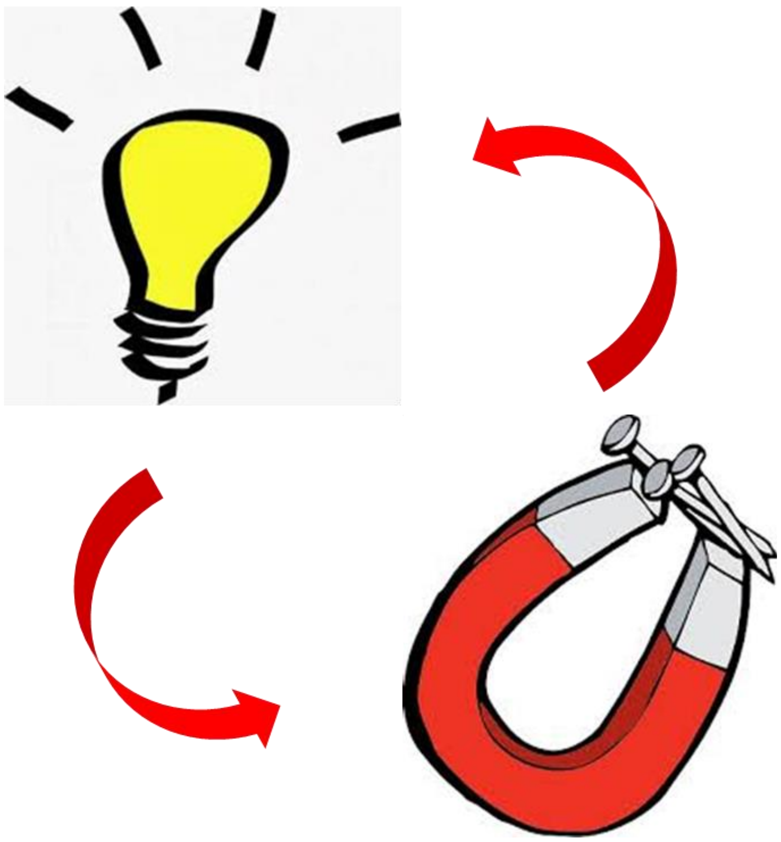
Class 8 : Magnetic Fields and Forces

- Phenomenon of magnetism produced by currents
- Magnetic force between wires
- Computing the magnetic field using the Biot-Savart Law
- Shape of the magnetic field in some simple cases

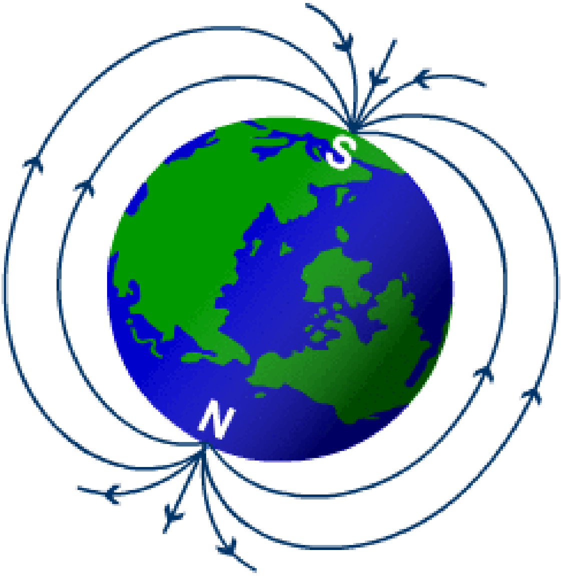
Recap

- We have so far focussed on how **charges give rise to electric fields**, which **exert forces on other charges**
- Moving charge constitutes a **current**, and creates new phenomena not described by electric forces : **magnetism**
- We will now begin to explore *deep connections* between these different phenomena



Magnetic Fields

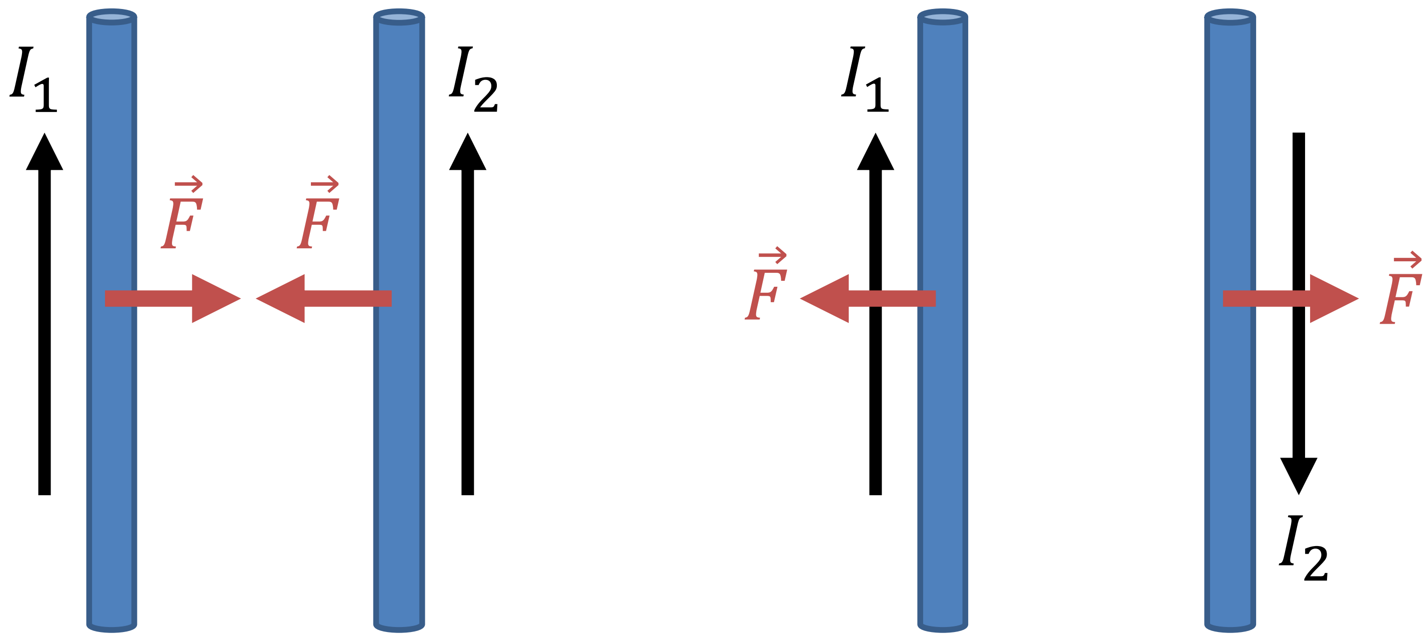
- **Magnetism** makes us think of a “classical magnet” with a north and south pole



- However, more fundamentally it is a phenomenon produced by **electrical currents**

Magnetic Fields

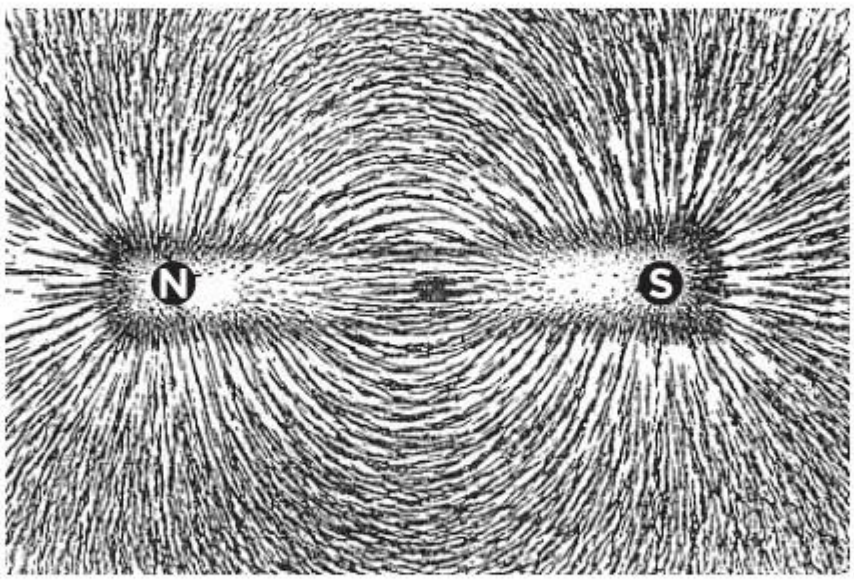
- There are **forces between current-carrying wires** – even though those wires are electrically neutral!

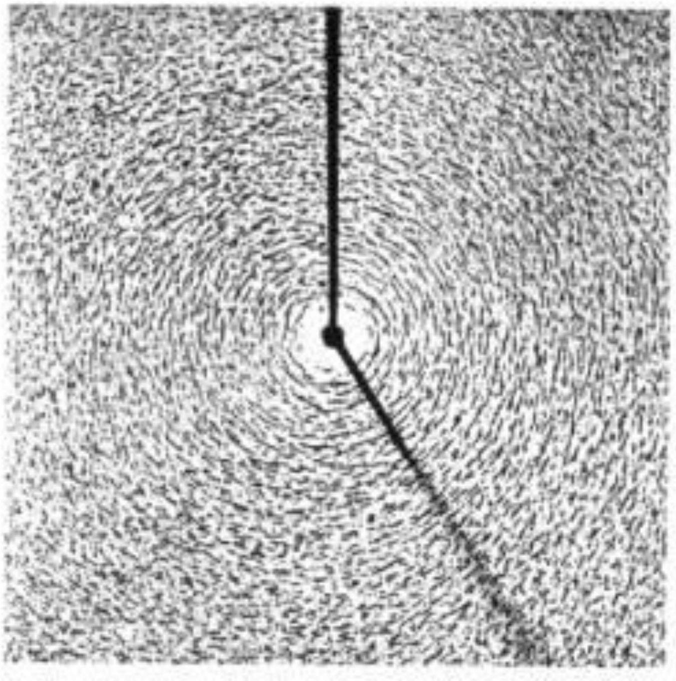


- These are known as **magnetic forces**

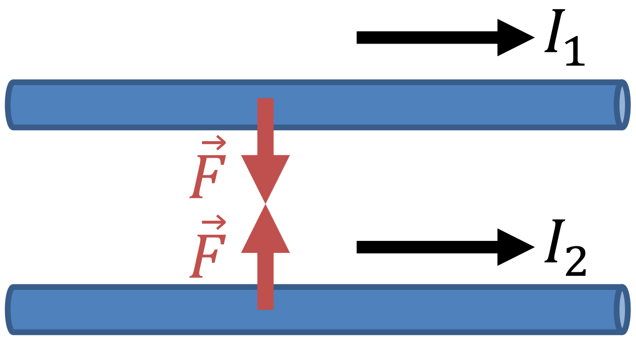
Magnetic Fields

- We say that a current (or magnet) sets up a **magnetic field** in the space around it. This field causes another current (or magnet) to feel a force.





Magnetic Fields



- **Charges** set up an *electric field* $\vec{E}$ in the space around them
- An **electric field** $\vec{E}$ is a region of space in which a *charge* experiences a force

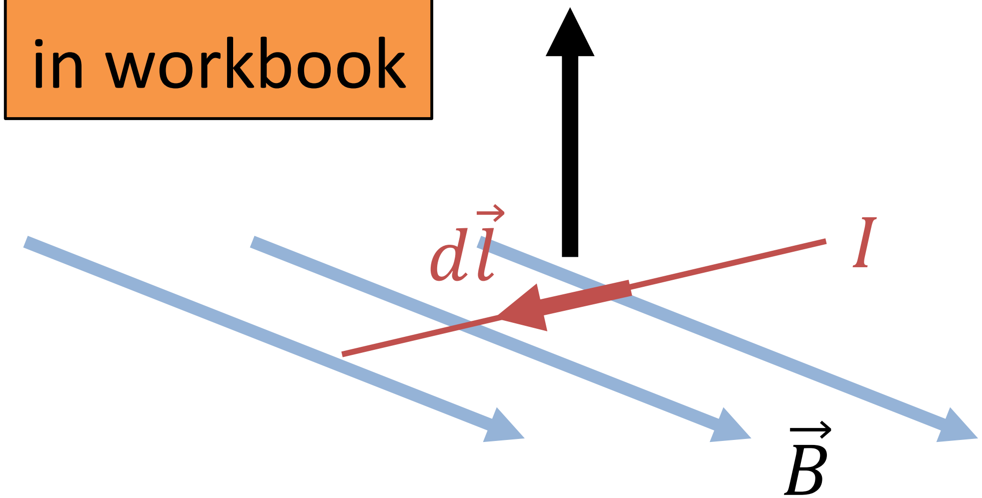
- **Currents** set up a *magnetic field* $\vec{B}$ in the space around them
- A **magnetic field** $\vec{B}$ is a region of space in which a *current* experiences a force

Magnetic Fields

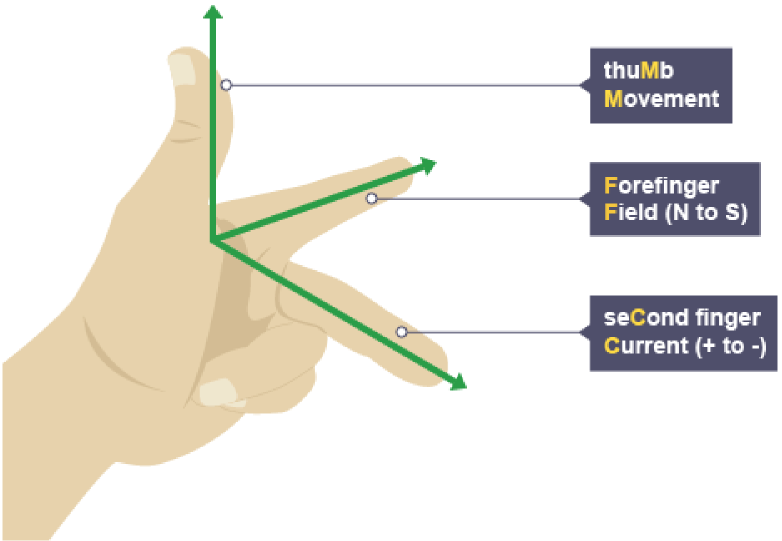
- The force felt by the current is **perpendicular to both the current and the magnetic field**

Please note in workbook

$$\vec{F}_B = I d\vec{l} \times \vec{B}$$



"Fleming's left-hand rule"

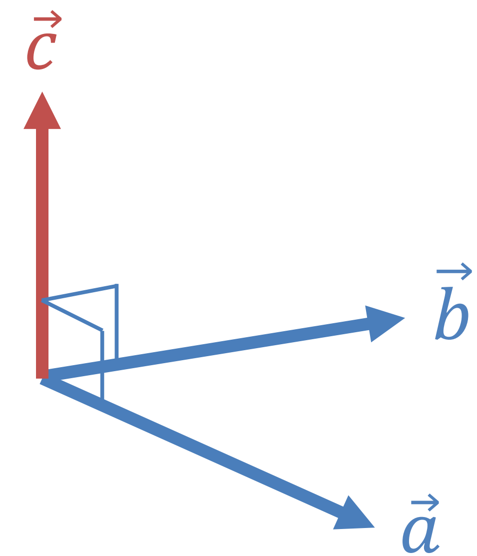


- $I\, d\vec{l}$ is known as a **current element**

Magnetic Fields

- Uses the concept of a **cross product of two vectors**, which is a vector perpendicular to each

- The mathematical definition:



$$\mathbf{c} = \mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} =$$

$$= (a_2 b_3 - a_3 b_2)\mathbf{i} + (a_3 b_1 - a_1 b_3)\mathbf{j} + (a_1 b_2 - a_2 b_1)\mathbf{k}$$

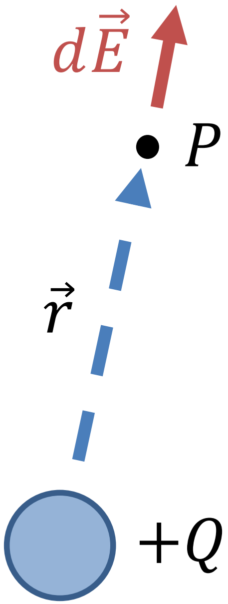
Magnetic Fields

How do we determine the strength of the magnetic field from a given current?

Units: magnetic field strength is measured in ***Tesla*** *(T)*

Biot-Savart Law

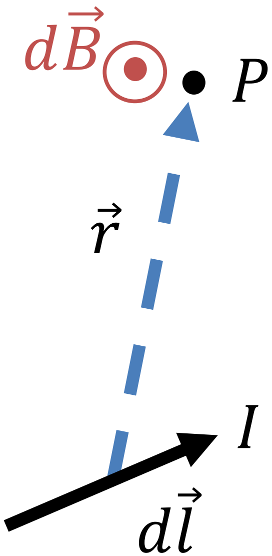
Coulomb's Law for $\vec{E}$



$$d\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{\vec{r}}$$

- $\vec{E}$ is parallel to $\vec{r}$
- Force depends on the *permittivity of free space* ε_0 where $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$
- Principle of superposition applies

Biot-Savart Law for $\vec{B}$



$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\, d\vec{l} \times \hat{\vec{r}}}{r^2}$$

Please note in workbook

- $\vec{B}$ is perpendicular to both $d\vec{l}$ and $\vec{r}$
- Force depends on the *permeability of free space* $\mu_0 = 4\pi \times 10^{-7}$
- Principle of superposition applies

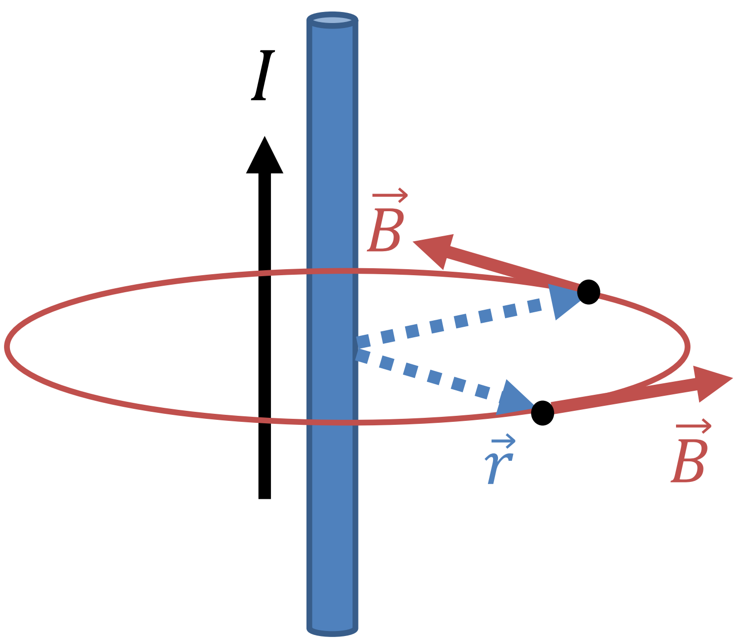
Biot-Savart Law

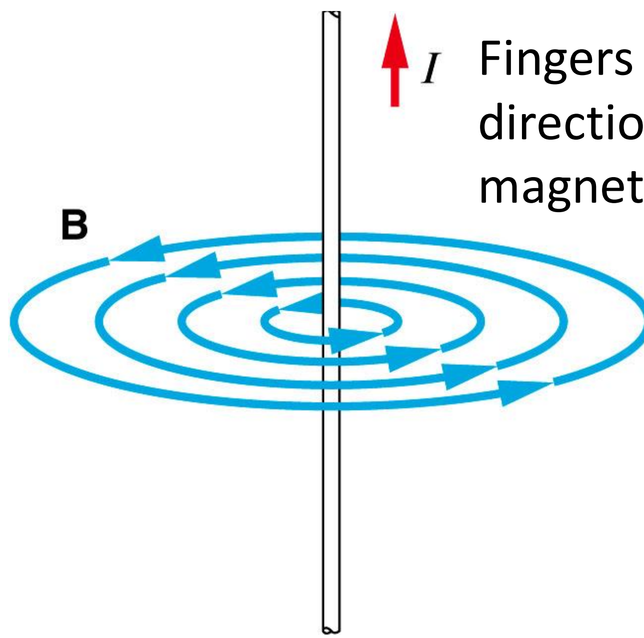
- What is the $\vec{B}$-field around a wire? $d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\hat{\vec{r}}}{r^2}$

What's the direction of the field?

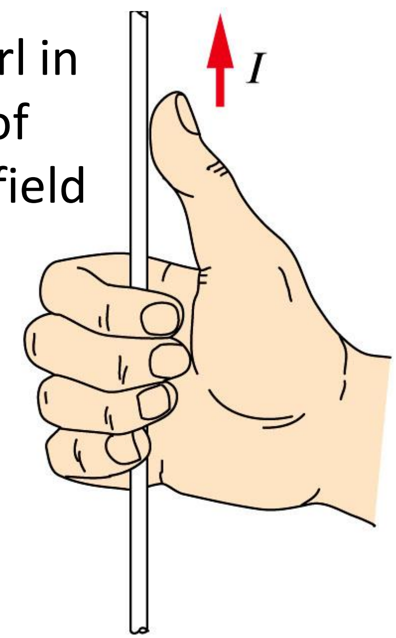
Direction can be recalled using the right-hand rule …

Thumb points in direction of current flow





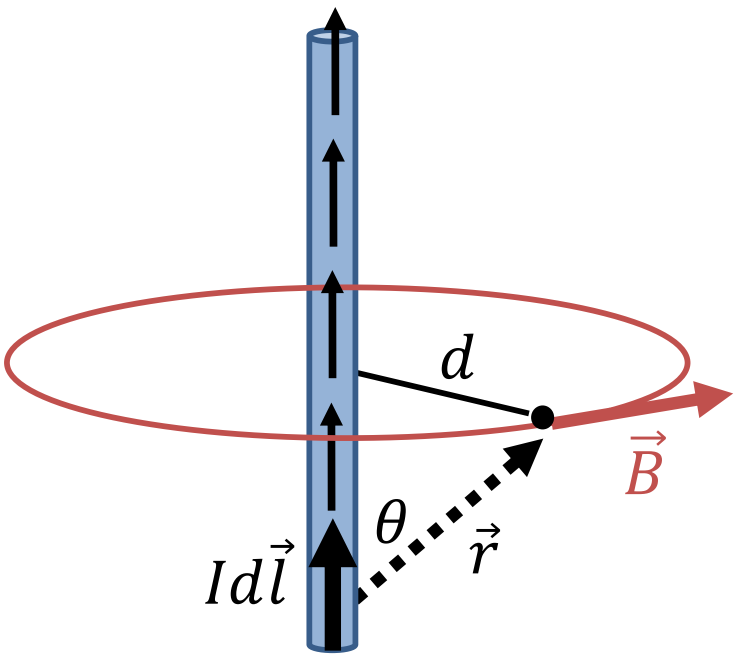
Fingers curl in direction of magnetic field



Biot-Savart Law

- What is the $\vec{B}$-field around a wire?

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l} \times \hat{\vec{r}}}{r^2}$$

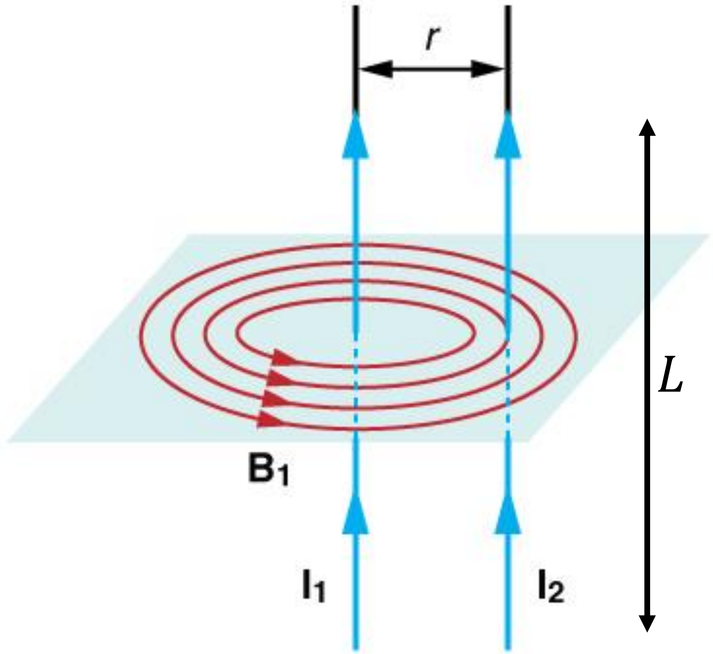


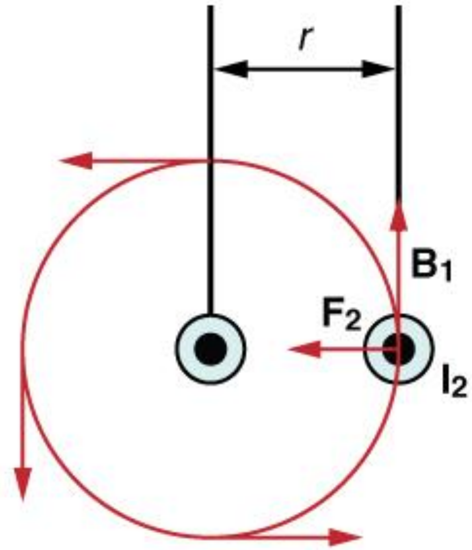
- We apply the Biot-Savart Law by splitting the wire into chunks $d\vec{l}$
- $d\vec{B}$ from each chunk is parallel, so we can sum up the magnitudes $\int dB$
- Biot-Savart : $dB = \frac{\mu_0}{4\pi}\frac{Idl \sin\theta}{r^2}$ from this chunk, where $d = r\sin\theta$ and $l = r\cos\theta$
- Substituting in for r and l and integrating over θ from $0°$ to $180°$, we find

$$B = \frac{\mu_0 I}{2\pi d}$$

Force between currents

- Force between 2 current-carrying wires:



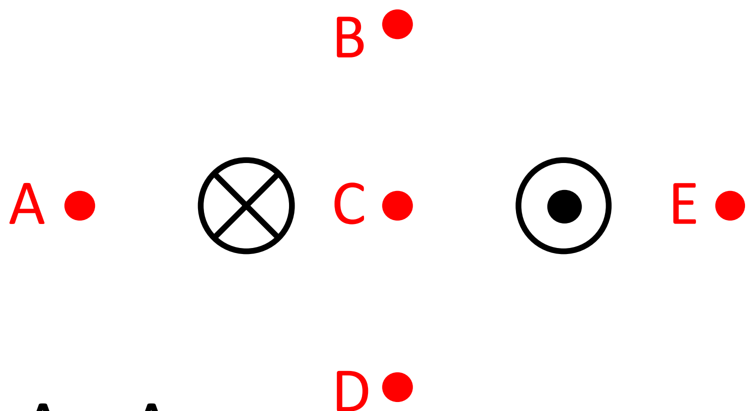


Magnetic field set up by wire 1 is $B_1 = \frac{\mu_0 I_1}{2\pi r}$

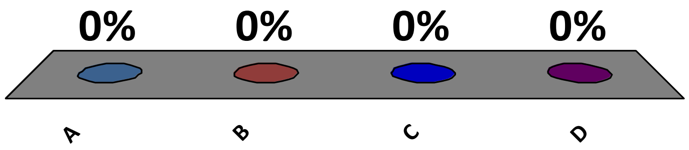
Force felt by wire 2 is $F = B_1 I_2 L = \frac{\mu_0 I_1 I_2 L}{2\pi r}$

Clicker question

A magnetic field is created by two parallel currents flowing in opposite directions. At what point is the field strongest?

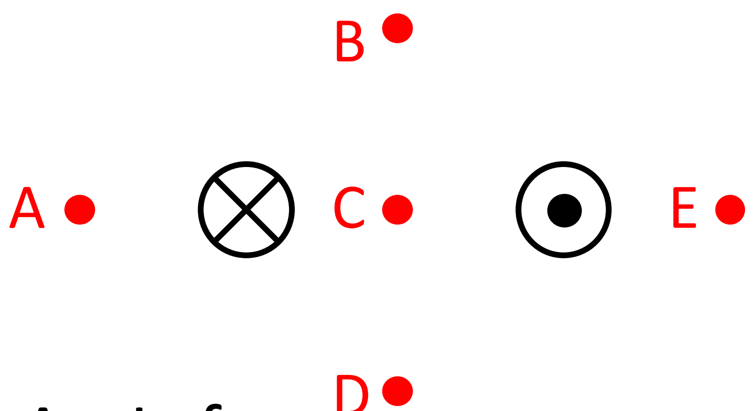


A. A
B. B
C. C
D. D

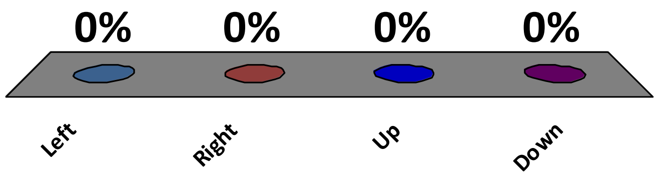


Clicker question

What is the direction of the net magnetic field at B?

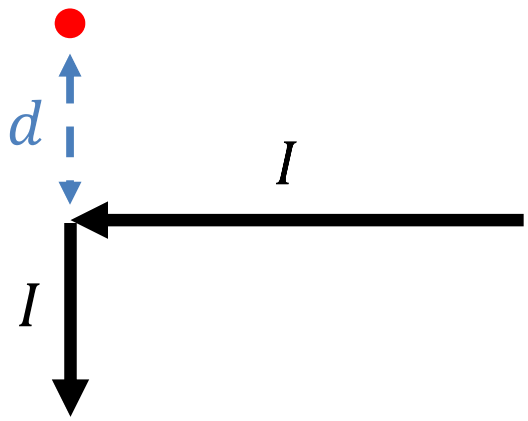


A. Left
B. Right
C. Up
D. Down

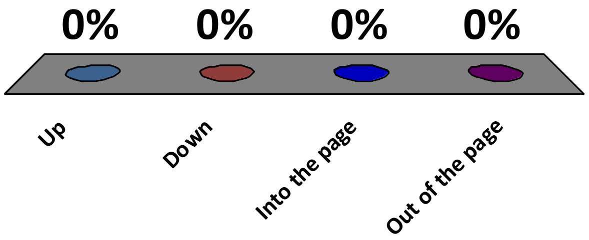


Clicker question

What is the magnetic field direction at the point shown?

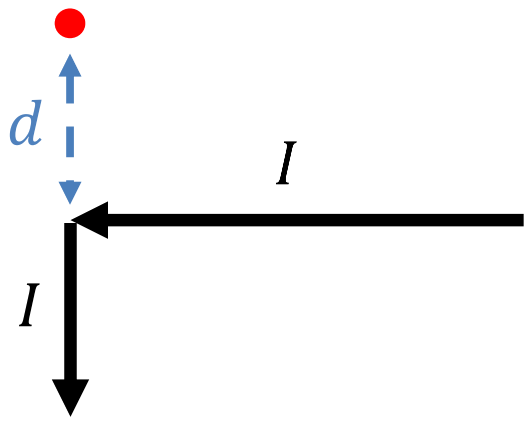


A. Up
B. Down
C. Into the page
D. Out of the page

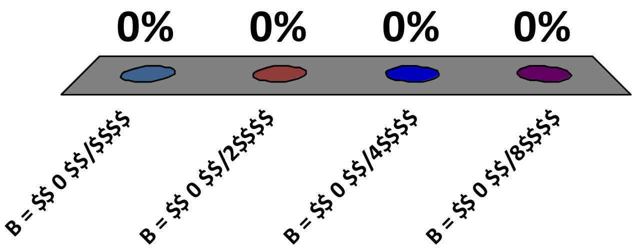


Clicker question

What is the magnetic field strength B at the point shown?

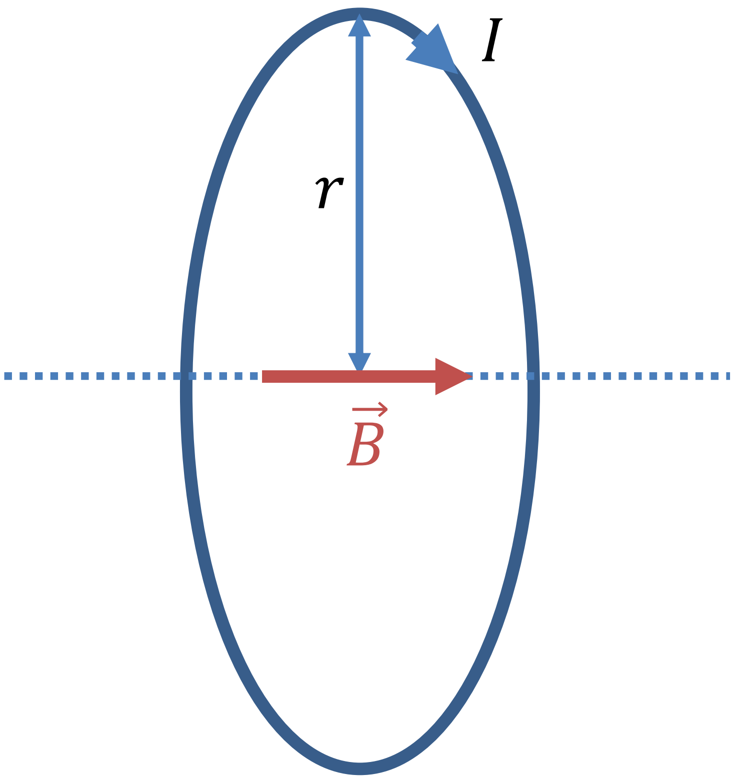


A. $B = \mu_0 I/\pi d$
B. $B = \mu_0 I/2\pi d$
C. $B = \mu_0 I/4\pi d$
D. $B = \mu_0 I/8\pi d$



Determining magnetic fields

- What is the $\vec{B}$ field at the centre of a current loop?



- Biot-Savart Law : $d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{\vec{r}}}{r^2}$
- The contribution $d\vec{B}$ from all current elements reinforce
- Hence $B = \int dB = \frac{\mu_0}{4\pi} \frac{I}{r^2} \int dl$ where $\int dl = 2\pi r$
- We find: $B = \frac{\mu_o I}{2r}$

Summary

- An electric current I (or magnet) sets up a **magnetic field** $\vec{B}$ around it; this causes another current (or magnet) to feel a force

- A current element $Id\vec{l}$ in a magnetic field $\vec{B}$ feels a **force** $\boldsymbol{\vec{F}_B = I\,d\vec{l} \times \vec{B}}$

- The magnetic field $d\vec{B}$ due to a current element $Id\vec{l}$ is given by the **Biot-Savart Law** $d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\hat{\vec{r}}}{r^2}$