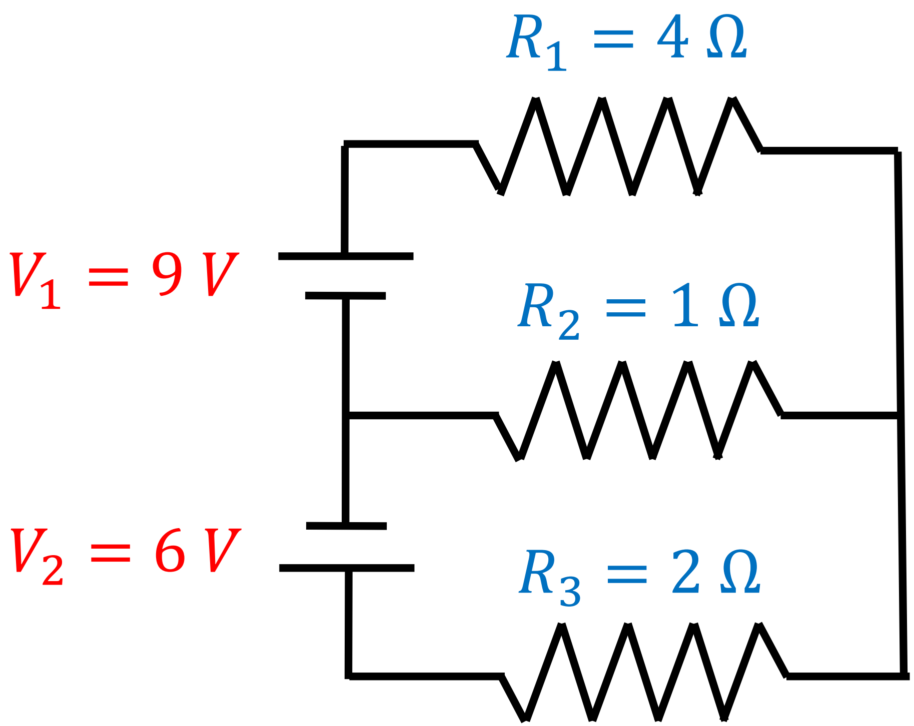
$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2}$$



$$C_{total} = C_1 + C_2$$

Kirchoff's rules

- Sometimes we might need to analyse more complicated circuits, for example …

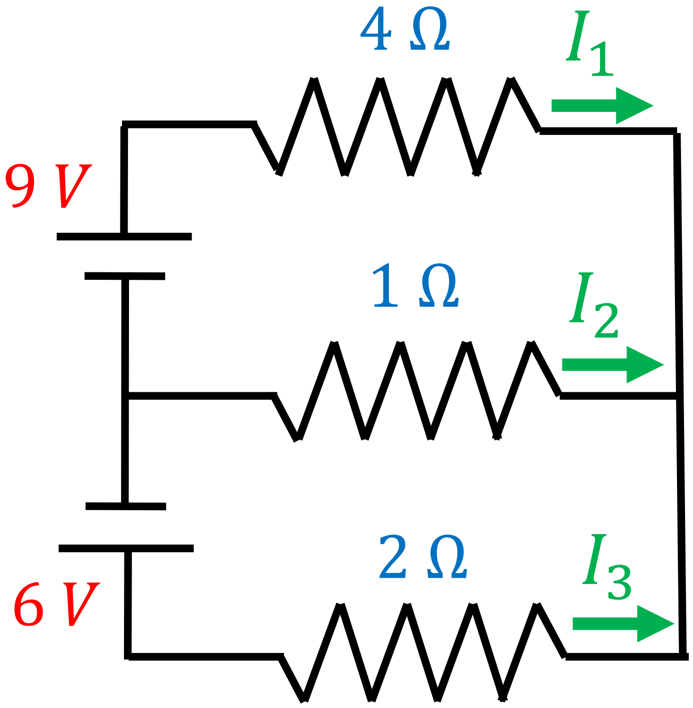


Q) What are the currents flowing in the 3 resistors?

- Kirchoff's rules give us a systematic method

Kirchoff's rules

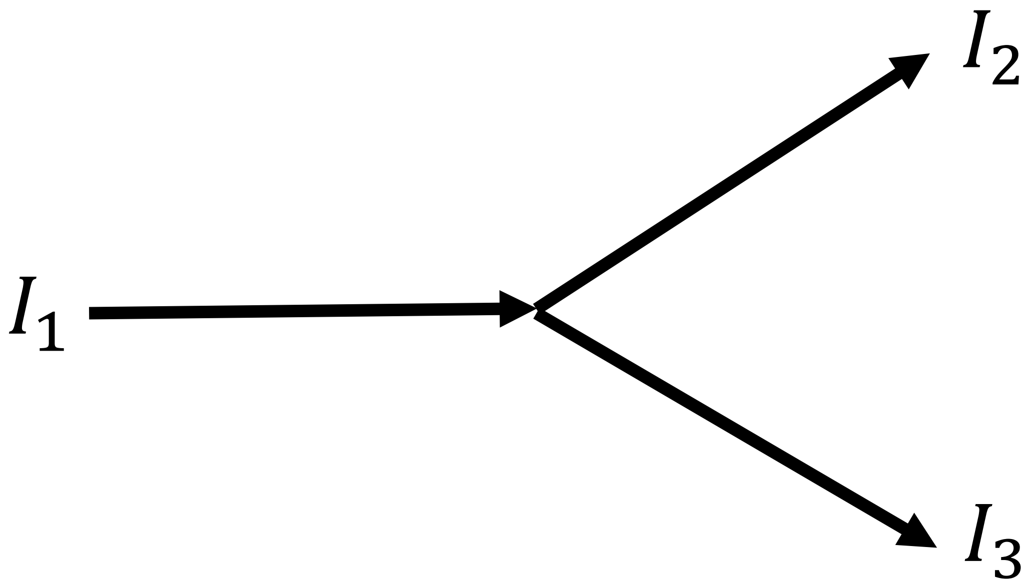
- What are the currents flowing in the 3 resistors?



Kirchoff's junction rule : the sum of currents at any junction is zero

Kirchoff's rules

- The sum of currents at any junction is zero
- Watch out for directions : into a junction is positive, out of a junction is negative

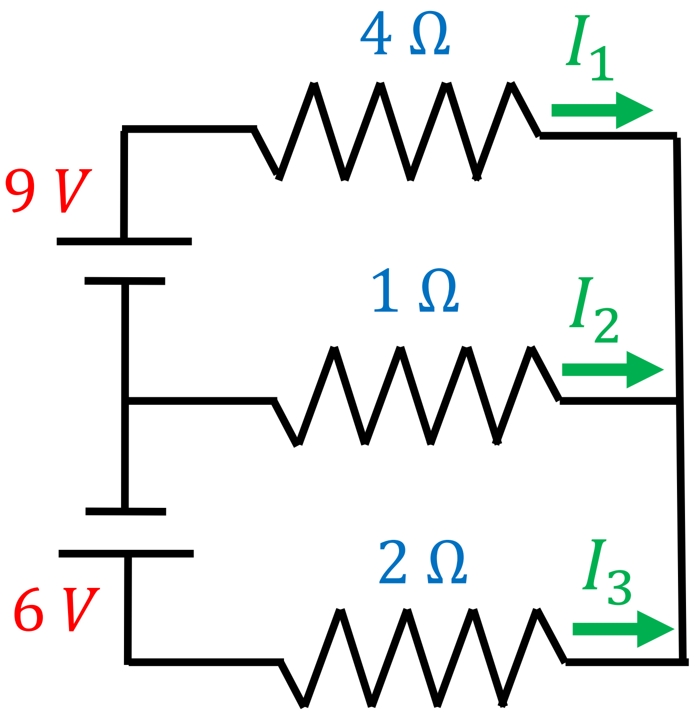


$$I_1 - I_2 - I_3 = 0$$

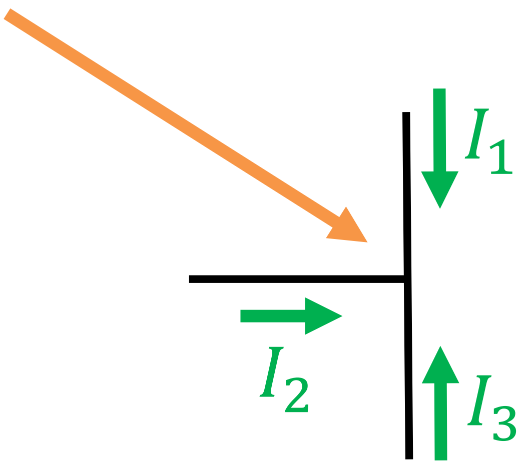
$$I_1 = I_2 + I_3$$

Kirchoff's rules

- What are the currents flowing in the 3 resistors?



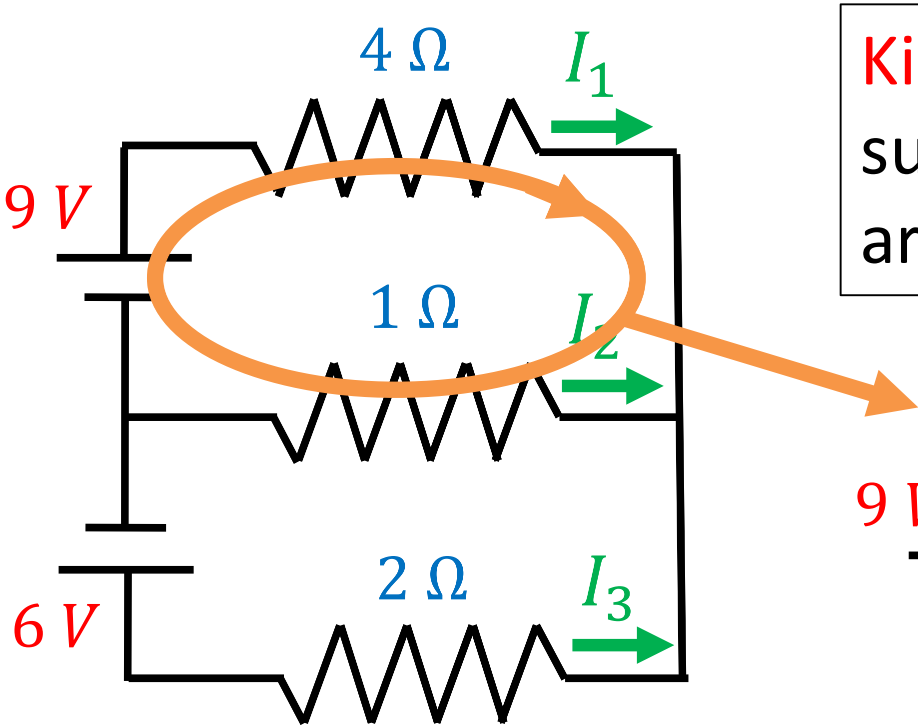
Kirchoff's junction rule :
the sum of currents at any junction is zero



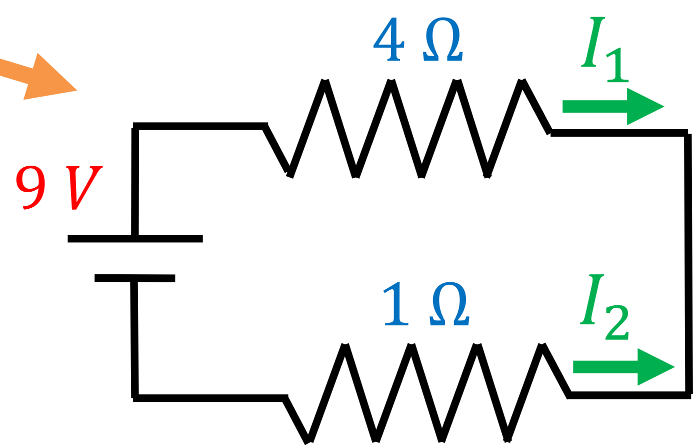
$$I_1 + I_2 + I_3 = 0$$

Kirchoff's rules

- What are the currents flowing in the 3 resistors?

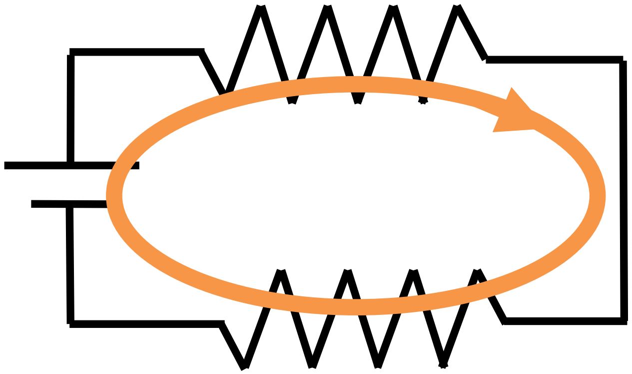


Kirchoff's loop rule : the sum of voltage changes around a closed loop is zero



Kirchoff's rules

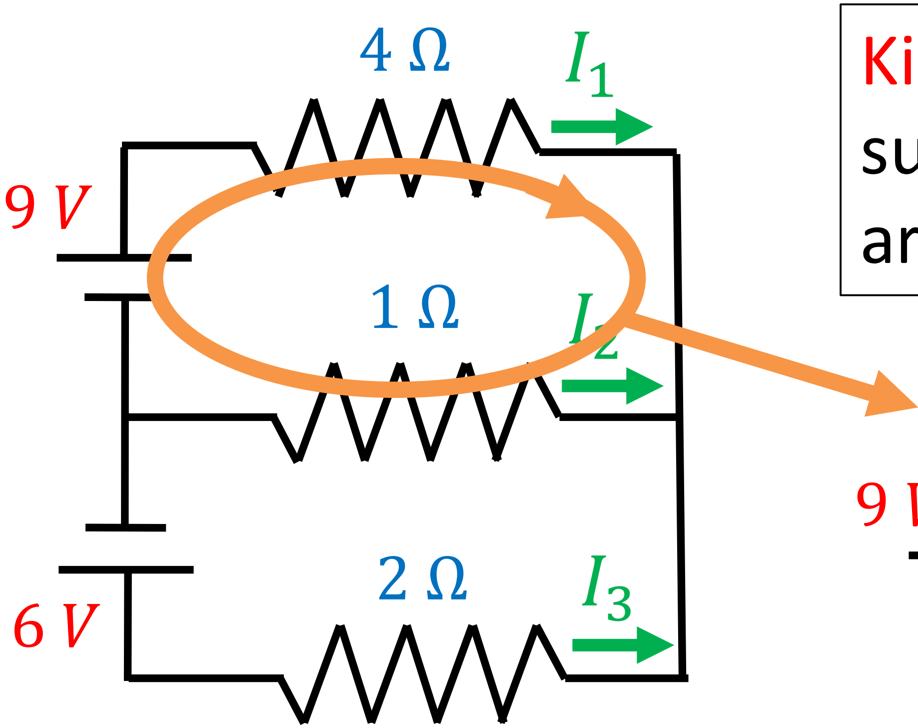
- Sum of voltage changes around a closed loop is zero



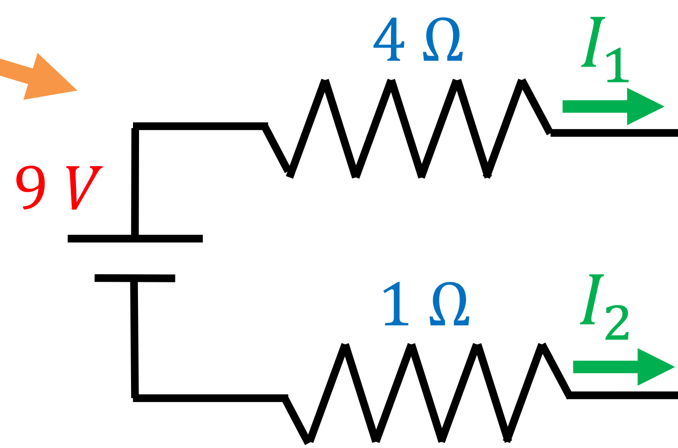
- Consider a unit charge (Q=1 Coulomb) going around this loop
- It gains energy from the battery (voltage change +V)
- It loses energy in the resistor (voltage change - I R)
- Conservation of energy : V - I R = 0 (or as we know, V = I R)

Kirchoff's rules

- What are the currents flowing in the 3 resistors?



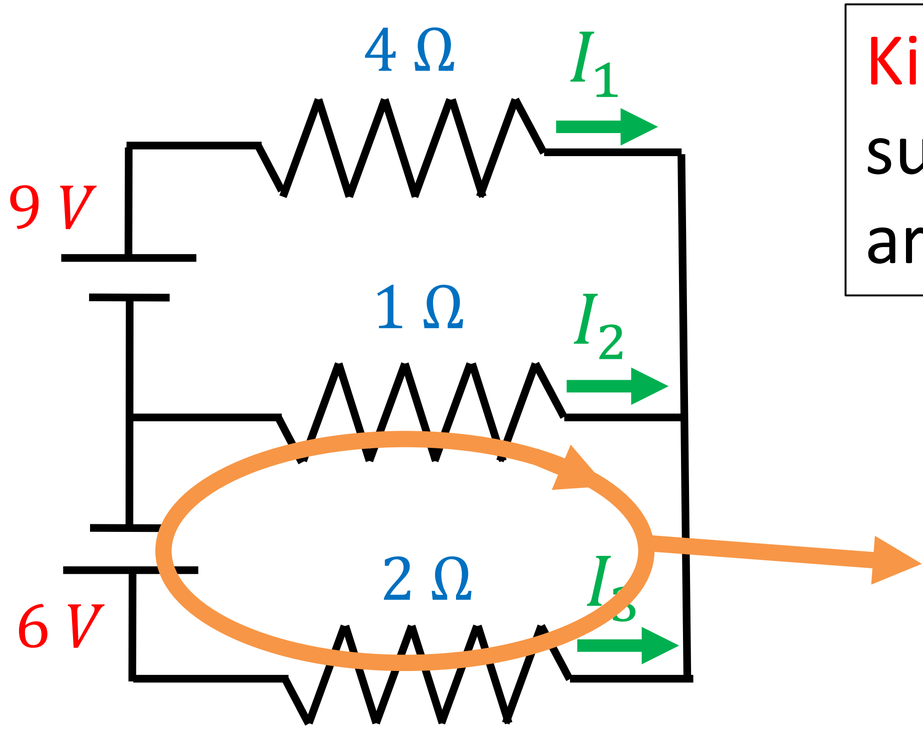
Kirchoff's loop rule : the sum of voltage changes around a closed loop is zero



$$9 - 4\,I_1 + 1\,I_2 = 0$$

Kirchoff's rules

- What are the currents flowing in the 3 resistors?

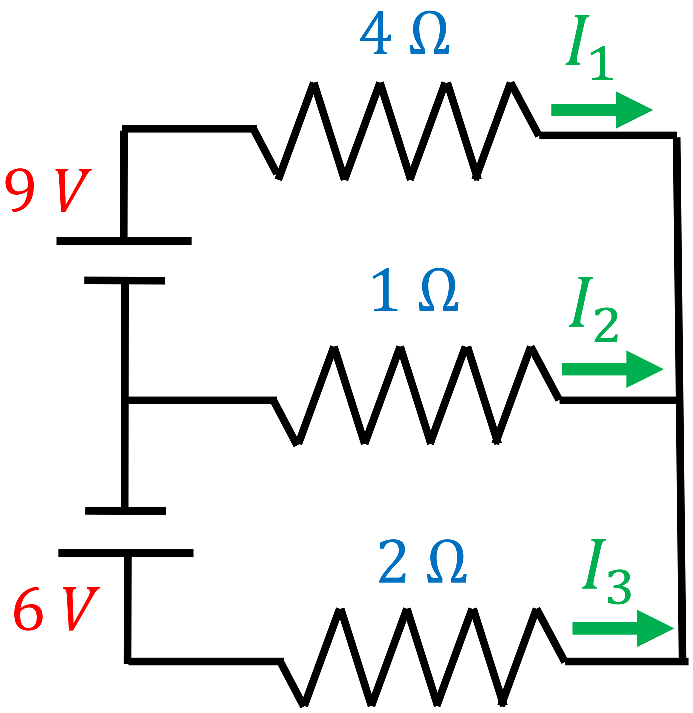


Kirchoff's loop rule : the sum of voltage changes around a closed loop is zero

$$-6 - 1\,I_2 + 2\,I_3 = 0$$

Kirchoff's rules

- What are the currents flowing in the 3 resistors?



We now have 3 equations:

$$I_1 + I_2 + I_3 = 0 \quad (1)$$
$$9 - 4I_1 + I_2 = 0 \quad (2)$$
$$-6 - I_2 + 2I_3 = 0 \quad (3)$$

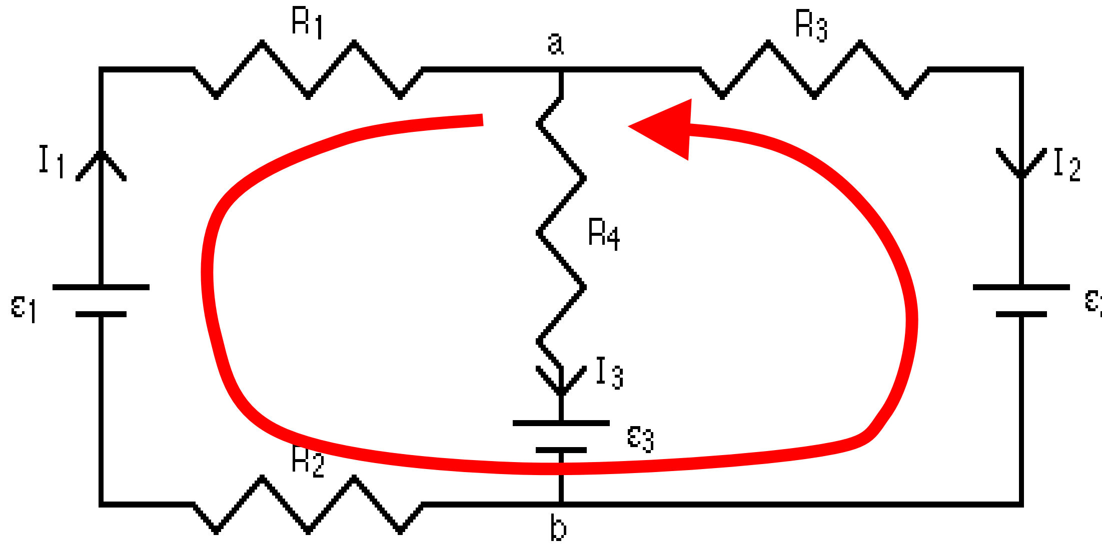
To solve for I_1 we can use algebra to eliminate I_2 and I_3 :

$$(1) \rightarrow \quad I_3 = -I_1 - I_2$$

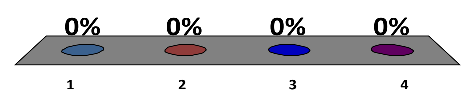
$$Sub.in\ (3) \rightarrow I_2 = -2 - \tfrac{2}{3}I_1$$

$$Sub.in\ (2) \rightarrow I_1 = 1.5\ A$$

Consider the loop shown in the circuit. The correct Kirchoff loop equation, starting at “a” is



1. $I_1R_1 + \varepsilon_1 + I_1R_2 + \varepsilon_2 + I_2R_3 = 0$
2. $-I_1R_1 - \varepsilon_1 - I_1R_2 - \varepsilon_3 - I_3R_4 = 0$
3. $I_1R_1 - \varepsilon_1 + I_1R_2 - \varepsilon_2 - I_2R_3 = 0$
4. $I_1R_1 - \varepsilon_1 + I_1R_2 + \varepsilon_2 + I_2R_3 = 0$



Chapter 25 summary

- Components in a series circuit all carry the same current
- Components in a parallel circuit all experience the same potential difference
- Capacitors are parallel plates which store equal & opposite charge $Q = C V$
- Kirchoff's junction rule and loop rule provide a systematic method for analysing circuits