Chapter 25: Electric circuits

- Voltage and current
- Series and parallel circuits
- Resistors and capacitors
- Kirchhoff’s rules for analysing circuits
Electric circuits

- Closed loop of electrical components around which current can flow, driven by a potential difference

- Current (in Amperes A) is the rate of flow of charge

- Potential difference (in volts V) is the work done on charge
Electric circuits

- May be represented by a circuit diagram.
  Here is a simple case:

- R is the resistance (in Ohms $\Omega$) to current flow
Electric circuits

• Same principles apply in more complicated cases!
Electric circuits

• How do we deal with a more complicated case?

What is the current flowing from the battery?
Electric circuits

• When components are connected in series, the same electric current flows through them.

• Charge conservation: current cannot disappear!
Electric circuits

- When components are connected in parallel, the same potential difference drops across them.

- Points connected by a wire are at the same voltage!
Electric circuits

- When there is a junction in the circuit, the inward and outward currents to the junction are the same.

\[ I_1 = I_2 + I_3 \]

- Charge conservation: current cannot disappear!
Consider the currents $I_1$, $I_2$ and $I_3$ as indicated on the circuit diagram. If $I_1 = 2.5\, \text{A}$ and $I_2 = 4\, \text{A}$, what is the value of $I_3$?

1. 6.5 A
2. 1.5 A
3. $-1.5\, \text{A}$
4. 0 A
5. The situation is not possible
Consider the currents $I_1$, $I_2$ and $I_3$ as indicated on the circuit diagram. If $I_1 = 2.5 \text{ A}$ and $I_2 = 4 \text{ A}$, what is the value of $I_3$?

\[
I_3 = I_1 - I_2
\]

\[
I_3 = 2.5 - 4 = -1.5 \text{ A}
\]

(Negative sign means opposite direction to arrow.)
A 9.0 V battery is connected to a 3 Ω resistor. Which is the **incorrect** statement about potential differences (voltages)?

1. $V_b - V_a = 9.0 \text{ V}$
2. $V_b - V_c = 0 \text{ V}$
3. $V_c - V_d = 9.0 \text{ V}$
4. $V_d - V_a = 9.0 \text{ V}$
Resistors in circuits

- Resistors are the basic components of a circuit that determine current flow: Ohm’s law $I = \frac{V}{R}$
If two resistors are connected in series, what is the total resistance?

- Potential drop $V_1 = I R_1$
- Potential drop $V_2 = I R_2$
- Total potential drop $V = V_1 + V_2 = I R_1 + I R_2 = I (R_1 + R_2)$
Resistors in series/parallel

• If two resistors are connected in series, what is the total resistance?

Potential drop \( V = I R_{total} = I (R_1 + R_2) \)

\[ R_{total} = R_1 + R_2 \]

• Total resistance increases in series!
Resistors in series/parallel

- Total resistance increases in series!
Resistors in series/parallel

- If two resistors are connected in parallel, what is the total resistance?
If two resistors are connected in parallel, what is the total resistance?

Total current $I = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2} = V \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$
Resistors in series/parallel

• If two resistors are connected in parallel, what is the total resistance?

\[ R_{total} \]

\[ I \]

Current \( I = \frac{V}{R_{total}} = V \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \)

\[ \frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} \]

• Total resistance decreases in parallel!
Resistors in series/parallel

• Total resistance decreases in parallel!
• What’s the current flowing?

(1) Combine these 2 resistors in parallel:

\[
\frac{1}{R_{\text{pair}}} = \frac{1}{30} + \frac{1}{50}
\]

\[R_{\text{pair}} = 18.75 \, \Omega\]

(2) Combine all the resistors in series:

\[R_{\text{total}} = 20 + 18.75 + 20 = 58.75 \, \Omega\]

(3) Current \[I = \frac{V}{R_{\text{total}}} = \frac{10}{58.75} = 0.17 \, A\]
If an additional resistor, $R_2$, is added in series to the circuit, what happens to the **power dissipated** by $R_1$?

1. Increases
2. Decreases
3. Stays the same

$$V = IR \quad P = VI = I^2R = \frac{V^2}{R}$$
If an additional resistor, $R_3$, is added in parallel to the circuit, what happens to the total current, $I$?

1. Increases
2. Decreases
3. Stays the same
4. Depends on $R$ values

Parallel resistors: reciprocal effective resistance is sum of reciprocal resistances

$$\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$
Series vs. Parallel

**CURRENT**
- Same current through all series elements
- Current “splits up” through parallel branches

**VOLTAGE**
- Voltages add to total circuit voltage
- Same voltage across all parallel branches

**RESISTANCE**
- Adding resistance increases total R
- Adding resistance reduces total R

String of Christmas lights – connected in *series*
Power outlets in house – connected in *parallel*
Voltage divider

Consider a circuit with several resistors in series with a battery.

Current in circuit: \[ I = \frac{V}{R_{total}} = \frac{V}{R_1 + R_2 + R_3} \]

The potential difference across one of the resistors (e.g. \( R_1 \))

\[ V_1 = IR_1 = V \frac{R_1}{R_1 + R_2 + R_3} \]

The fraction of the total voltage that appears across a resistor in series is the ratio of the given resistance to the total resistance.
What must be the resistance $R_1$ so that $V_1 = 2.0 \text{ V}$?

1. 0.80 $\Omega$
2. 1.2 $\Omega$
3. 6.0 $\Omega$
4. 30 $\Omega$
A capacitor is a device in a circuit which can be used to store charge.

It’s charged by connecting it to a battery ...

A capacitor consists of two charged plates ...

Electric field $E$
Capacitors

• A capacitor is a device in a circuit which can be used to store charge

Example: store and release energy...
Capacitors

- The capacitance $C$ measures the amount of charge $Q$ which can be stored for a given potential difference $V$.

\[ C = \frac{Q}{V} \]

\[ Q = C \cdot V \]

(Value of $C$ depends on geometry...)

- Unit of capacitance is Farads [F]
Resistor-capacitor circuit

• Consider the following circuit with a resistor and a capacitor in series

What happens when we connect the circuit?
Resistor-capacitor circuit

- When the switch is connected, the battery charges up the capacitor

- Move the switch to point a

- Initial current flow $I=\frac{V}{R}$

- Charge $Q$ flows from battery onto the capacitor

- Potential across the capacitor $V_C=\frac{Q}{C}$ increases

- Potential across the resistor $V_R$ decreases

- Current decreases to zero
Resistor-capacitor circuit

- When the switch is connected, the battery charges up the capacitor
Resistor-capacitor circuit

- When the battery is disconnected, the capacitor pushes charge around the circuit

- Move the switch to point b
- Initial current flow $I = \frac{V_C}{R}$
- Charge flows from one plate of capacitor to other
- Potential across the capacitor $V_C = \frac{Q}{C}$ decreases
- Current decreases to zero
When the battery is disconnected, the capacitor pushes charge around the circuit.
If two capacitors are connected in series, what is the total capacitance?

Potential drop $V_1 = \frac{Q}{C_1}$

Potential drop $V_2 = \frac{Q}{C_2}$

Total potential drop $V = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2} = Q \left( \frac{1}{C_1} + \frac{1}{C_2} \right)$

Same charge must be on every plate!
Capacitors in series/parallel

- If two capacitors are connected in series, what is the total capacitance?

\[ Q = C \cdot V \]

Potential drop \( V = \frac{Q}{C_{total}} = Q \left( \frac{1}{C_1} + \frac{1}{C_2} \right) \)

- Total capacitance decreases in series!
Capacitors in series/parallel

- If two capacitors are connected in parallel, what is the total capacitance?

\[ Q = CV \]

Total charge \( Q = Q_1 + Q_2 = C_1V + C_2V = (C_1 + C_2) V \)
Capacitors in series/parallel

- If two capacitors are connected in parallel, what is the total capacitance?

\[ Q = C \times V \]

Total charge \( Q = C_{total} \times V = (C_1 + C_2) \times V \)

\[ C_{total} = C_1 + C_2 \]

- Total capacitance increases in parallel!
Two 5.0 F capacitors are in series with each other and a 1.0 V battery. Calculate the charge on each capacitor (Q) and the total charge drawn from the battery (Q_{total}).

1. Q = 5.0 C, Q_{total} = 5.0 C
2. Q = 0.25 C, Q_{total} = 0.50 C
3. Q = 2.5 C, Q_{total} = 2.5 C
4. Q = 2.5 C, Q_{total} = 5.0 C

\[ Q = CV \]

\[
\frac{1}{C_{\text{total}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots
\]
Two 5.0 F capacitors are in series with each other and a 1.0 V battery. Calculate the charge on each capacitor (Q) and the total charge drawn from the battery ($Q_{total}$).

Potential difference across each capacitor = 0.5 V

Charge on each capacitor $Q = CV = 5 \times 0.5 = 2.5 \, C$

\[
\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} \rightarrow \frac{1}{C_{total}} = \frac{1}{5} + \frac{1}{5} = \frac{2}{5} \rightarrow C_{total} = 2.5 \, F
\]

$Q_{total} = C_{total} \times V = 2.5 \times 1 = 2.5 \, C$
Two 5.0 F capacitors are in parallel with each other and a 1.0 V battery. Calculate the charge on each capacitor (Q) and the total charge drawn from the battery (Q_{total}).

1. Q = 5.0 C, Q_{total} = 5.0 C
2. Q = 0.2 C, Q_{total} = 0.4 C
3. Q = 5.0 C, Q_{total} = 10 C
4. Q = 2.5 C, Q_{total} = 2.5 C

\[ Q = CV \] \[ C_{total} = C_1 + C_2 + C_3 + \cdots \]
Two 5.0 F capacitors are in parallel with each other and a 1.0 V battery. Calculate the charge on each capacitor \( Q \) and the total charge drawn from the battery \( Q_{\text{total}} \).

Potential difference across each capacitor = 1 V

Charge on each capacitor \( Q = CV = 5 \times 1 = 5 \, C \)

\[ Q_{\text{total}} = 10 \, C \]
Resistors vs. Capacitors

\[ R_{\text{total}} = R_1 + R_2 \]

\[ \frac{1}{C_{\text{total}}} = \frac{1}{C_1} + \frac{1}{C_2} \]
Resistors vs. Capacitors

\[ R_1 \]
\[ R_2 \]

\[ \frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} \]

\[ C_{total} = C_1 + C_2 \]
Kirchoff’s rules

• Sometimes we might need to analyse more complicated circuits, for example ...

\[ R_1 = 4 \, \Omega \]
\[ R_2 = 1 \, \Omega \]
\[ R_3 = 2 \, \Omega \]

\[ V_1 = 9 \, V \]
\[ V_2 = 6 \, V \]

Q) What are the currents flowing in the 3 resistors?

• Kirchoff’s rules give us a systematic method
Kirchoff’s rules

- What are the currents flowing in the 3 resistors?

Kirchoff’s junction rule: the sum of currents at any junction is zero
Kirchoff's rules

• The sum of currents at any junction is zero

• Watch out for directions: into a junction is positive, out of a junction is negative

\[ I_1 - I_2 - I_3 = 0 \]

\[ I_1 = I_2 + I_3 \]
Kirchoff’s rules

- What are the currents flowing in the 3 resistors?

Kirchoff’s junction rule: the sum of currents at any junction is zero

\[ I_1 + I_2 + I_3 = 0 \]
Kirchoff’s rules

• What are the currents flowing in the 3 resistors?

Kirchoff’s loop rule: the sum of voltage changes around a closed loop is zero.
Kirchoff’s rules

• Sum of voltage changes around a closed loop is zero

Consider a unit charge (Q=1 Coulomb) going around this loop

• It gains energy from the battery (voltage change +V)
• It loses energy in the resistor (voltage change - IR)

Conservation of energy: V - IR = 0 (or as we know, V = IR)
What are the currents flowing in the 3 resistors?

Kirchoff’s loop rule: the sum of voltage changes around a closed loop is zero.

\[ 9 - 4I_1 + 1I_2 = 0 \]
Kirchoff’s rules

What are the currents flowing in the 3 resistors?

Kirchoff’s loop rule: the sum of voltage changes around a closed loop is zero

\[-6 - 1 I_2 + 2 I_3 = 0\]
Kirchoff’s rules

• What are the currents flowing in the 3 resistors?

We now have 3 equations:

\[ I_1 + I_2 + I_3 = 0 \quad (1) \]
\[ 9 - 4I_1 + I_2 = 0 \quad (2) \]
\[ -6 - I_2 + 2I_3 = 0 \quad (3) \]

To solve for \( I_1 \) we can use algebra to eliminate \( I_2 \) and \( I_3 \):

\[(1) \rightarrow I_3 = -I_1 - I_2 \]

Sub. in (3) \( \rightarrow I_2 = -2 - \frac{2}{3} I_1 \)

Sub. in (2) \( \rightarrow I_1 = 1.5 \, A \)
Consider the loop shown in the circuit. The correct Kirchoff loop equation, starting at “a” is

1. \[ I_1 R_1 + \varepsilon_1 + I_1 R_2 + \varepsilon_2 + I_2 R_3 = 0 \]

2. \[ -I_1 R_1 - \varepsilon_1 - I_1 R_2 - \varepsilon_3 - I_3 R_4 = 0 \]

3. \[ I_1 R_1 - \varepsilon_1 + I_1 R_2 - \varepsilon_2 - I_2 R_3 = 0 \]

4. \[ I_1 R_1 - \varepsilon_1 + I_1 R_2 + \varepsilon_2 + I_2 R_3 = 0 \]
Chapter 25 summary

• Components in a **series** circuit all carry the **same** current

• Components in a **parallel** circuit all experience the **same** potential difference

• **Capacitors** are parallel plates which store equal & opposite charge \( Q = C \ V \)

• **Kirchoff’s junction rule** and **loop rule** provide a systematic method for analysing circuits