Class 7 : Electric Current

- Microscopic description of current
- Current density
- Expression of charge conservation
- Ohm’s Law, resistance and power

Recap

- So far we have considered **stationary charges** with density ρ, which produce an $\vec{E}$-field described by $\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0}$ and $\vec{\nabla} \times \vec{E} = \vec{0}$

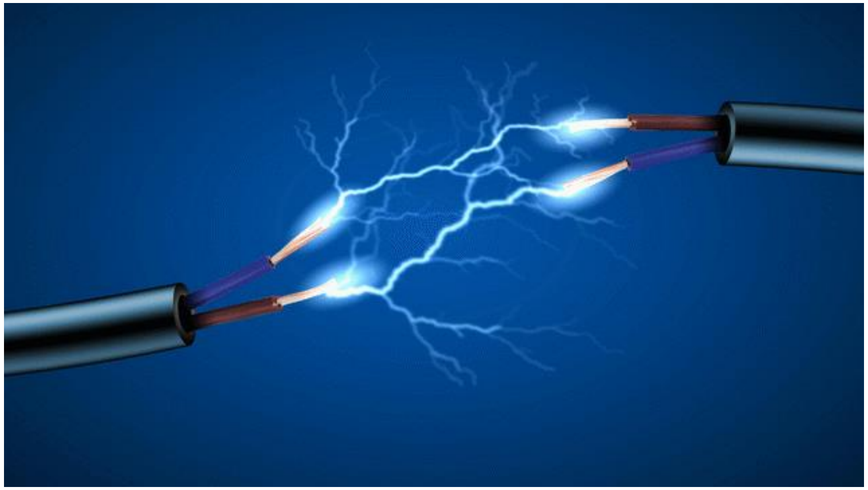
- This $\vec{E}$-field may be described as the gradient of an **electrostatic potential** V, where $\vec{E} = -\vec{\nabla}V$

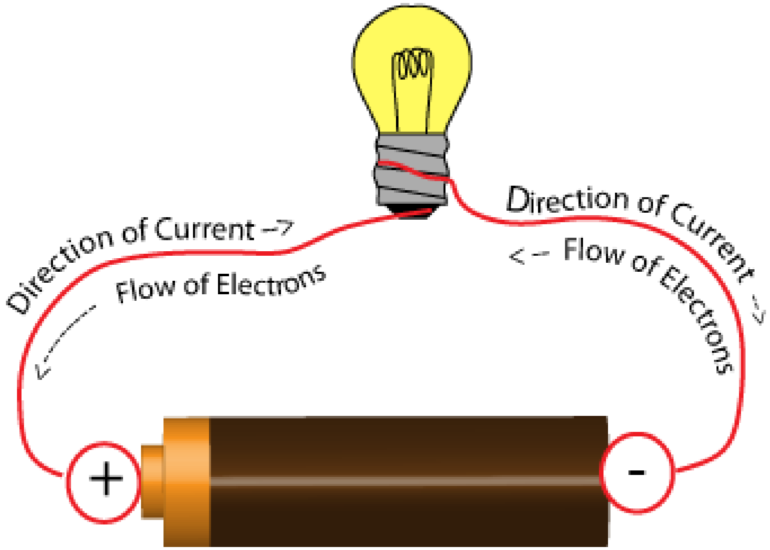
- The **potential difference** is the work done in moving a unit charge between 2 points, $\Delta V = -\int \vec{E}.d\vec{l}$

WE WERE GOING TO USE THE TIME MACHINE TO PREVENT THE ROBOT APOCALYPSE, BUT THE GUY WHO BUILT IT WAS AN ELECTRICAL ENGINEER.

Current

- **Electric current** is the motion of charge from one place to another (driven by a potential difference, for example)

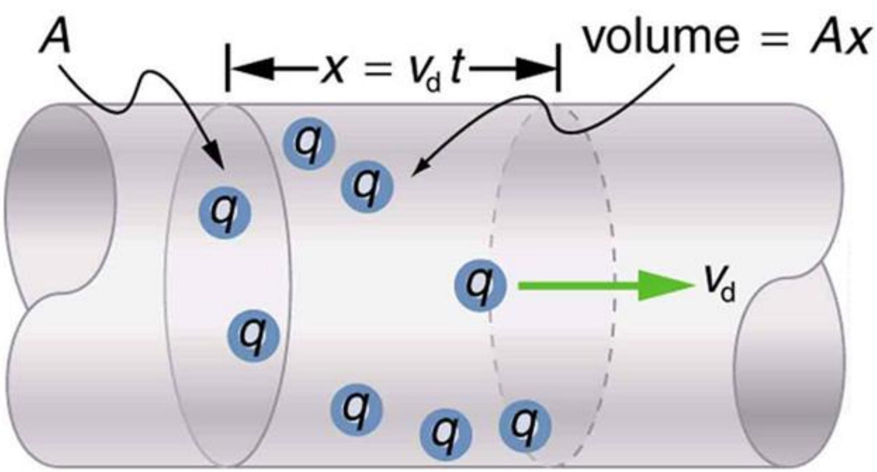




- Current I is the charge flowing per unit time, $I = \frac{dq}{dt}$, and is measured in *Amperes* where 1 Amp = 1 C/s

Current

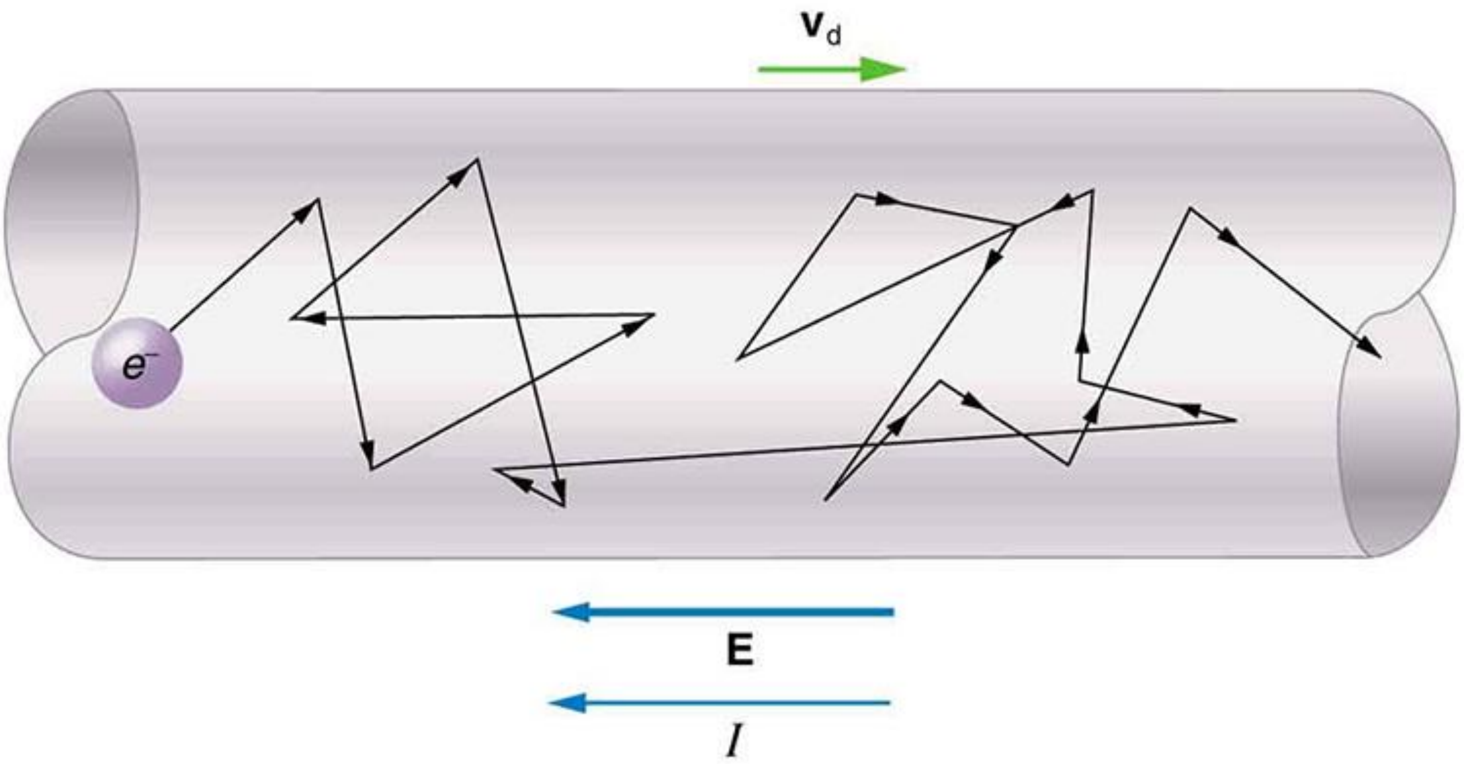
- On a *microscopic scale*, current is created by the motion of individual particles of charge (e.g., electrons)



- Consider charges q, with number density n, drifting along the wire with speed v_d
- In time t, the charges passing a vertical plane are contained within a volume $Av_d t$
- The total charge is then $Q = n \times Av_d t \times q$, corresponding to **current** $\boldsymbol{I = \frac{Q}{t} = nAqv_d}$

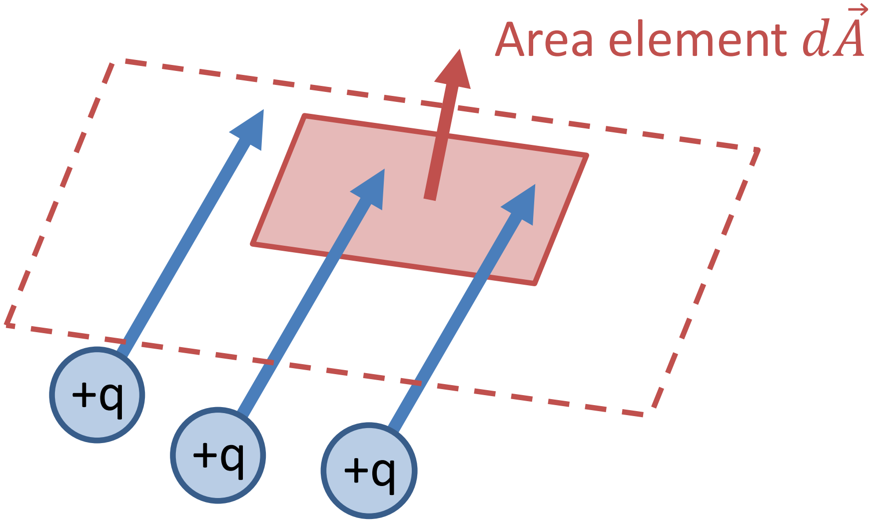
Current

- The **drift velocity** v_d results from a balance between the applied electric field and the inter-atomic collisions



Current

- In general, currents are **extended in space** rather than flowing along a wire. The **current density** $\vec{J}$ describes the general distribution and direction of flowing charge



- Current flowing through area element is $dI = \vec{J}.d\vec{A}$

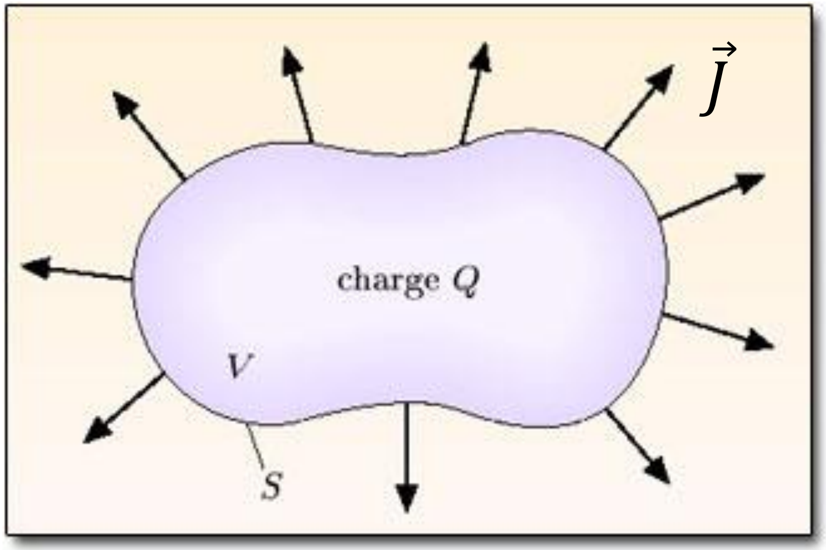
- Total current flowing through surface S is $I = \int \vec{J}.d\vec{A}$

Please note in workbook

- $\vec{J}$ is the *current per unit area normal to the flow* and is related to the microscopic charges as $\vec{J} = nq\vec{v} = \rho\vec{v}$

Current

- **Charge conservation** implies that the rate of charge leaving a volume V is equal to the current flowing across the surface S

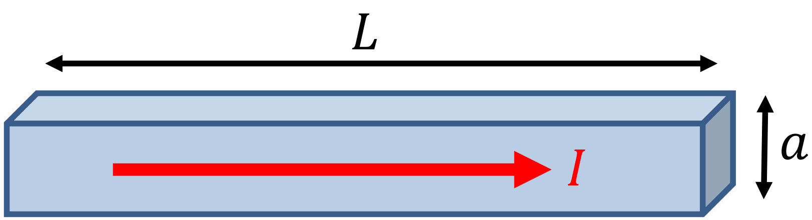


- Mathematically, $\frac{\partial Q}{\partial t} = -\int \vec{J}.d\vec{A}$
- But charge $Q = \int \rho\, dV$, where $\rho(\vec{x})$ is the charge density
- Also by the divergence theorem: $\int \vec{J}.d\vec{A} = \int (\vec{\nabla}.\vec{J})\, dV$
- Hence: $\frac{\partial}{\partial t}\left(\int \rho\, dV\right) = -\int (\vec{\nabla}.\vec{J})\, dV$

- We find the **charge continuity equation :** $\boldsymbol{\vec{\nabla}.\vec{J} + \frac{\partial \rho}{\partial t} = 0}$

Clicker question

Current I uniformly flows down a wire of length L and square cross-section of side a. What is the current density J?

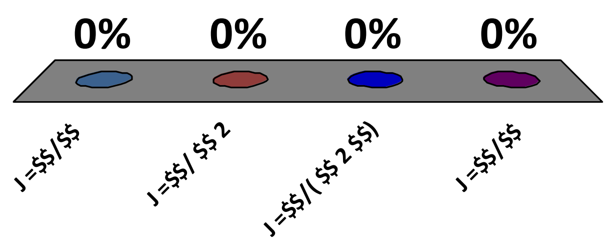


A. $J = I/a$

B. $J = I/a^2$

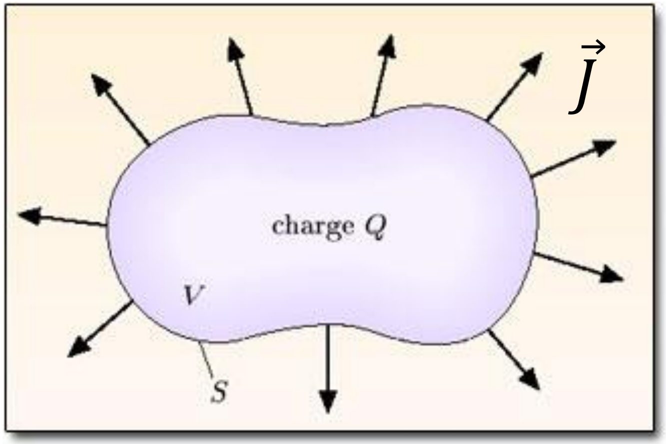
C. $J = I/(a^2 L)$

D. $J = I/L$



Clicker question

Which of the following is a correct statement of charge conservation?

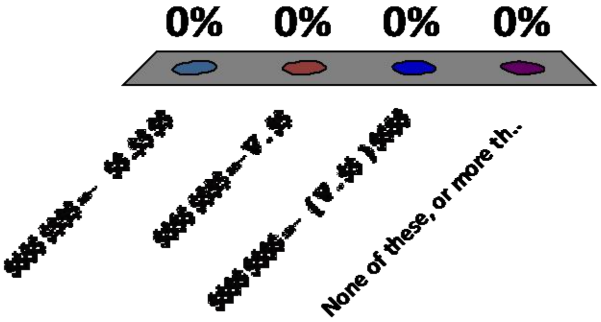


A. $\frac{dQ}{dt} = -\int \vec{J}.d\vec{A}$

B. $\frac{dQ}{dt} = -\vec{\nabla}.\vec{J}$

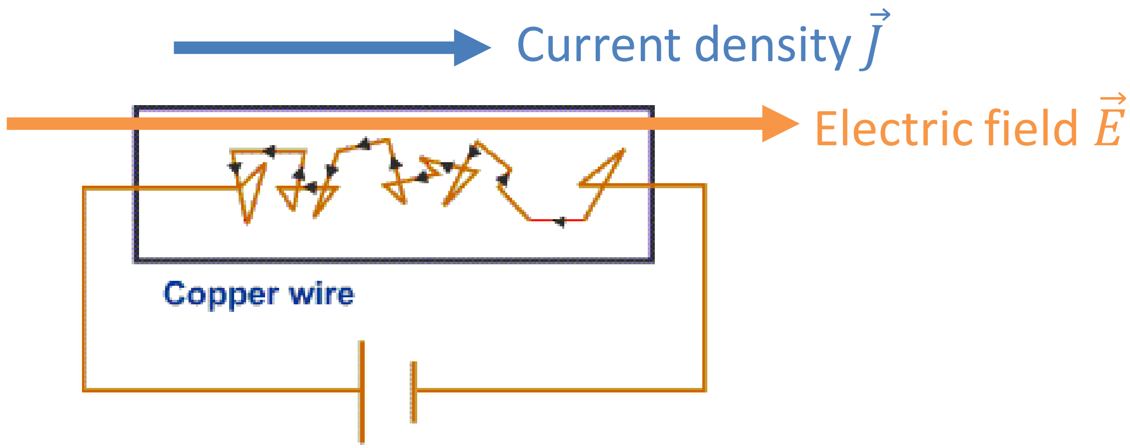
C. $\frac{dQ}{dt} = -\int (\vec{\nabla}.\vec{J})\, dV$

D. None of these, or more than one



Ohm’s Law

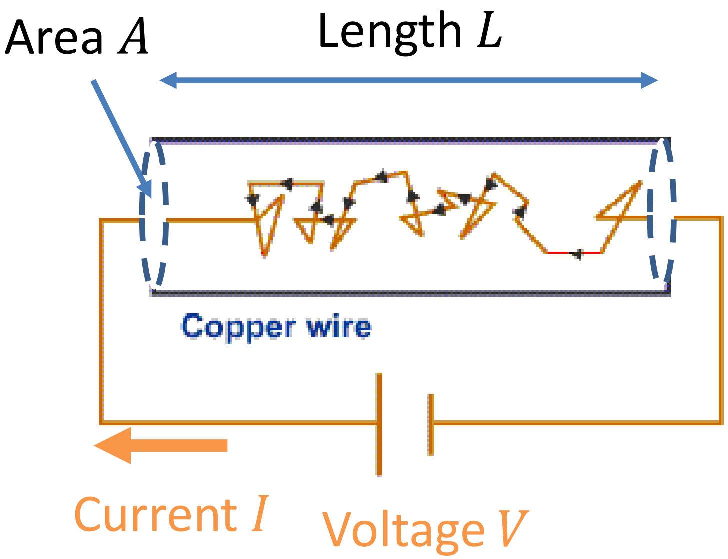
- Conductors exhibit a **resistance** to the flow of charge, due to inter-atomic collisions



- Experimentally we find **Ohm’s Law** : the current density is proportional to the applied electric field, $\vec{\boldsymbol{J}} = \boldsymbol{\sigma}\vec{\boldsymbol{E}}$, where σ is known as the **conductivity** of the material

Ohm's Law

- Ohm's Law can be written in a more familiar circuit form: **the current flowing I is proportional to the applied voltage V**



- Electric field is the potential gradient : $E = \frac{V}{L}$
- Current density is $J = \frac{I}{A}$
- Ohm's Law: $J = \sigma E$
- Hence $I = \left(\frac{\sigma A}{L}\right) V$

- This is often written $V = I\ R$, in terms of the **resistance** R

Ohm's Law

- The resistance to current flow results in a **dissipation of heat energy** in the conductor

- The energy transferred due to the transport of charge dQ across potential difference V is $dW = V\,dQ$

- Hence the power dissipated is $\frac{dW}{dt} = V\frac{dQ}{dt} = V\,I$

- **Joule's Law** : the power dissipated in a circuit is $VI = I^2R$

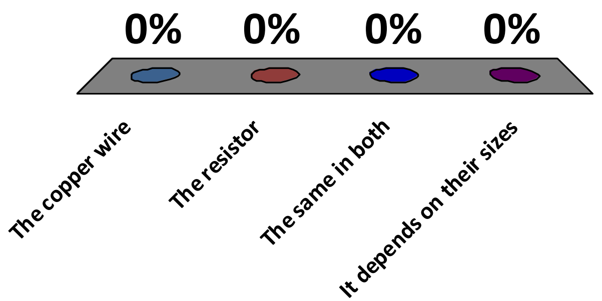
Ohm's Law

- We can derive a *more general form* of the power dissipated when current density $\vec{J}$ is flowing in an electric field $\vec{E}$
- The current density is a flow of n charges q per unit volume with velocity $\vec{v}$, where $\vec{J} = nq\vec{v}$
- **The power dissipated by a force is $\vec{F}.\vec{v}$, hence in unit volume this is $nq\vec{E}.\vec{v} = \vec{J}.\vec{E}$**

Clicker question

A current flows along a copper wire (high conductivity) into a resistor (low conductivity). In which material is the $\vec{E}$-field largest?

A. The copper wire
B. The resistor
C. The same in both
D. It depends on their sizes



Clicker question

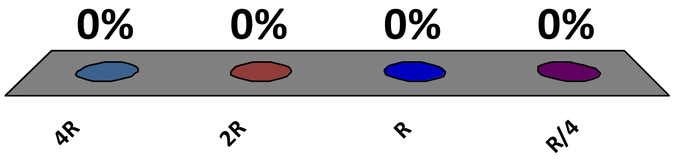
A resistor of resistance *R* has its length and cross-sectional area both doubled. Its resistance will become:

A. 4*R*

B. 2*R*

C. *R*

D. *R*/4



Summary

- **Current** is moving charge; either along a wire ($I = \frac{dq}{dt}$) or across space with **current density** $\vec{J}$ (where $I = \int \vec{J}.d\vec{A}$)
- The fundamental principle of **charge conservation** implies a **continuity equation** $\vec{\nabla}.\vec{J} + \frac{\partial \rho}{\partial t} = 0$ for charge density ρ
- **Ohm's Law** is an experimental observation that $\vec{J} \propto \vec{E}$ (or in circuit form, $I = V/R$. The power dissipated when current flows is given by **Joule's Law**, $\vec{J}.\vec{E}$, or in circuit form, VI