Class 9 : Computing the Magnetic Field

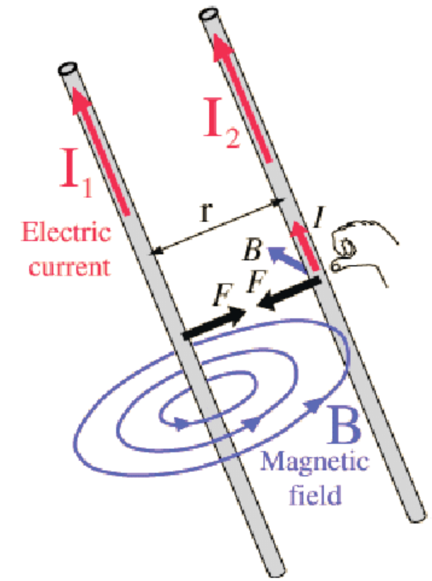
- Ampere’s Law
- Application of Ampere’s Law to a long-straight wire
- Solenoids

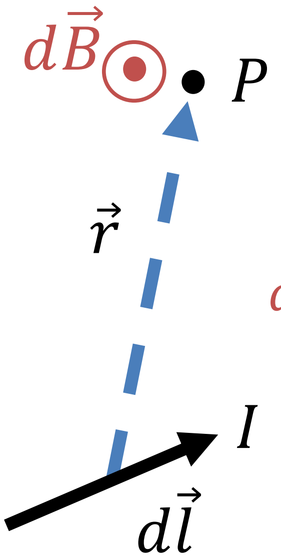
Recap

- **Magnetic forces** are a fundamental phenomenon of nature generated by electric currents

- We say that a current (or magnet) sets up a **magnetic field** $\vec{B}$ around it, which causes another current (or magnet) to feel a force

- The **Biot-Savart Law** describes how magnetic fields $\vec{B}$ are generated by a current I





$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{l} \times \hat{r}}{r^2}$$

Ampere's Law

- Because of its vector nature, the Biot-Savart law is complicated to apply! Luckily there is a simpler formulation (analogous to Gauss's Law)

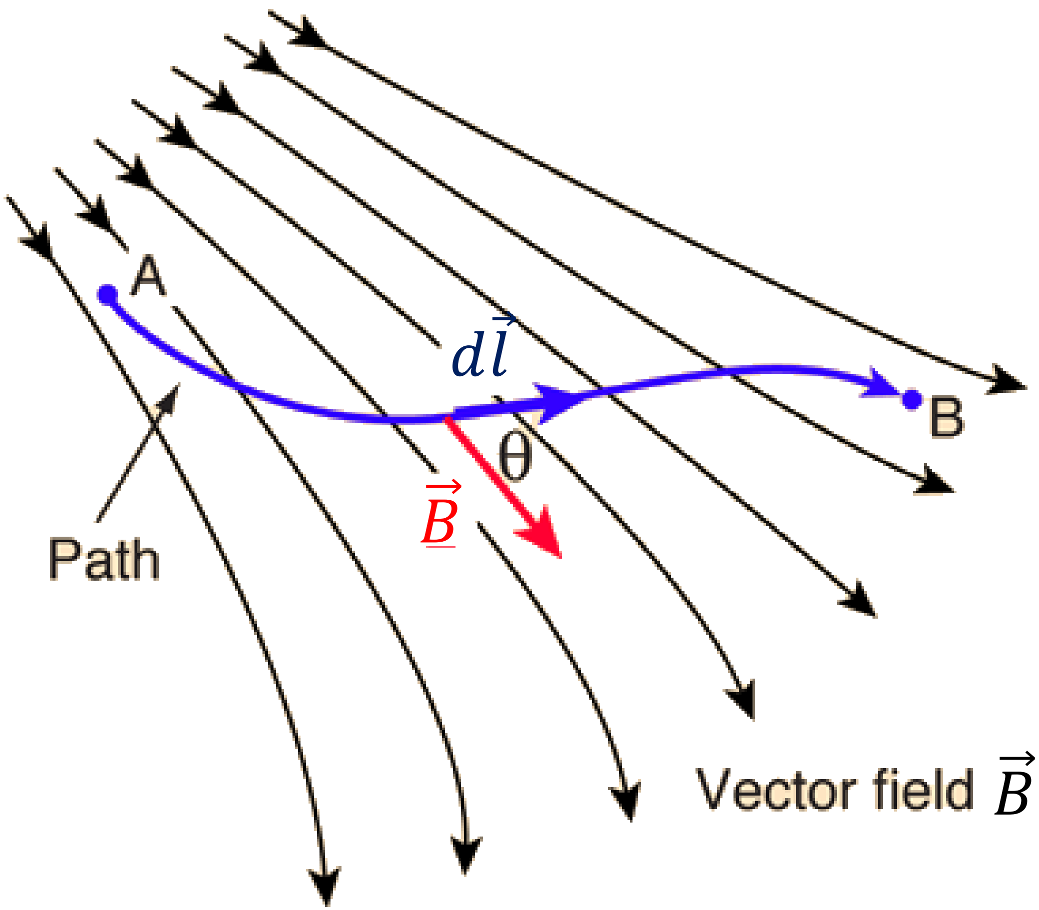
- **Ampere's Law** states that *the line integral of the magnetic field* $\vec{B}$ *around a closed loop is equal to the current enclosed multiplied by* μ_0

- In mathematical terms : $\oint \vec{B}.d\vec{l} = \mu_0 \, I_{enclosed}$

Please note in workbook

Ampere's Law

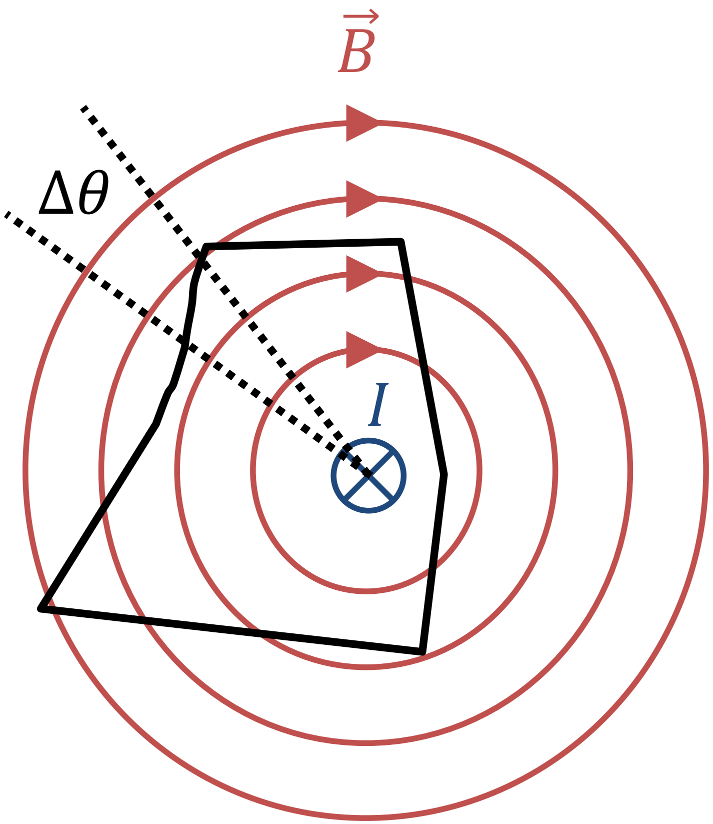
What is meant by a line integral $\int \vec{B}.d\vec{l}$ of a vector field $\vec{B}$?



- Along a curve joining two points, sum up the elements $\vec{B}.d\vec{l} = B\ dl\ \cos\theta$
- If the line forms a **closed loop** (i.e. has no beginning or end) then we write the integral as $\oint \vec{B}.d\vec{l}$

Ampere's Law

Why does Ampere's law $\oint \vec{B}.d\vec{l} = \mu_0 I$ work?

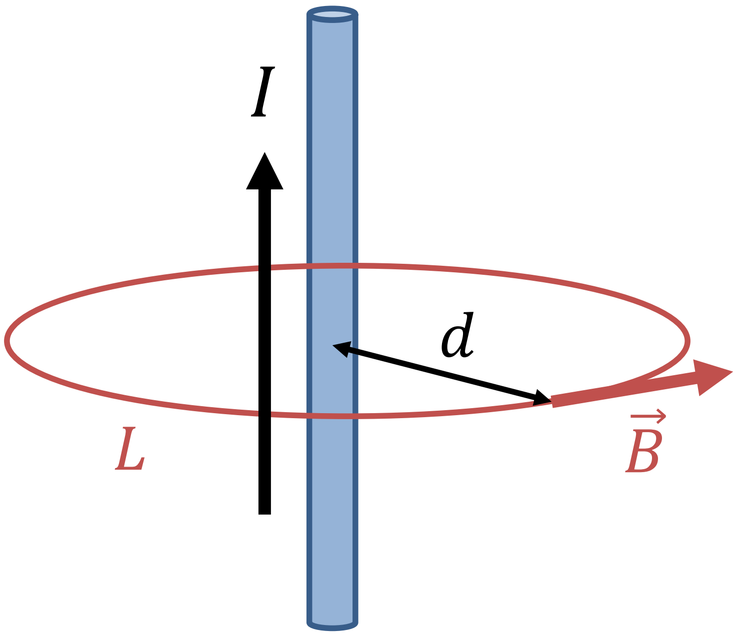


- Consider a current I which sets up magnetic field $B = \frac{\mu_0 I}{2\pi r}$
- Draw any closed loop around I
- Consider the bit of the loop passing between two radii separated by angle $d\theta$
- The contribution $\vec{B}.d\vec{l} = \frac{\mu_0 I}{2\pi r}.r\, d\theta$
- Total $\oint \vec{B}.d\vec{l} = \frac{\mu_0 I}{2\pi} \int d\theta = \mu_0 I$

Ampere's Law

- What is the $\vec{B}$-field around a wire?

$$\oint \vec{B}.d\vec{l} = \mu_0 I_{enc}$$



- We apply Ampere's Law around a circular closed loop L of radius d
- By symmetry, $\vec{B}$ has the same strength around L
- $\vec{B}$ and $d\vec{l}$ are parallel around L, such that $\oint \vec{B}.d\vec{l} = B \times 2\pi d$
- The current enclosed by L is I
- Ampere's Law: $B \times 2\pi d = \mu_0 I$

$$B = \frac{\mu_0 I}{2\pi d}$$

Ampere’s Law

Electric fields

- The electric field around a charge is given by **Coulomb’s Law** $d\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{\vec{r}}$
- The equivalent **Gauss’s Law** $\int \vec{E}.d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$ is easier to apply in practice
- This integral is over a **closed surface** enclosing charge Q_{enc}

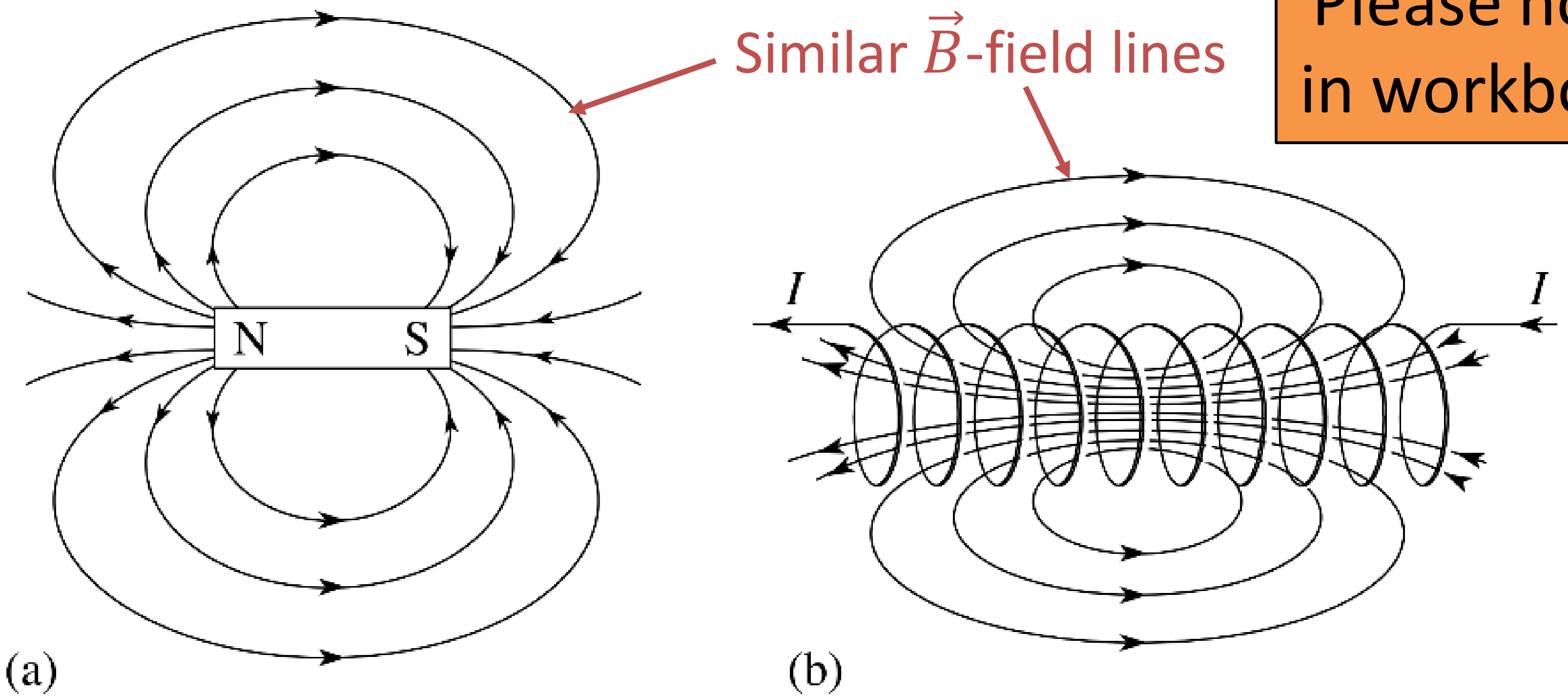
Magnetic fields

- The magnetic field around a current is given by the **Biot-Savart Law** $d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l} \times \hat{\vec{r}}}{r^2}$
- The equivalent **Ampere’s Law** $\oint \vec{B}.d\vec{l} = \mu_0 I_{enc}$ is easier to apply in practice
- The integral is around a **closed loop** enclosing current I_{enc}

Solenoids

- We now introduce the **wire coil of N turns** (or solenoid), which is equivalent to a bar magnet

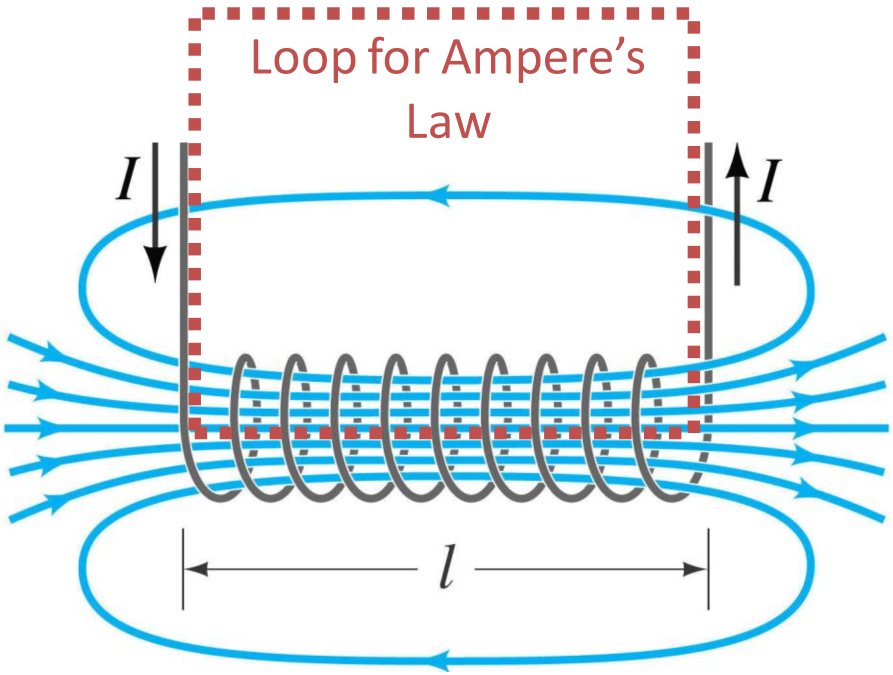
Please note in workbook



- A solenoid **concentrates the $\vec{B}$-field inside it**

Solenoids

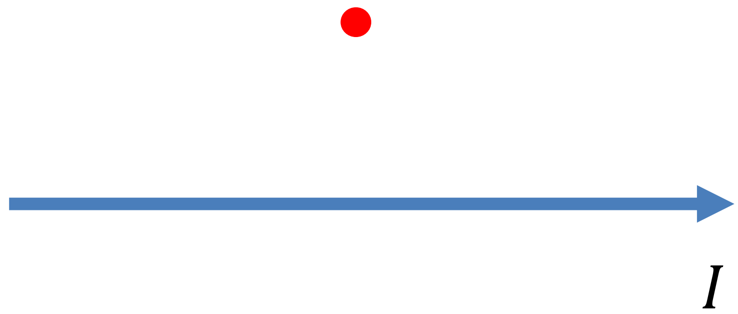
- Now, consider a solenoid of length l with N turns carrying current I. What is the $\vec{B}$-field in the middle?

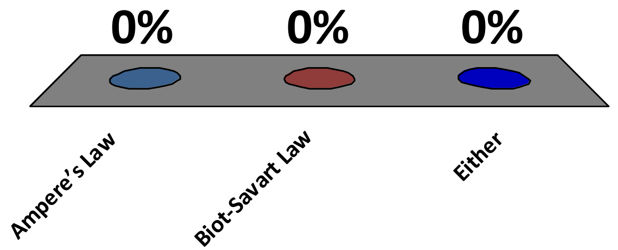


- Apply Ampere's Law $\oint \vec{B}\,.\,d\vec{l} = \mu_0 I_{enc}$ to the closed loop shown
- $\oint \vec{B}.d\vec{l} = B \times l,$ neglecting $\vec{B}$ outside coil
- $I_{enc} = N \times I$
- Hence $B = \frac{\mu_0\, N\, I}{l}$

Clicker question

What is the easiest way to compute the magnetic field near a **long current-carrying wire**?





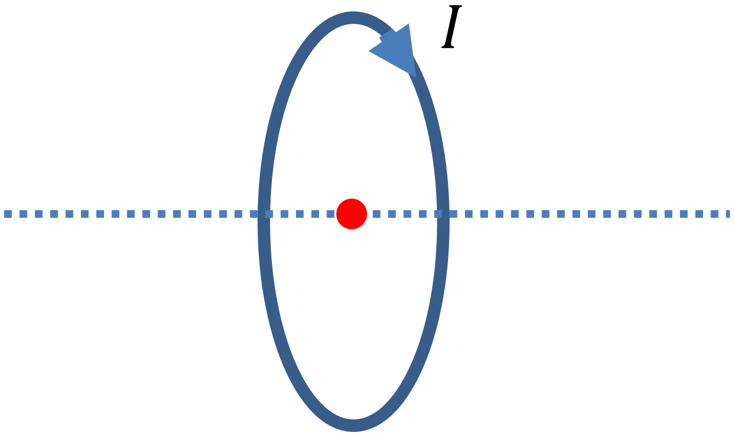
A. Ampere's Law

B. Biot-Savart Law

C. Either

Clicker question

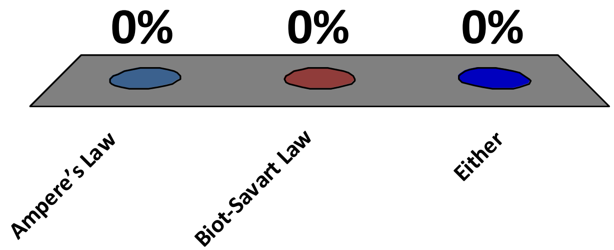
What is the easiest way to compute the magnetic field at the **centre of a circular current loop**?



A. Ampere's Law

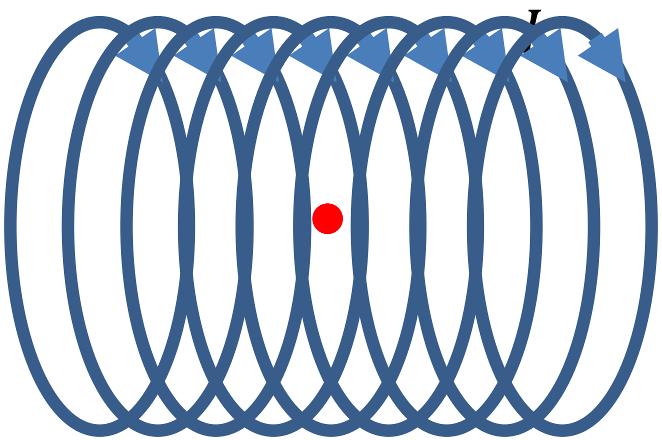
B. Biot-Savart Law

C. Either



Clicker question

What is the easiest way to compute the magnetic field **inside a solenoid**?



A. Ampere's Law

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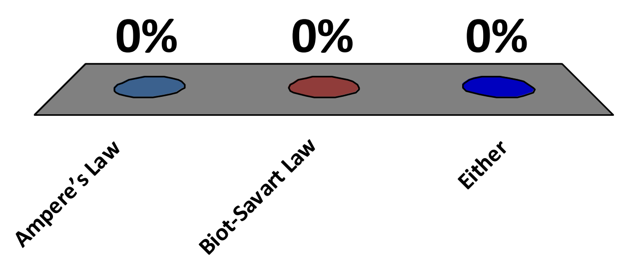

Summary

- The magnetic field around a symmetric current distribution can be conveniently determined using **Ampere's Law** $\oint \vec{B}.d\vec{l} = \mu_0 I_{enc}$ evaluated around a closed loop

- In particular, Ampere's Law allows us to compute the magnetic field inside a coil of wire or **solenoid**