Revision : exam tips

Introduction to Physics exam

- 180 minutes – 60% of course assessment
- Section A : 20 multiple choice questions for 15 marks – circle responses on answer sheet
- Section B : 16 basic problems for 30 marks – write answers in the spaces provided
- Section C : 5 advanced problems for 15 marks – write answers in the spaces provided

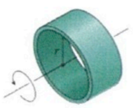
Introduction to Physics exam

- Only allowed to take in Standard Exam Calculator ("TI-30XB")
- Formula sheet provided (see next slide)
- Make sure you show all working for Sections B and C
- Note : all exam questions are taken from the textbook!

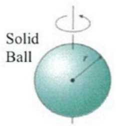
Formula sheet

LINEAR MECHANICS		ROTATIONAL MECHANICS	
$v = v_0 + at$	$x - x_0 = \frac{1}{2}(v_0 + v)t$	$\omega = \omega_0 + \alpha t$	$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$
$v^2 = v_0^2 + 2a(x - x_0)$	$x - x_0 = v_0 t + \frac{1}{2}at^2$	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$	$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$
$\vec{F}_{net} = m\vec{a} = \frac{d\vec{p}}{dt}$	$\vec{w} = m\vec{g}$	$\vec{\tau}_{net} = I\vec{\alpha} = \frac{d\vec{L}}{dt}$	$s = r\theta \quad \omega = \frac{d\theta}{dt} \quad \alpha = \frac{d\omega}{dt}$
$W = \vec{F} \cdot \Delta\vec{r} = F\Delta r \cos\theta$	$f_s \le \mu_s n \quad f_k = \mu_k n$	$\lvert\vec{\tau}\rvert = \lvert\vec{r} \times \vec{F}\rvert = rF\sin\theta$	$\vec{F}_r = m\vec{a}_r = \frac{mv^2}{r}$
$W = \int_{x_1}^{x_2} F\,dx$	$F_s = -kx$ $\Delta U_s = \frac{1}{2}k(x_f^2 - x_i^2)$	$I = \sum_i m_i r_i^2$	$v = r\omega \quad a_t = r\alpha$ $\vec{a}_{net} = \vec{a}_r + \vec{a}_t$
$W_{net} = \Delta K = \frac{1}{2}m(v_f^2 - v_i^2)$	$W_c = -\Delta U \quad U_g = mgy$	$K_R = \frac{1}{2}I\omega^2$	$K_{roll} = \frac{1}{2}I_{cm}\omega^2 + \frac{1}{2}mv_{cm}^2$
$\Delta K + \Delta U = W_{nc} = -F_{fric}d$	$P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$	$P_R = \frac{dW}{dt} = \vec{\tau} \cdot \vec{\omega}$	$x_{cm} = \frac{\sum_i m_i x_i}{\sum_i m_i}$
$\vec{p} = m\vec{v}$ $\vec{p}_{1,i} + \vec{p}_{2,i} = \vec{p}_{1,f} + \vec{p}_{2,f}$	$\vec{J} = \int_{t_1}^{t_2} \vec{F}\,dt = \Delta\vec{p} = \vec{F}\Delta t$	$\vec{L} = I\vec{\omega}$ $\vec{L}_{1,i} + \vec{L}_{2,i} = \vec{L}_{1,f} + \vec{L}_{2,f}$	$\lvert\vec{L}\rvert = \lvert\vec{r} \times \vec{p}\rvert = mvr\sin\theta$

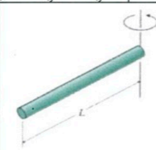
$I = \frac{1}{12}ML^2$



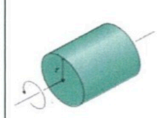
$I = MR^2$



$I = \frac{2}{5}MR^2$



$I = \frac{1}{3}ML^2$



$I = \frac{1}{2}MR^2$

FLUID MECHANICS			
$p = \frac{F}{A} \quad F_B \propto \rho V g$	$p = p_0 + \rho g h \quad \rho = \frac{m}{V}$	$p + \frac{1}{2}\rho v^2 + \rho g y = const$	$A_1 v_1 = A_2 v_2 = const$

THERMODYNAMICS			
$\frac{\Delta L}{L} = \alpha\Delta T \quad \frac{\Delta V}{V} = \beta\Delta T$	$pV = nRT = Nk_B T$	$\frac{1}{2}m\overline{v^2} = \frac{3}{2}k_B T$	$n = \frac{N}{N_A} = \frac{m}{M}$
$Q = mc\Delta T \quad Q = m\,L$	$PV = \frac{1}{3}m\overline{v^2}$	$H = \frac{Q}{\Delta t} = -kA\frac{dT}{dx}$	$P_{net} = \sigma Ae(T^4 - T_{amb}^4)$

ELECTRICITY			
$F = k_e \frac{q_1 q_2}{r^2}$	$E = k_e \frac{q}{r^2} = \frac{F_e}{q}$	$i = \frac{\Delta q}{\Delta t}\ ,\ i = \frac{V}{R}$	$P = Vi = i^2 R = \frac{V^2}{R}$
$V_b - V_a = \frac{1}{q}(U_b - U_a) = \frac{-W_{ba}}{q}$	$E = -\frac{V_b - V_a}{d}$	$q = CV$	$v = \sqrt{\frac{F}{\mu}} \quad f_n = \frac{n}{2L}v$
$\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$ parallel	$R_{eff} = R_1 + R_2 + R_3 + \cdots$ series	$C_{eff} = C_1 + C_2 + C_3 + \cdots$ parallel	$\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$ parallel

Formula sheet

LINEAR MECHANICS	
$v = v_0 + at$	$x - x_0 = \frac{1}{2}(v_0 + v)t$
$v^2 = v_0^2 + 2a(x - x_0)$	$x - x_0 = v_0 t + \frac{1}{2}at^2$
$\vec{F}_{net} = m\vec{a} = \frac{d\vec{p}}{dt}$	$\vec{w} = m\vec{g}$
$W = \vec{F} \cdot \Delta\vec{r} = F\Delta r \cos\theta$	$f_s \leq \mu_s n \quad f_k = \mu_k n$
$W = \int_{x_1}^{x_2} F\, dx$	$F_s = -kx$ $\Delta U_s = \frac{1}{2}k(x_f^2 - x_i^2)$
$W_{net} = \Delta K = \frac{1}{2}m(v_f^2 - v_i^2)$	$W_c = -\Delta U \quad U_g = mgy$
$\Delta K + \Delta U = W_{nc} = -F_{fric}d$	$P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$
$\vec{p} = m\vec{v}$ $\vec{p}_{1,i} + \vec{p}_{2,i} = \vec{p}_{1,f} + \vec{p}_{2,f}$	$\vec{J} = \int_{t_1}^{t_2} \vec{F}\, dt = \Delta\vec{p} = \vec{F}\Delta t$

Formula sheet

Acceleration due to gravity at the earth's surface	g	$9.80\ m/s^2$
Avogadro's constant	N_A	$6.02 \times 10^{23}\ mol^{-1}$
Boltzmann's constant	k_B	$1.38 \times 10^{-23}\ J/K$
Ideal gas constant	R	$8.31\ J/mol\ K$
Stefan constant	σ	$5.67 \times 10^{-8}\ W/m^2\ K^4$
Atomic Mass Unit	u	$1.66 \times 10^{-27}\ kg$
Density of water		$1.00 \times 10^3\ kg/m^3$
Density of helium		$0.18\ kg/m^3$
Density of concrete		$2200\ kg/m^3$
Density of Styrofoam		$160\ kg/m^3$
Co-efficient of linear expansion of steel		12×10^{-6}
Specific heat of aluminium		$900\ J/kg\ ^oC$
Specific heat of ice		$2050\ J/kg\ ^oC$
Specific heat of iron		$447\ J/kg\ ^oC$
Specific heat of Styrofoam		$1300\ J/kg\ ^oC$
Specific heat of water		$4186\ J/kg\ ^oC$
Specific heat of wood		$1400\ J/kg\ ^oC$
Latent heat of fusion of ice		$3.33 \times 10^5\ J/kg$
Latent heat of vaporisation of water		$2.26 \times 10^6\ J/kg$
Thermal Conductivity of iron		$80.4\ W/m\ ^oC$
Thermal Conductivity of water		$0.61\ W/m\ ^oC$
Thermal Conductivity of Styrofoam		$0.029\ W/m\ ^oC$
Thermal Conductivity of wood		$0.11\ W/m\ ^oC$
Atomic mass of argon, Ar		40 u
Molecular mass of hydrogen, H_2		2.0 u
Molecular mass of nitrogen, N_2		28.0 u
Molecular mass of oxygen, O_2		32.0 u

Conversion factors

1 atm $\equiv$ 1.013×10^5 Pa

$K = {}^oC + 273$

1 litre = $10^{-3}\ m^3$

1 revolution per minute = 2π radians per 60 seconds

Problem-solving tips (1/4)

- Convert all numbers to S.I. units!
- e.g. Distance = m, Time = s, Mass = kg, Force = N, Energy = J, Angle = rad, Pressure = Pa, Temperature = K, Charge = C, Current = A, Resistance = Ω, etc.

- Watch out for unit prefixes and powers!
- $8\ cm = 8\ \times 10^{-2}\ m = 0.08\ m$
- $0.1\ g = 0.1 \times 10^{-3}\ kg = 10^{-4}\ kg$
- $2\ kPa = 2 \times 10^{3}\ Pa$
- $100\ mA = 100\ \times 10^{-3}\ A = 0.1\ A$
- $5\ cm^2 = 5 \times 10^{-4}\ m^2$ (because $1\ cm = \ 10^{-2}\ m$)
- 1 litre $(L) = 1000\ cm^3 = 1000 \times 10^{-6}\ m^3 = 10^{-3}\ m^3$

Problem-solving tips (1/4)

- e.g. you are given a speed of $100\ km/h$
- This is not an S.I. unit so we need to convert!
- $1\ km = 1000\ m$
- $1\ h = 3600\ s$
- $100\frac{km}{h} = 100 \times \frac{1000\ m}{3600\ s} = 27.8\ m/s$

- e.g. you are given a rotation rate of $21\ rpm$
- This is not an S.I. unit so we need to convert!
- 1 revolution $= 2\pi\ rad = 6.28\ rad$
- $1\ m = 60\ s$
- $21\ rpm = 21 \times \frac{6.28\ rad}{60\ s} = 2.2\ rad/s$

Problem-solving tips (2/4)

- Determine what topic the problem is about

    1. Linear mechanics
    2. Rotational mechanics
    3. Fluid mechanics
    4. Thermodynamics
    5. Electricity

- This will help identify the appropriate section of the formula sheet

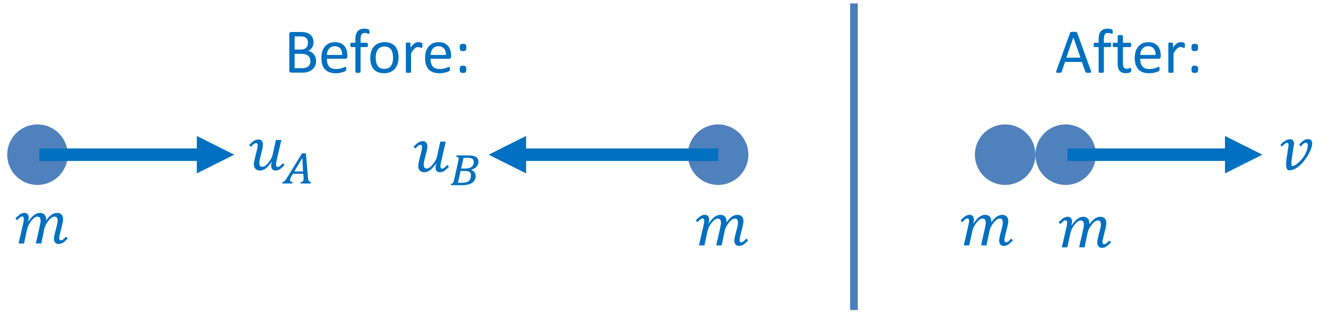
- This will help with symbol confusion, e.g. in mechanics p=momentum, in fluids p=pressure

Problem-solving tips (3/4)

- Draw a simple diagram

1. Two identical masses A and B undergo collide. Initially, A is travelling at 4 m/s in the forward direction; B is travelling at 3 m/s in the backward direction.

After the eventual collision, A and B stick together and move at velocity v, where v equals .



$u_A = 4\, m\, s^{-1}$, $\quad u_B = 3\, m\, s^{-1}$ $\quad v = ??$

- Subconscious starts working on the problem!

Problem-solving tips (4/4)

- Do as much algebra as possible before substituting in numbers

- Entering numbers in the calculator is prone to error!

- Sometimes variables will cancel, so produce as simple an expression as you can

- e.g. conservation of energy problem $mgh = \frac{1}{2}mv^2$, you are given m and h and asked to find the speed v

- Can first re-arrange to give $v = \sqrt{2gh}$, no need to evaluate the total energy

Problem-solving tips (summary)

- Watch out that all numbers are in S.I. units
- What topic is the problem about? (linear / rotational / fluids / thermodynamics / electricity)
- Draw a diagram! Which variables have you been given and which are unknown?
- Do as much algebra as possible before substituting in numbers

Revision :
linear mechanics

Linear Mechanics key facts (1/8)

- Displacement x [unit is m]
- Velocity $v = \frac{Displacement\ change}{Time} = \frac{\Delta x}{\Delta t}$ [unit is $m\ s^{-1}$]
- Acceleration $a = \frac{Velocity\ change}{Time} = \frac{\Delta v}{\Delta t}$ [unit is $m\ s^{-2}$]

Instantaneous velocity = rate of change of x at a given t

Average velocity = (Total displacement)/(Total time)

Can be vectors e.g. $\vec{v} = (v_x, v_y) \rightarrow |\vec{v}| = \sqrt{{v_x}^2 + {v_y}^2}$

Linear Mechanics key facts (2/8)

- 1D motion with constant acceleration a : what is the displacement x and velocity v at time t?

$$x = x_0 + v_0\, t + \frac{1}{2} a\, t^2$$

$$v = v_0 + a\, t$$

$$v^2 = {v_0}^2 + 2\, a\, (x - x_0)$$

x_0 = initial displacement

v_0 = initial velocity

Sometimes a = acceleration due to gravity $g = 9.8\, m\, s^{-2}$

Acceleration will be negative if it's in the direction opposite x

Linear Mechanics key facts (3/8)

- Newton's Laws define the concept of force, measured in Newtons [N]

1. Forces balance in equilibrium
2. Net force causes mass m to accelerate : $F = m\,a$
3. Forces arranged in action/reaction pairs

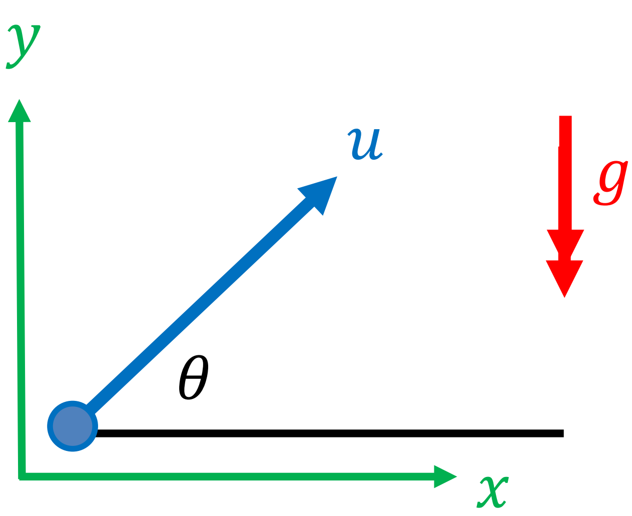
Force under gravity (weight) : $W = m\,g$

Force from a stretched spring $= k\,x$

Linear Mechanics key facts (4/8)

- Motion in 2D : apply equations of motion, or $F = ma$, to both components

e.g. projectile motion …



Acceleration $a_x = 0,\ a_y = -g$

$$v = v_0 + a\,t$$

$$v_x = u\cos\theta$$

$$v_y = u\sin\theta - g\,t$$

$$x = x_0 + v_0\,t + \tfrac{1}{2}a\,t^2$$

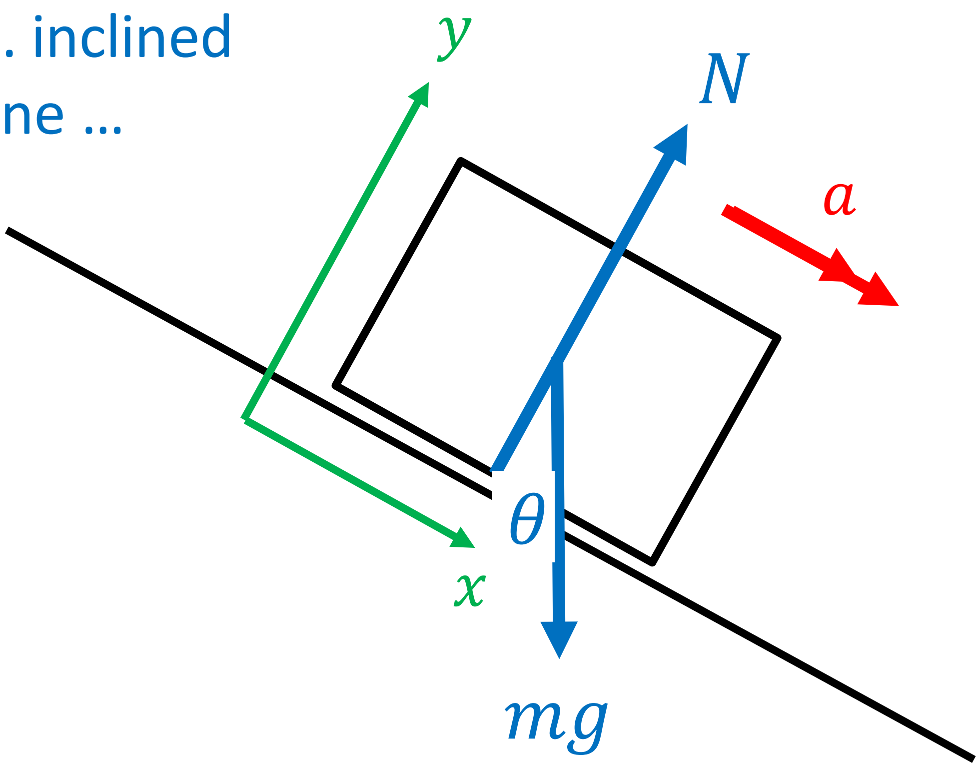
$$x = (u\cos\theta)\,t$$

$$y = (u\sin\theta)\,t - \tfrac{1}{2}g\,t^2$$

Linear Mechanics key facts (4/8)

- Motion in 2D : apply equations of motion, or $F = ma$, to both components

e.g. inclined plane …



x-direction:

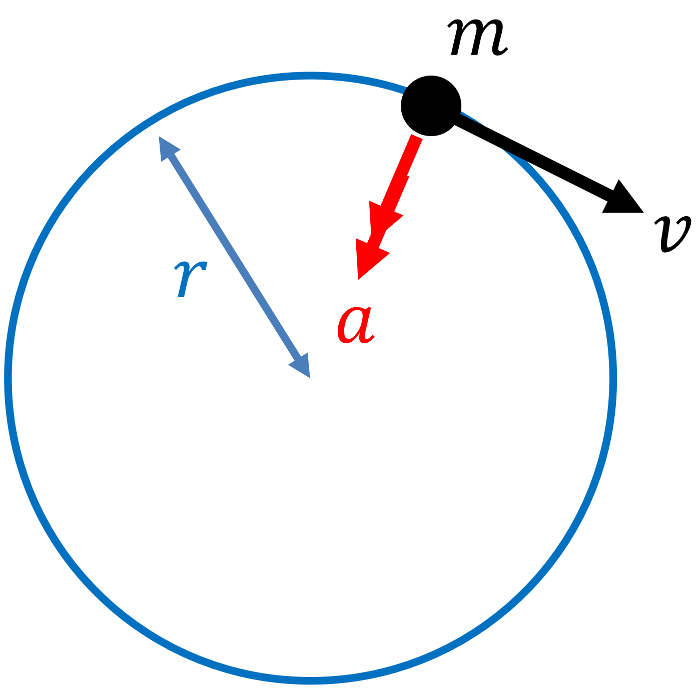
$$mg \sin\theta = ma$$

y-direction:

$$mg \cos\theta - N = 0$$

Linear Mechanics key facts (5/8)

- Motion in a circle :



Centripetal acceleration

$$a = \frac{v^2}{r}$$

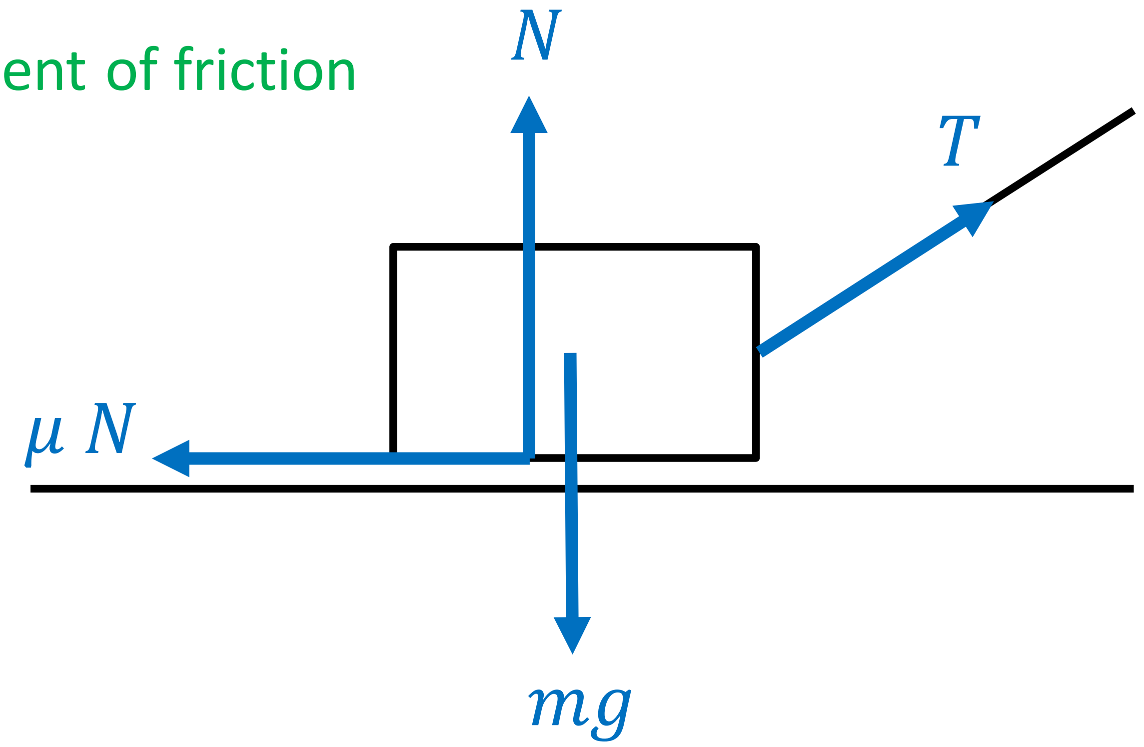
Centripetal force

$$F = \frac{m\,v^2}{r}$$

Linear Mechanics key facts (6/8)

- Friction force opposes relative motion of surfaces in contact $= \mu \times$ Normal Force

$\mu =$ coefficient of friction



Linear Mechanics key facts (7/8)

- Conservation of energy is a quick way of solving many problems. Energy is measured in Joules [J]

- Energy of work done = Force x Distance $= \vec{F} . \overrightarrow{\Delta x}$
- Kinetic energy $= \frac{1}{2} m v^2$
- Gravitational potential energy $= mgh$
- Energy of stretching a spring $= \frac{1}{2} k x^2$
- Power is rate of doing work $= \frac{\Delta W}{\Delta t} = \frac{F \, \Delta x}{\Delta t} = F \, v$

Linear Mechanics key facts (8/8)

- Momentum of a particle $p = mv$

- Collisions of particles (1) : momentum is always conserved

$$m_1 u + 0 = (m_1 + m_2)\, v$$

- Collisions of particles (2) : kinetic energy is only conserved for elastic collisions (otherwise lost)

Practice exam questions

1. Two identical masses A and B undergo collide. Initially, A is travelling at 4 m/s in the forward direction; B is travelling at 3 m/s in the backward direction.

After the eventual collision, A and B stick together and move at velocity v, where v equals .

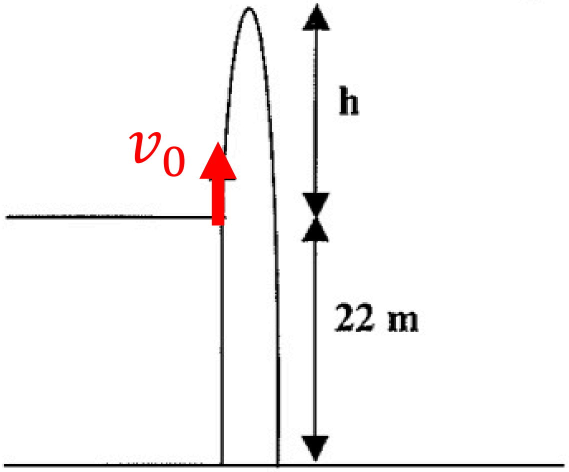
A. 0.5 m/s in the backward direction.
B. 0.5 m/s in the forward direction
C. 5 m/s in the forward direction
D. 6 m/s in the forward direction

Conservation of momentum : $mu_A - mu_B = 2mv$

$v = \frac{1}{2}(u_A - u_B) = \frac{1}{2}(4 - 3) = 0.5\, m\, s^{-1}$ [B]

Practice exam questions

1. (a) A 0.17 kg ball is thrown vertically upwards with a velocity of 34 m/s at the edge of a 22 m cliff.
The diagram is not drawn to scale.
Neglect air resistance in all of the following.



Calculate

(i) the height to which the ball rises above the cliff, h.

Conservation of energy: $\frac{1}{2}mv_0{}^2 = mgh$

$$\rightarrow h = \frac{v_0{}^2}{2g} = \frac{34^2}{2 \times 9.8} = 59\ m$$

1

Practice exam questions

(ii) the total time after release for the ball to reach the ground.

$x_0 = 22\, m\, , v_0 = 34\, ms^{-1}\, , a = -9.8\, ms^{-2}$

2

Final position: $x = 0\, m$, what is v and t?

$v^2 = {v_0}^2 + 2a(x - x_0)$

$v = \sqrt{34^2 + 2 \times (-9.8) \times (-22)} = -39.8\, ms^{-1}$

$v = v_0 + at$ Re-arrange: $t = \frac{v - v_0}{a} = \frac{-39.8 - 34}{-9.8} = 7.5\, s$

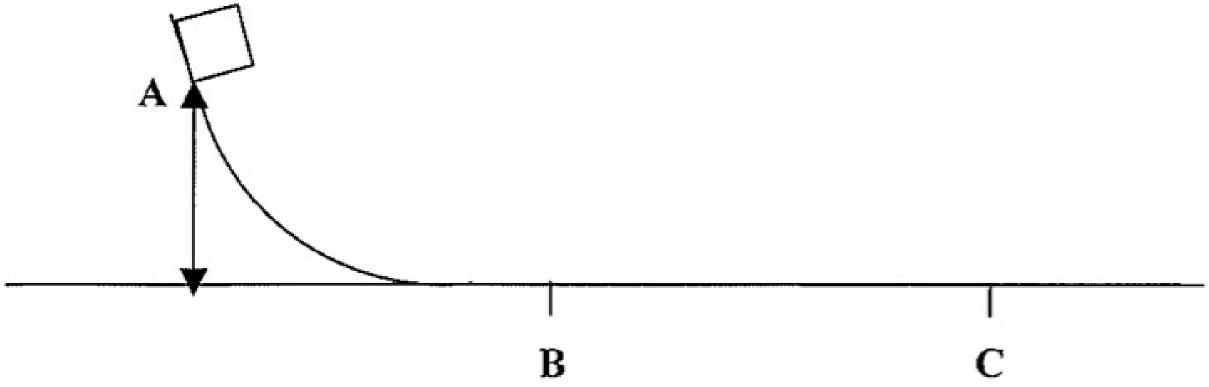
(iii) the velocity of the ball when it reaches the ground.

1

See working above: 39.8 ms^{-1} downwards

Practice exam questions

(b) A 2.0 kg block is released from point A 0.96 m above the ground.
The track is frictionless up to point B; after B the rest of the track has a rough surface.
The block stops at point C; the distance between points B and C is 1.89 m.
The diagram is not drawn to scale.



Calculate

(i) the decrease in gravitational energy as the block goes from point A to point B.

$$mgh = 2 \times 9.8 \times 0.96 = 18.8\, J$$

1

Practice exam questions

(ii) the kinetic energy of the block at point B.

Kinetic energy gained = Potential energy lost $= 18.8\ J$

1

(iii) the speed of the block at point B

$$KE = \frac{1}{2}mv^2 \quad \rightarrow \quad v = \sqrt{\frac{2\ KE}{m}} = \sqrt{\frac{2 \times 18.8}{2}} = 4.34\ m\ s^{-1}$$

1

(iv) the work done by friction in bringing the block to rest.

Work done = Kinetic energy lost $= 18.8\ J$

1

Practice exam questions

(v) the friction force between B and C, f_k.

Work = Force x Distance $\rightarrow$ $f_k = \frac{Work}{Distance} = \frac{18.8}{1.89} = 9.96\ N$

1

(vi) the coefficient of kinetic friction between points B and C, μ_k.

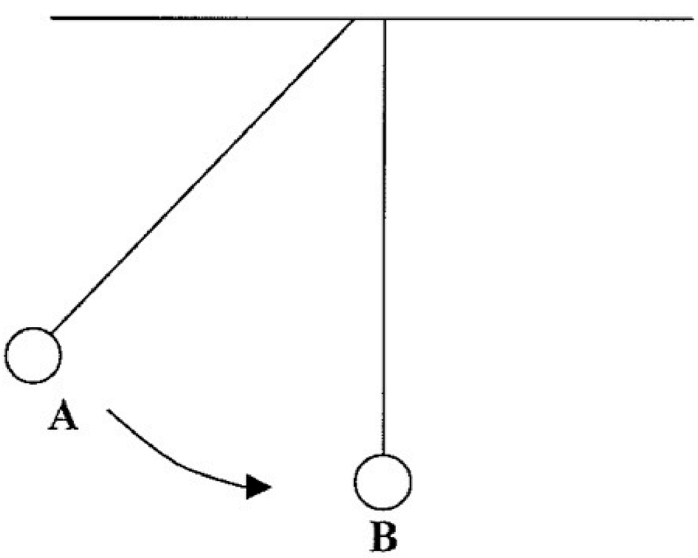
$$f_k = \mu_k N = \mu_k mg$$

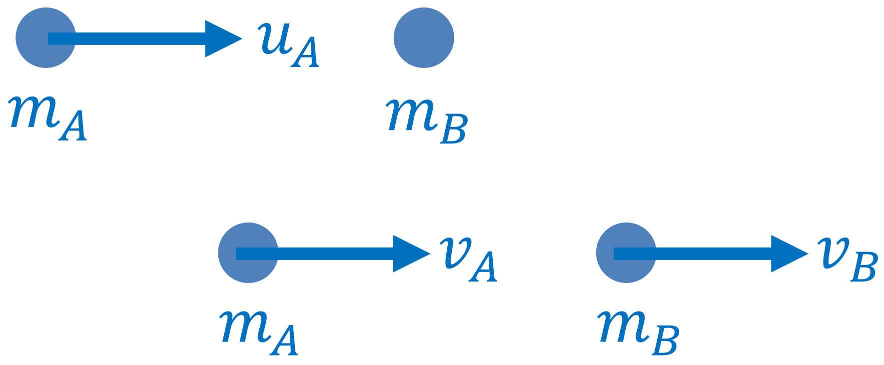
$$\mu_k = \frac{f_k}{mg} = \frac{9.96}{2 \times 9.8} = 0.51$$

1

Practice exam questions

2.(a) Ball A is released from rest. It collides with the stationary ball B with a velocity 3.2 m/s; **immediately** after the collision ball A travels in the same direction with velocity 2.3 m/s.
Ball A has mass 0.26 kg; ball B has mass 0.07 kg.





Conservation of momentum :

$$m_A u_A = m_A v_A + m_B v_B$$

Calculate
(i) the velocity of ball B **immediately** after the collision..

$$v_B = \frac{m_A(u_A - v_A)}{m_B} = \frac{0.26(3.2 - 2.3)}{0.07} = 3.34\, m\, s^{-1}$$

2

Practice exam questions

(ii) the maximum height reached by ball B.

2

Conservation of energy : $\frac{1}{2}m_B{v_B}^2 = m_B gh$

$$h = \frac{{v_B}^2}{2g} = \frac{(3.34)^2}{2 \times 9.8} = 0.57\,m$$

Practice exam questions

(b) A driver is travelling at a constant speed of 15.4 m/s in a 1800 kg car.

At this speed he then enters a large empty car park, and makes a U-turn, travelling in a complete half-circle of radius r.

The friction force between the tyres and the ground is 12.4 kN.

Calculate r.

2

Centripetal force $F = \frac{mv^2}{r}$

$v = 15.4\, ms^{-1}, m = 1800\, kg, F = 12.4\, kN = 12400\, N$

$$r = \frac{m\, v^2}{F} = \frac{1800 \times (15.4)^2}{12400} = 34.4\, m$$

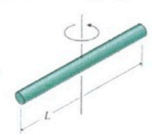
Next steps

- Make sure you are comfortable with unit conversions
- Review the linear mechanics key facts
- Familiarize yourself with the linear mechanics section of the formula sheet
- Try questions from the sample exam papers on Blackboard and/or the textbook

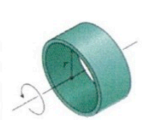
Revision :
rotational mechanics

Formula sheet

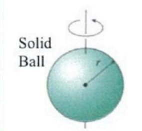
LINEAR MECHANICS		ROTATIONAL MECHANICS	
$v = v_0 + at$	$x - x_0 = \frac{1}{2}(v_0 + v)t$	$\omega = \omega_0 + \alpha t$	$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$
$v^2 = v_0^2 + 2a(x - x_0)$	$x - x_0 = v_0 t + \frac{1}{2}at^2$	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$	$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$
$\vec{F}_{net} = m\vec{a} = \frac{d\vec{p}}{dt}$	$\vec{w} = m\vec{g}$	$\vec{\tau}_{net} = I\vec{\alpha} = \frac{d\vec{L}}{dt}$	$s = r\theta \quad \omega = \frac{d\theta}{dt} \quad \alpha = \frac{d\omega}{dt}$
$W = \vec{F} \cdot \Delta\vec{r} = F\Delta r \cos\theta$	$f_s \le \mu_s n \quad f_k = \mu_k n$	$\lvert\vec{\tau}\rvert = \lvert\vec{r} \times \vec{F}\rvert = rF\sin\theta$	$\vec{F}_r = m\vec{a}_r = \frac{mv^2}{r}$
$W = \int_{x_1}^{x_2} F\,dx$	$F_s = -kx$ $\Delta U_s = \frac{1}{2}k(x_f^2 - x_i^2)$	$I = \sum_i m_i r_i^2$	$v = r\omega \quad a_t = r\alpha$ $\vec{a}_{net} = \vec{a}_r + \vec{a}_t$
$W_{net} = \Delta K = \frac{1}{2}m(v_f^2 - v_i^2)$	$W_c = -\Delta U \quad U_g = mgy$	$K_R = \frac{1}{2}I\omega^2$	$K_{roll} = \frac{1}{2}I_{cm}\omega^2 + \frac{1}{2}mv_{cm}^2$
$\Delta K + \Delta U = W_{nc} = -F_{fric}d$	$P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$	$P_R = \frac{dW}{dt} = \vec{\tau} \cdot \vec{\omega}$	$x_{cm} = \frac{\sum_i m_i x_i}{\sum_i m_i}$
$\vec{p} = m\vec{v}$ $\vec{p}_{1,i} + \vec{p}_{2,i} = \vec{p}_{1,f} + \vec{p}_{2,f}$	$\vec{J} = \int_{t_1}^{t_2} \vec{F}\,dt = \Delta\vec{p} = \vec{F}\Delta t$	$\vec{L} = I\vec{\omega}$ $\vec{L}_{1,i} + \vec{L}_{2,i} = \vec{L}_{1,f} + \vec{L}_{2,f}$	$\lvert\vec{L}\rvert = \lvert\vec{r} \times \vec{p}\rvert = mvr\sin\theta$



$I = \frac{1}{12}ML^2$

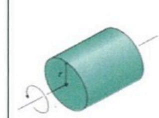


$I = MR^2$



$I = \frac{2}{5}MR^2$

$I = \frac{1}{3}ML^2$



$I = \frac{1}{2}MR^2$

FLUID MECHANICS			
$p = \frac{F}{A} \quad F_B \propto \rho V g$	$p = p_0 + \rho g h \quad \rho = \frac{m}{V}$	$p + \frac{1}{2}\rho v^2 + \rho g y = const$	$A_1 v_1 = A_2 v_2 = const$

THERMODYNAMICS			
$\frac{\Delta L}{L} = \alpha\Delta T \quad \frac{\Delta V}{V} = \beta\Delta T$	$pV = nRT = Nk_BT$	$\frac{1}{2}m\,\overline{v^2} = \frac{3}{2}k_BT$	$n = \frac{N}{N_A} = \frac{m}{M}$
$Q = mc\Delta T \quad Q = m\,L$	$PV = \frac{1}{3}m\,\overline{v^2}$	$H = \frac{Q}{\Delta t} = -kA\frac{dT}{dx}$	$P_{net} = \sigma Ae(T^4 - T_{amb}^4)$

ELECTRICITY			
$F = k_e\frac{q_1 q_2}{r^2}$	$E = k_e\frac{q}{r^2} = \frac{F_e}{q}$	$i = \frac{\Delta q}{\Delta t}, \ i = \frac{V}{R}$	$P = Vi = i^2R = \frac{V^2}{R}$
$V_b - V_a = \frac{1}{q}(U_b - U_a) = \frac{-W_{ba}}{q}$	$E = -\frac{V_b - V_a}{d}$	$q = CV$	$v = \sqrt{\frac{F}{\mu}} \quad f_n = \frac{n}{2L}v$
$\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$ parallel	$R_{eff} = R_1 + R_2 + R_3 + \cdots$ series	$C_{eff} = C_1 + C_2 + C_3 + \cdots$ parallel	$\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$ parallel

Formula sheet

ROTATIONAL MECHANICS	
$\omega = \omega_0 + \alpha t$	$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$
$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$	$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$
$\vec{\tau}_{net} = I\vec{\alpha} = \frac{d\vec{L}}{dt}$	$s = r\theta \quad \omega = \frac{d\theta}{dt} \quad \alpha = \frac{d\omega}{dt}$
$\lvert\vec{\tau}\rvert = \lvert\vec{r}\times\vec{F}\rvert = rF\sin\theta$	$\vec{F}_r = m\vec{a}_r = \frac{mv^2}{r}$
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$\vec{L} = I\vec{\omega}$ $\vec{L}_{1,i} + \vec{L}_{2,i} = \vec{L}_{1,f} + \vec{L}_{2,f}$	$\lvert\vec{L}\rvert = \lvert\vec{r}\times\vec{p}\rvert = mvr\sin\theta$

Formula sheet

LINEAR MECHANICS		ROTATIONAL MECHANICS	
$v = v_0 + at$	$x - x_0 = \frac{1}{2}(v_0 + v)t$	$\omega = \omega_0 + \alpha t$	$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$
$v^2 = v_0^2 + 2a(x - x_0)$	$x - x_0 = v_0 t + \frac{1}{2}at^2$	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$	$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$
$\vec{F}_{net} = m\vec{a} = \frac{d\vec{p}}{dt}$	$\vec{w} = m\vec{g}$	$\vec{\tau}_{net} = I\vec{\alpha} = \frac{d\vec{L}}{dt}$	$s = r\theta \quad \omega = \frac{d\theta}{dt} \quad \alpha = \frac{d\omega}{dt}$
$W = \vec{F}\cdot\Delta\vec{r} = F\Delta r\cos\theta$	$f_s \le \mu_s n \quad f_k = \mu_k n$	$\lvert\vec{\tau}\rvert = \lvert\vec{r}\times\vec{F}\rvert = rF\sin\theta$	$\vec{F}_r = m\vec{a}_r = \frac{mv^2}{r}$
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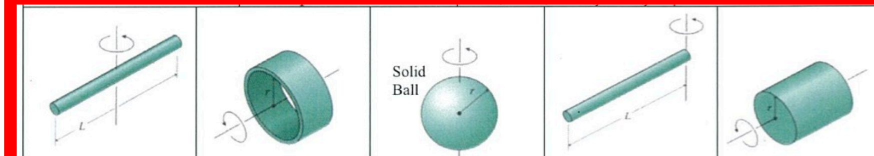

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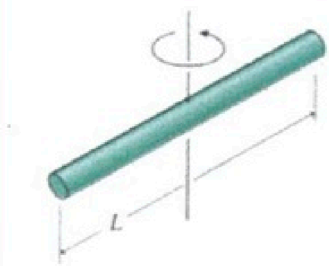
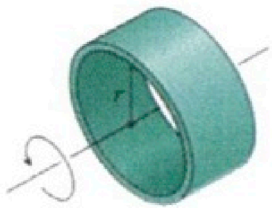
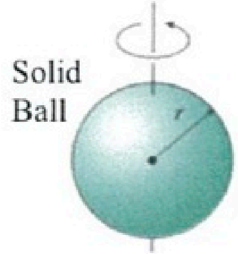
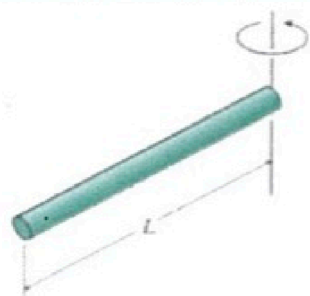
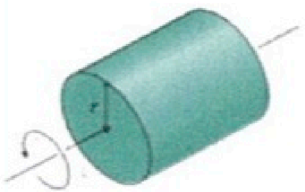
Formula sheet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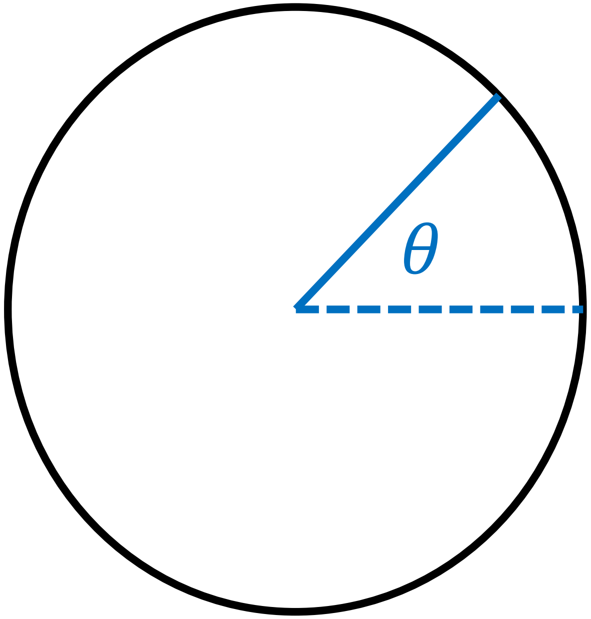
Rotational Mechanics key facts (1/8)

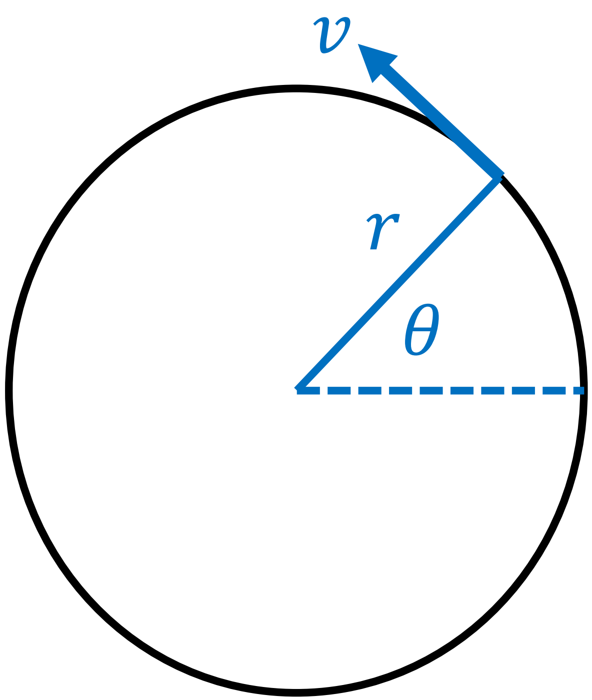
- Analogous formulae to linear mechanics apply, where linear quantities are replaced by rotational quantities



- Displacement x is equivalent to angle swept out θ
- Angle is measured in radians, where 2π is a complete circle
- 1 revolution = 360 degrees = 2π radians

Rotational Mechanics key facts (1/8)

- Analogous formulae to linear mechanics apply, where linear quantities are replaced by rotational quantities



- Angular velocity $\omega = \frac{\Delta\theta}{\Delta t}$ [units : $rad\ s^{-1}$]
- Angular acceleration $\alpha = \frac{\Delta\omega}{\Delta t}$ [units : $rad\ s^{-2}$]
- Linear velocity $v = r\omega$

Rotational Mechanics key facts (1/8)

- Analogous formulae to linear mechanics apply, where linear quantities are replaced by rotational quantities

- Angular velocity $\omega = \frac{\Delta\theta}{\Delta t}$ [units : $rad\ s^{-1}$]
- Linear velocity $v = \frac{\Delta x}{\Delta t}$ [units : $m\ s^{-1}$]
- Angular acceleration $\alpha = \frac{\Delta\omega}{\Delta t}$ [units : $rad\ s^{-2}$]
- Linear acceleration $a = \frac{\Delta v}{\Delta t}$ [units : $m\ s^{-2}$]

Rotational Mechanics key facts (2/8)

- Equations of constant angular acceleration

$$\theta = \theta_0 + \omega_0\, t + \tfrac{1}{2} \alpha\, t^2$$

$$\omega = \omega_0 + \alpha\, t$$

$$\omega^2 = {\omega_0}^2 + 2\, \alpha\, (\theta - \theta_0)$$

Analogous to linear case:

$$x = x_0 + v_0\, t + \tfrac{1}{2} a\, t^2$$

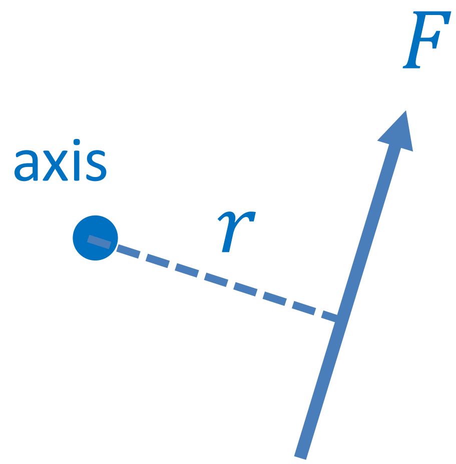
$$v = v_0 + a\, t$$

$$v^2 = {v_0}^2 + 2\, a\, (x - x_0)$$

- They are all on the formula sheet, or you can remember them by analogy with the linear case with $\theta \rightarrow x$, $\omega \rightarrow v$ and $\alpha \rightarrow a$.

Rotational Mechanics key facts (3/8)

- In linear motion, force causes acceleration
- In rotational motion, the torque of a force causes angular acceleration about an axis/pivot



Torque τ = force x perpendicular distance to the axis

$$\tau = F\, r$$

The units of torque are $N\ m$

Rotational Mechanics key facts (4/8)

- In linear motion, the acceleration is determined by the mass m : $F = ma$
- In rotational motion, the role of mass is played by the rotational inertia I of the body about the axis

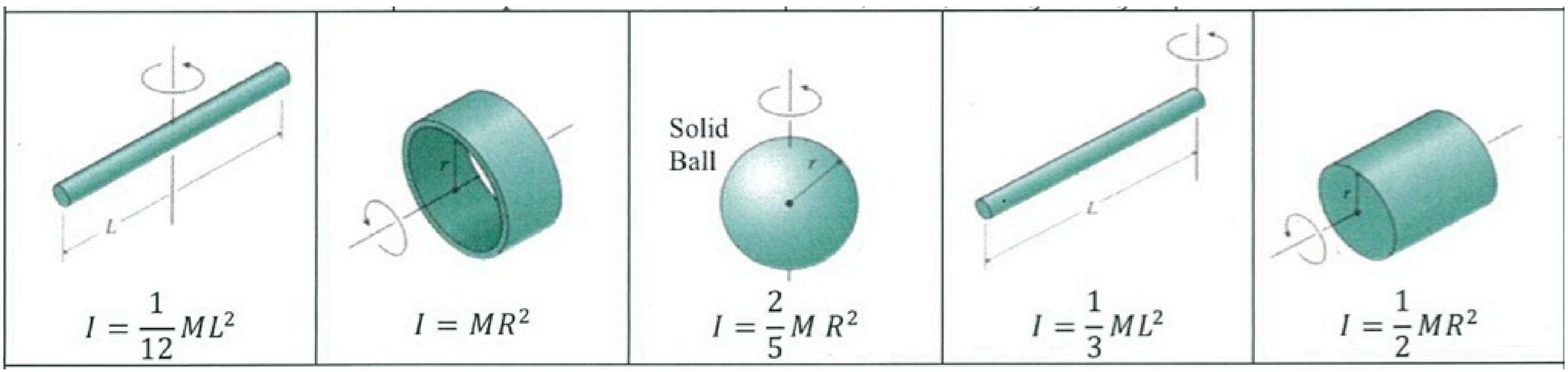
$$F = ma \rightarrow \qquad \boxed{\tau = I\,\alpha}$$

torque = rotational inertia x angular acceleration

Rotational Mechanics key facts (5/8)

- What is the rotational inertia about an axis?

- Different bodies have different rotational inertia depending on their mass M and radius R / length L



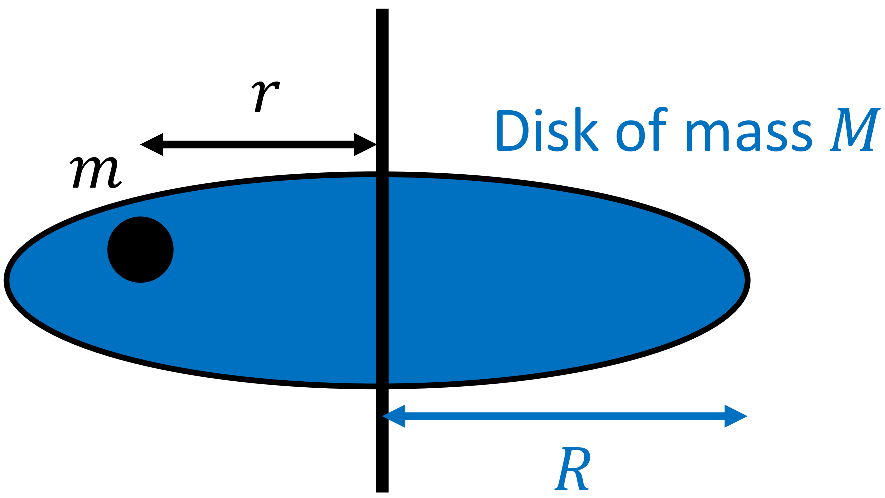
- General formula for a system of particles: $I = \sum_i m_i \, {r_i}^2$

Rotational Mechanics key facts (5/8)

- For a composite system, the rotational inertia about an axis is the sum of the components

$$I_{total} = I_1 + I_2 + \cdots$$

e.g. particle sitting on a disk …



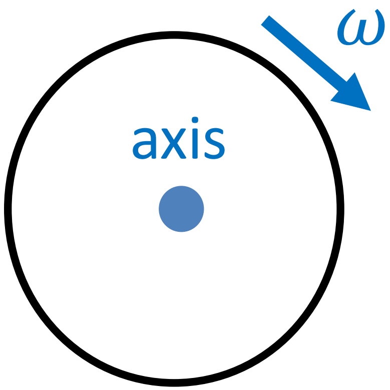
$$I_{total} = \frac{1}{2}MR^2 + mr^2$$

Rotational Mechanics key facts (6/8)

- Rotational energy

- In linear motion, kinetic energy $= \frac{1}{2}mv^2$

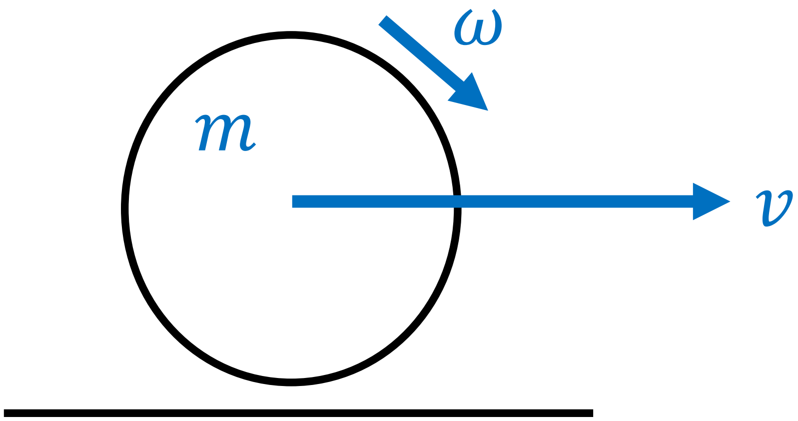
- In rotational motion, kinetic energy $= \frac{1}{2}I\omega^2$



rotational inertia
about axis $= I$

Rotational Mechanics key facts (6/8)

- Rotational energy
- In linear motion, kinetic energy $= \frac{1}{2}mv^2$
- In rotational motion, kinetic energy $= \frac{1}{2}I\omega^2$



Energy of rolling object $= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

Rotational Mechanics key facts (7/8)

- Angular momentum L

$p = mv \rightarrow$ Angular momentum $L = I\,\omega$

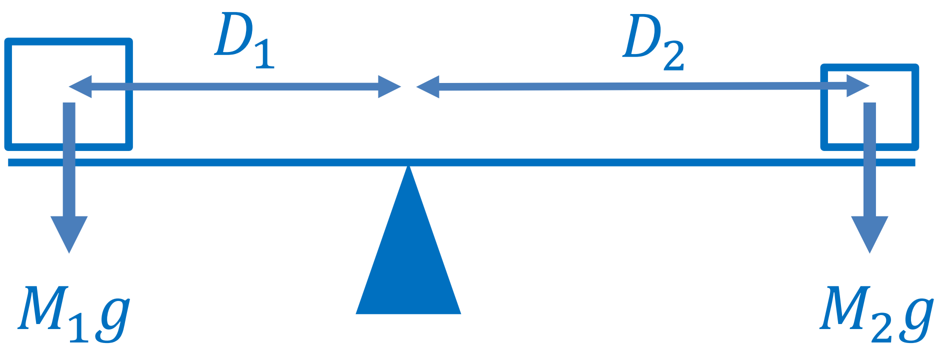
- In linear motion, momentum is conserved if there is no external force (e.g. colliding particles)
- In rotational motion, angular momentum is conserved if there is no external torque

Rotational Mechanics key facts (8/8)

- Rotational equilibrium

- In linear motion, a system is in equilibrium when the forces balance in all directions

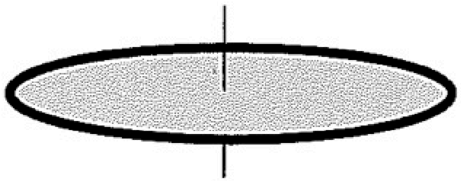
- In rotational motion, a system is in equilibrium when the torques balance



$$M_1 g \times D_1 = M_2 g \times D_2$$

Practice exam questions

3. (a) A disc of mass 10.0 kg and radius 20.0 cm accelerates uniformly from rest and reaches an angular velocity of 20 rad/s in 10.0 s.



Calculate

(i) the moment of inertia of the disc about its vertical rotation axis

$$I = \frac{1}{2}MR^2 = \frac{1}{2} \times 10 \times (0.2)^2 = 0.2\ kg\ m^2$$

1

(ii) the number of revolutions completed in 10.0 s

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{20}{10} = 2\ rad\ s^{-2}$$

$$\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2} \times 2 \times 10^2 = 100\ rad$$

$$\frac{100}{2\pi} = 15.9\ rev$$

(iii) the kinetic energy of the disc at this time.

$$KE = \frac{1}{2}I\omega^2 = \frac{1}{2} \times 0.2 \times 20^2 = 40\ J$$

1

Practice exam questions

(b) After 10.0 s, the disc in part (a) above is allowed to slow down uniformly under the influence of a frictional torque. It takes 120 s to come to rest.

Calculate

(i) the angular acceleration of the disc during this time.

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{-20}{120} = -0.167\ rad\ s^{-2}$$

1

(ii)the magnitude of the frictional torque.

$$\tau = I\ \alpha = 0.2 \times 0.167 = 0.033\ N\ m$$

1

Practice exam questions

A6. A ball is attached to a rod and swung in a horizontal circular path with angular velocity ω. The tension in the rod is T. If the ball is now made to rotate at 2ω, twice the original angular velocity, what will be the tension in the rod?

A. 0.5T

B. T

C. 2T

D. 4T

The tension is providing the centripetal force $F = \frac{mv^2}{r}$

$$v = r\omega$$

Double ω → Double v → Factor 4 increase in F → Option D

A8. Consider a uniform rod of length X and total mass M. The rotational inertia of this rod is

A. MX^2

B. $\frac{1}{3}MX^2$

C. $\frac{1}{12}MX^2$

D. Not enough information to determine.

Impossible to say because it depends on the axis – Option D

Practice exam questions

A9. Conservation of angular momentum only applies when

.

A. there are no external forces acting on a system

B. the rotational inertia of a system is constant

C. there is zero net torque on a system

D. all of the above must apply

Option C is correct

Practice exam questions

B5. A wind turbine's blades are 28 m long and rotate at 21 rpm.

(a) Calculate the angular speed of the blades in radians per second.

$$\omega = 21\,rpm = \frac{21 \times 2\pi\,rad}{60\,s} = 2.2\,rad\,s^{-1}$$

1

(b) Calculate the tangential speed at the tip of a blade.

$$v = r\,\omega = 28 \times 2.2 = 62\,m\,s^{-1}$$

1

Practice exam questions

B6. A wheel turns through 3.0 revolutions while accelerating from rest at 1.7 rad/s^2. Calculate the final angular velocity.

$$\theta - \theta_0 = 3\,rev = 3 \times 2\pi\,rad = 18.8\,rad$$

$$\alpha = 1.7\,rad\,s^{-2}$$

$$\omega_0 = 0\,rad\,s^{-1}$$

What is ω?

$$\omega^2 = {\omega_0}^2 + 2\alpha(\theta - \theta_0)$$

$$\omega = \sqrt{2 \times 1.7 \times 18.8} = 8.0\,rad\,s^{-1}$$

2

Practice exam questions

B7. A 660 g hoop with diameter 95 cm is rotating at 170 rpm about its central axis.

(a) Calculate the rotational inertia of the hoop about this axis.

$$I = M\,R^2 = 0.66 \times \left(\frac{0.95}{2}\right)^2 = 0.15\ kg\ m^2$$

1

(a) Calculate the angular momentum of the hoop.

$$L = I\,\omega$$

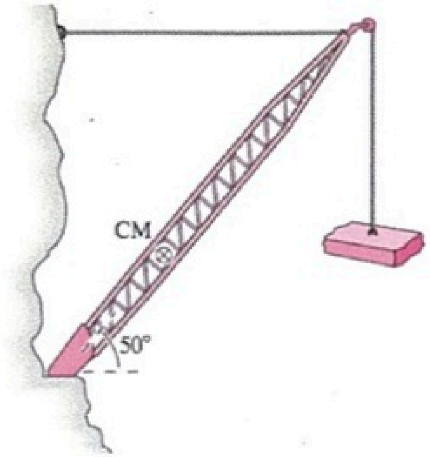
$$\omega = 170\ rpm = \frac{170 \times 2\pi\ rad}{60\ s} = 17.8\ rad\ s^{-1}$$

$$L = I\,\omega = 0.15 \times 17.8 = 2.7\ kg\ m^2\ s^{-1}$$

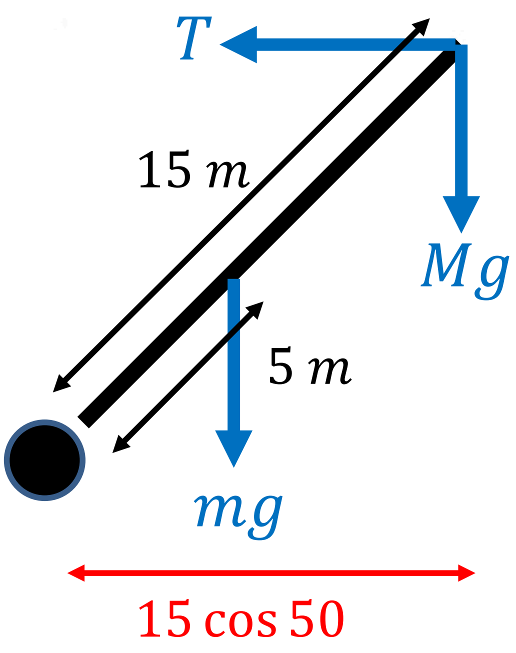
2

Practice exam questions

C2. A crane in a marble quarry is mounted on the quarry's rock walls and is supporting a 2500 kg marble slab as shown in the figure to the right. The centre of mass of the 700 kg boom is located one-third of the way from the pivot end of its 15 m length, as shown. Calculate the tension in the horizontal cable that supports the boom.



In equilibrium, torques about pivot balance



Torque = Force x perpendicular distance to pivot

$$T \times 15 \sin 50 = Mg \times 15 \cos 50 + mg \times 5 \cos 50$$

Using $M = 2500\ kg,\ m = 700\ kg$ and re-arranging the equation …

$$\rightarrow T = 22500\ N$$

Next steps

- Make sure you are comfortable with unit conversions, especially for radians/revolutions
- Review the rotational mechanics key facts
- Familiarize yourself with the rotational mechanics section of the formula sheet, including the rotational inertia panel
- Try questions from the sample exam papers on Blackboard and/or the textbook