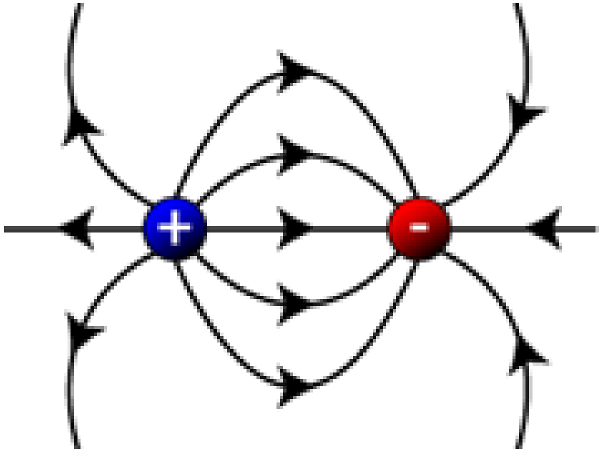
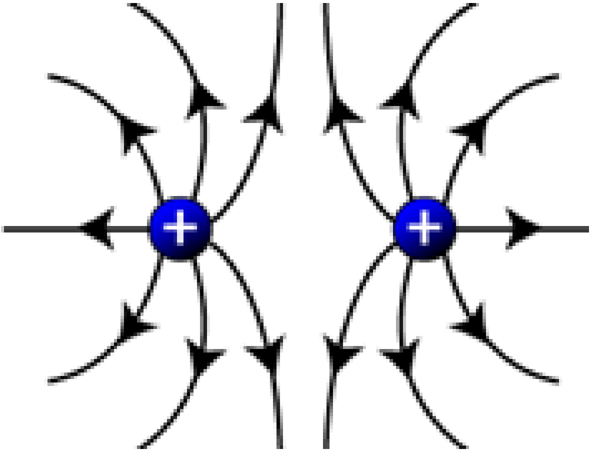
Class 6 : Insulating Materials

- What is an insulator?
- Electric dipoles
- Polarization of an insulator, and how it modifies electric field
- Electric displacement
- Boundary conditions for $\vec{E}$

Recap (1)

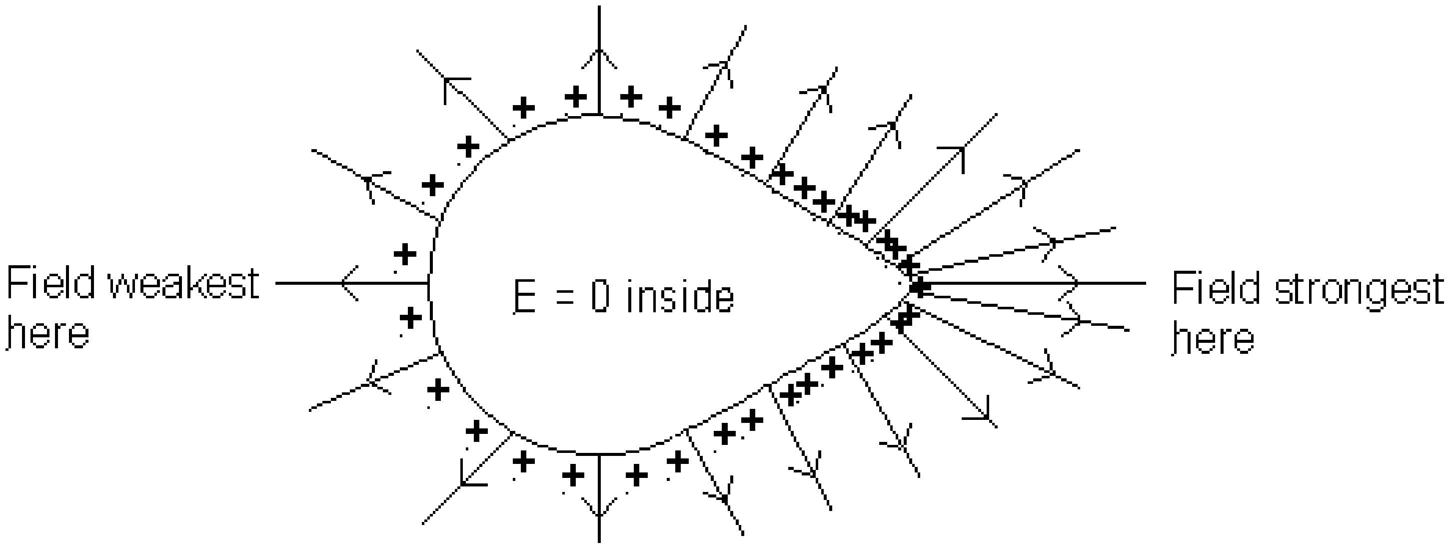
- **Maxwell's 1st equation** describes how the electric field $\vec{E}$ is related to the free charge density ρ_f, via $\vec{\nabla}.\vec{E} = \frac{\rho_f}{\varepsilon_0}$
- This is equivalent to **Gauss's Law**, $\int \vec{E}.d\vec{A} = Q_{enclosed}/\varepsilon_0$
- $\vec{E}$-field lines *start on +ve charges, end on –ve charges*, and *do not circulate* (**Maxwell's 2nd equation** $\vec{\nabla} \times \vec{E} = \vec{0}$)





Recap (2)

- **Conductors** are materials in which electric charges can flow freely. Inside a conductor $\vec{E} = \vec{0}$, and outside the field lines are perpendicular to the surface.

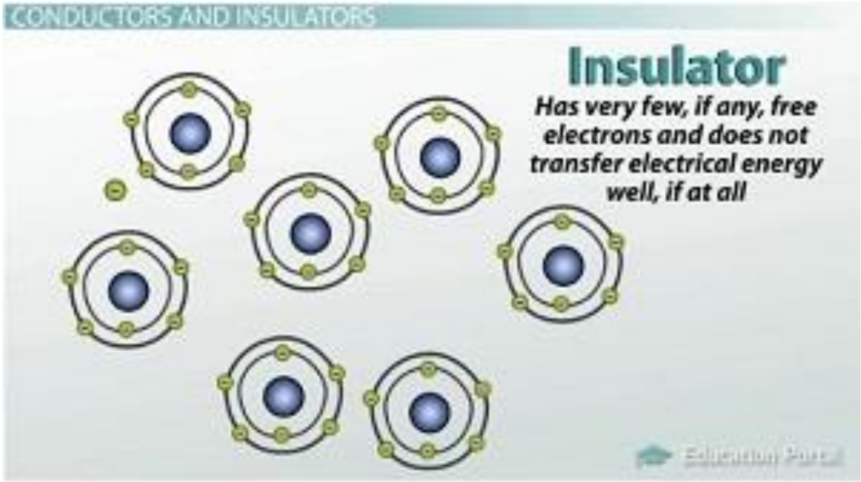


Insulators

- **Insulators** or **dielectrics** are materials in which *charge cannot flow freely*

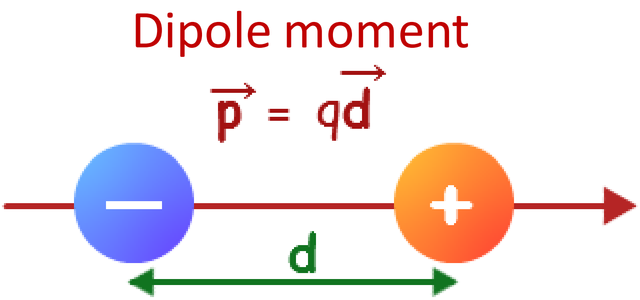
Insulators

- Atomic theory attributes this to whether or not **free electrons** are available to move within the atomic lattice to conduct electricity (or heat)

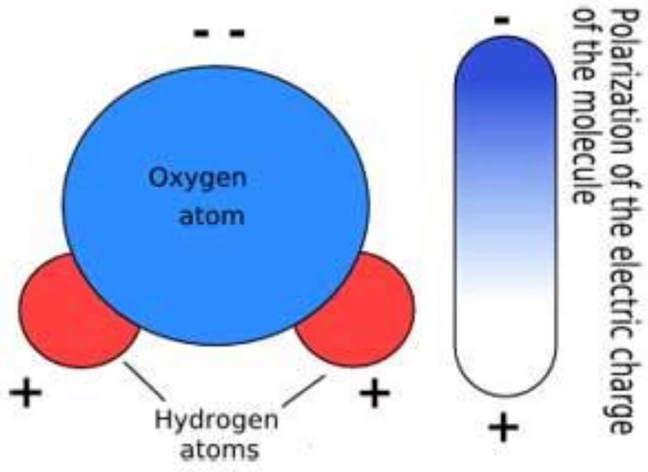


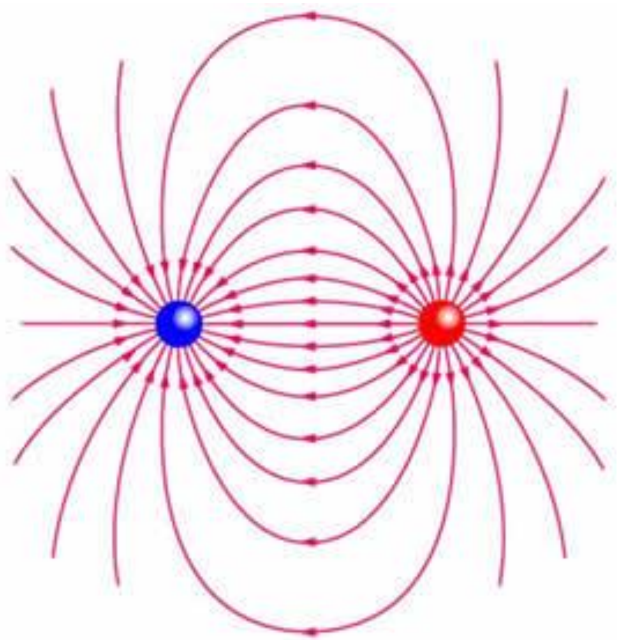
Electric dipoles

- An **electric dipole** consists of two charges $+q$ and $-q$ separated by distance d. It is neutral but produces an $\vec{E}$-field



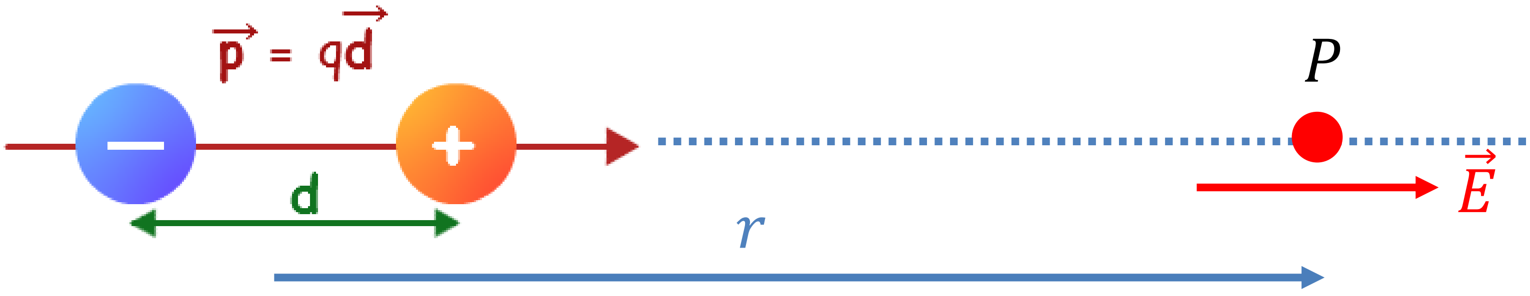
- A dipole is a good model for many molecules!





Electric dipoles

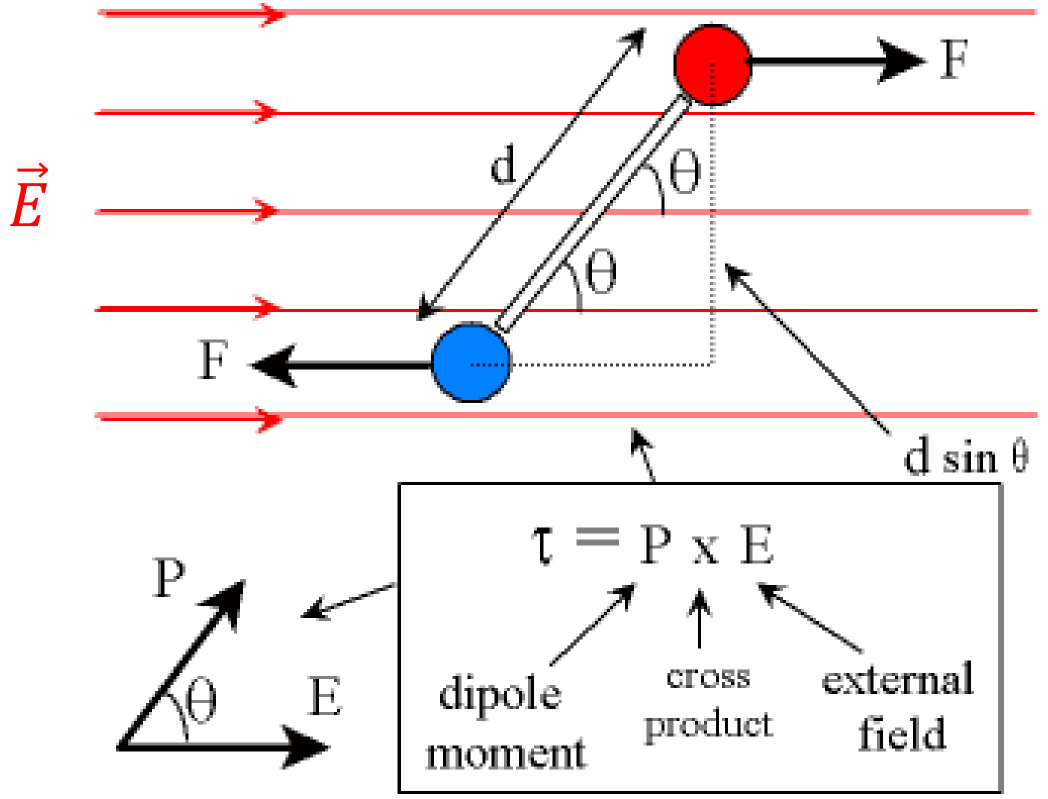
What is the electric field strength along the axis?



- Electric potential: $V_P = \frac{q}{4\pi\varepsilon_0(r-\frac{d}{2})} + \frac{-q}{4\pi\varepsilon_0(r+\frac{d}{2})} \approx \frac{p}{4\pi\varepsilon_0 r^2}$
- Electric field $E_x = -\frac{dV}{dr} = \frac{2p}{4\pi\varepsilon_0 r^3}$
- $E \propto \frac{1}{r^2}$ for a point charge, and $E \propto \frac{1}{r^3}$ for a dipole

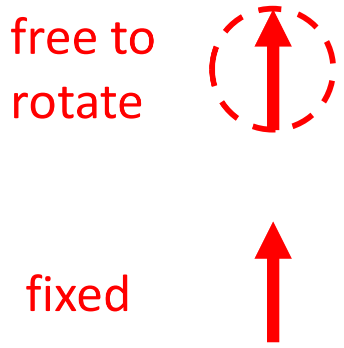
Electric dipoles

- When a dipole $\vec{p}$ is placed in an electric field $\vec{E}$, it feels no net force (since it is neutral) but it feels **a net torque** $\vec{\tau} = \vec{p} \times \vec{E}$

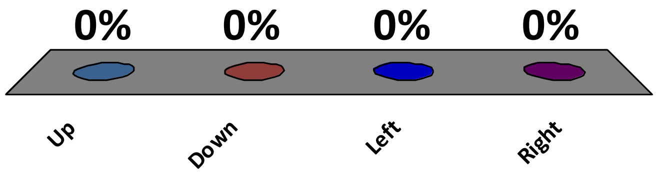


Clicker question

A dipole, free to rotate, is placed **above** a fixed dipole. In which direction does it point?



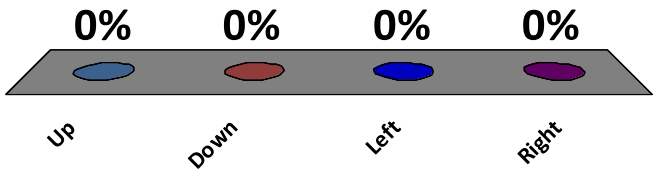
A. Up
B. Down
C. Left
D. Right



Clicker question

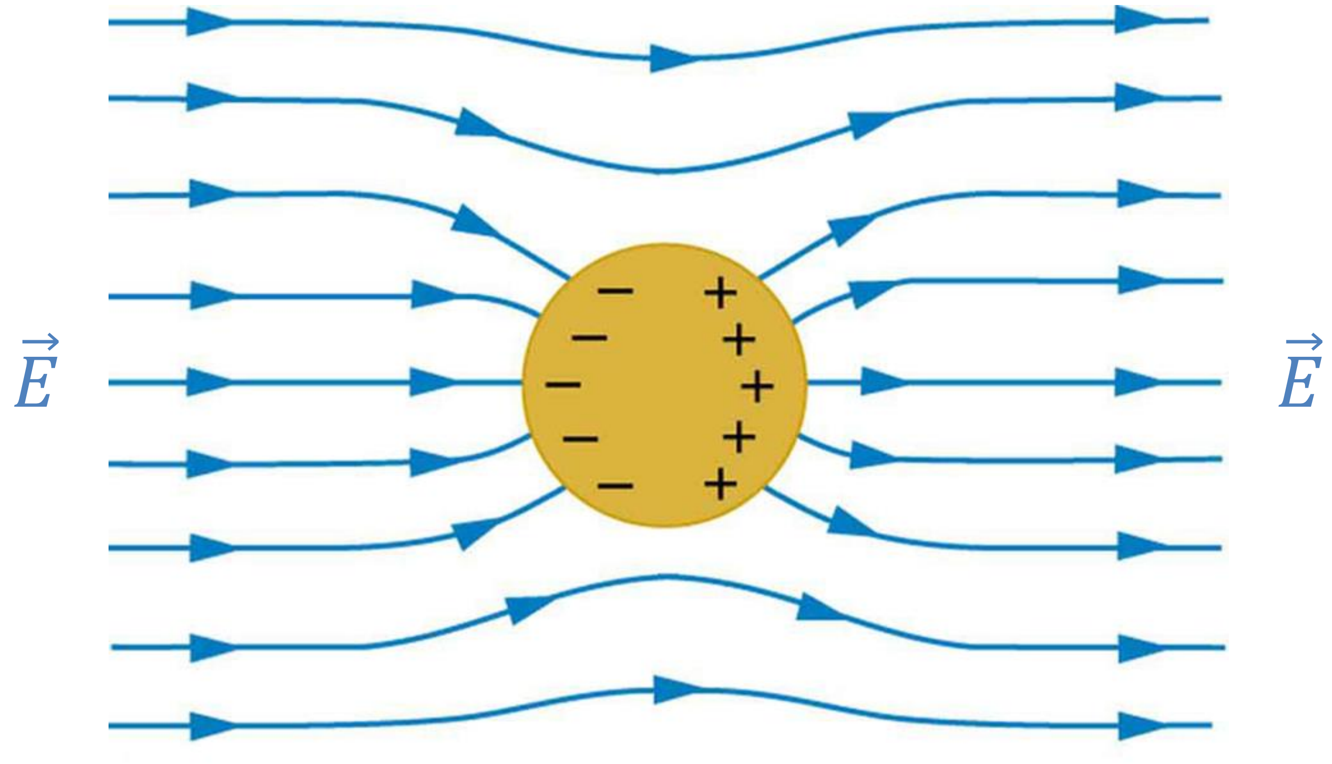
A dipole, free to rotate, is placed **beside** a fixed dipole. In which direction does it point?

A. Up
B. Down
C. Left
D. Right



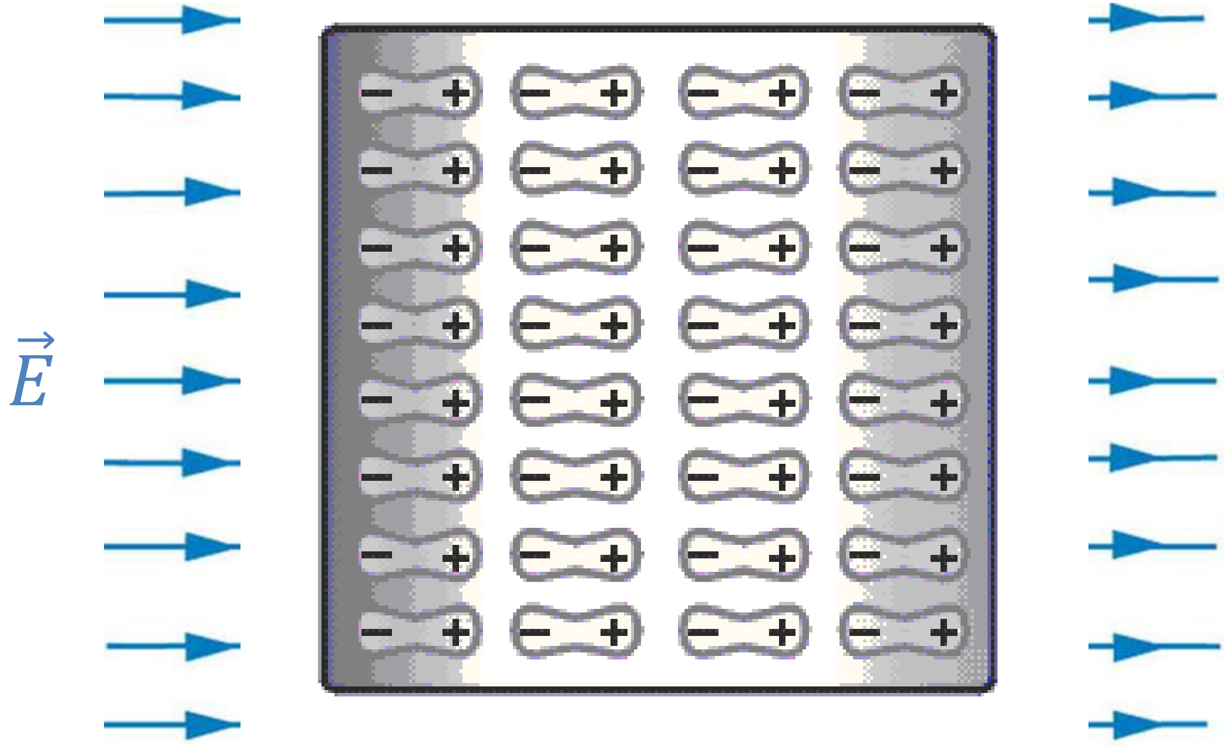
Polarization

- When a **conductor** is placed in an applied electric field, the *free charge moves to the surface to cancel the electric field inside the conductor*



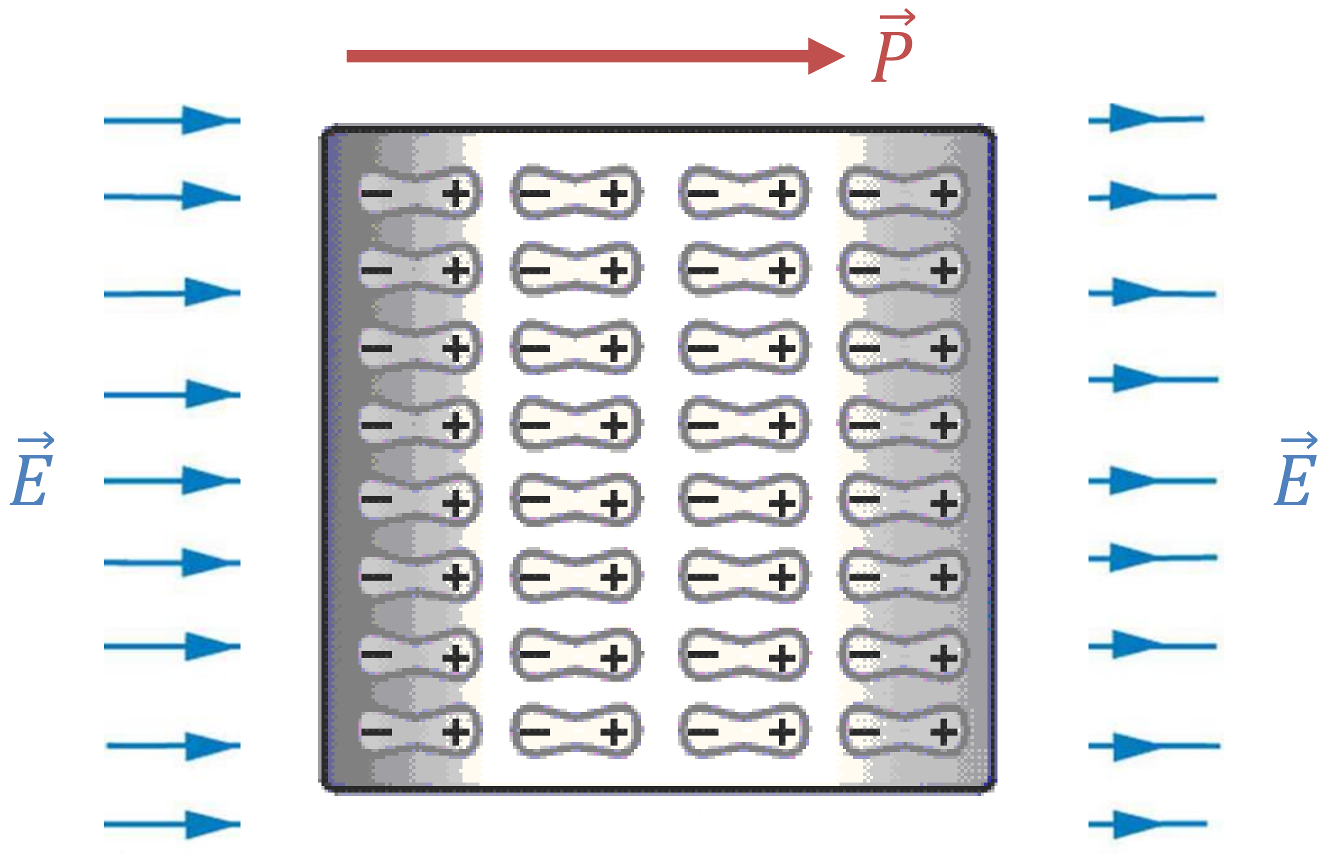
Polarization

- When an **insulator** is placed in an electric field, *the electron cloud and nucleus around each atom minutely displace in opposite directions,* creating **many electric dipoles**



Polarization

- This effect is called **electric polarization** and the polarization vector $\vec{P}$ is the *induced dipole moment per unit volume*



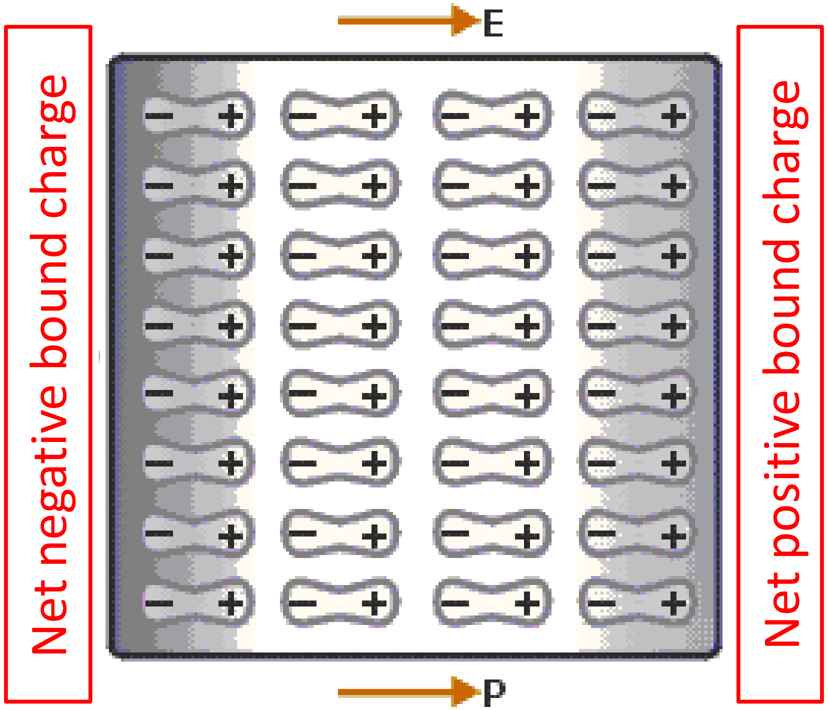
Polarization

- These dipoles **create an electric field in the opposite direction to the applied field**, which reduces the total electric field inside the insulator

- The polarization $\vec{P}$ is equivalent to a “bound charge” appearing within the insulator with charge density $-\vec{\nabla}.\vec{P}$

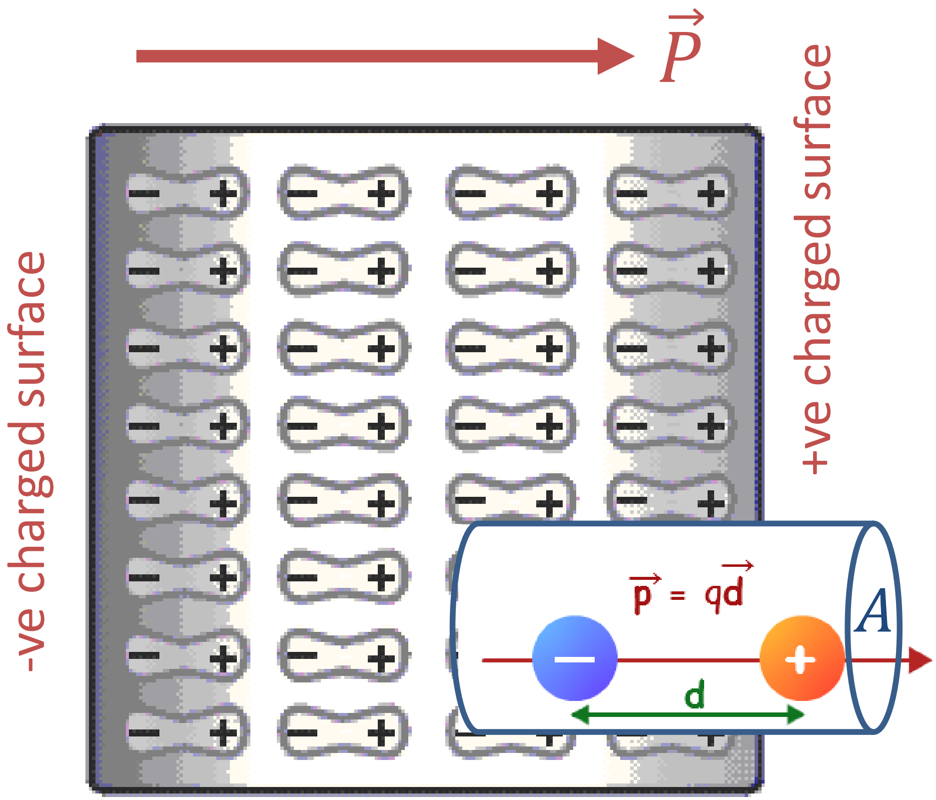
- (The next two slides are optional and explain this for students who would like to follow up)

Please note in workbook



Polarization

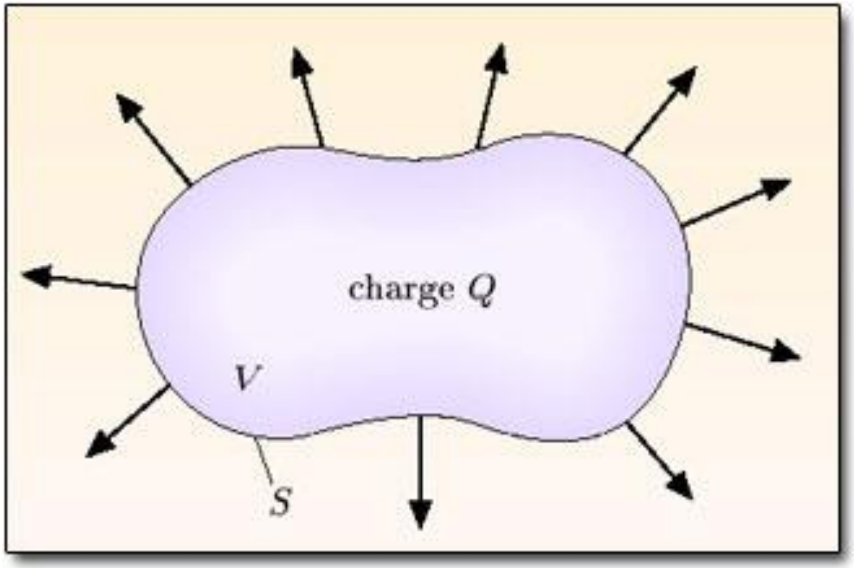
- The displacement of positive and negative charges creates a **net charge on the surfaces** known as a "bound charge"



- How much charge crosses an area A normal to the surface?
- If the dipoles are charges $\pm q$ separated by distance d, with number density n, the net charge crossing area A is $n \times q \times d \times A = P \times A$ (using $P = np = nqd$)
- Generalizing, **the charge crossing an area element $d\vec{A}$ is $\vec{P}.d\vec{A}$**

Polarization

- We can use the fact that the net charge crossing area $d\vec{A}$ of a polarized medium is $\vec{P}.d\vec{A}$ to derive a general formula



- The ***charge leaving*** volume V is equal to an integral over the surface S, $\int \vec{P}.d\vec{A}$
- By the divergence theorem, this can also be written $\int \vec{\nabla}.\vec{P}\,dV$
- Hence, the ***charge density in the volume reduces by*** $\boldsymbol{\vec{\nabla}.\vec{P}}$

- We must modify Maxwell's Equation to $\boldsymbol{\vec{\nabla}.\vec{E} = \frac{1}{\varepsilon_0}\left(\rho_f - \vec{\nabla}.\vec{P}\right)}$ where we include both free charge ρ_f and "bound charge"

Electric displacement

- In an insulator with polarization $\vec{P}$, Maxwell's 1st equation including the bound charge is $\vec{\boldsymbol{\nabla}}.\vec{\boldsymbol{E}} = \frac{\mathbf{1}}{\boldsymbol{\varepsilon_0}}\left(\boldsymbol{\rho_f} - \vec{\boldsymbol{\nabla}}.\vec{\boldsymbol{P}}\right)$

- We can re-arrange this in the form $\vec{\nabla}.\left(\varepsilon_0\vec{E} + \vec{P}\right) = \rho_f$

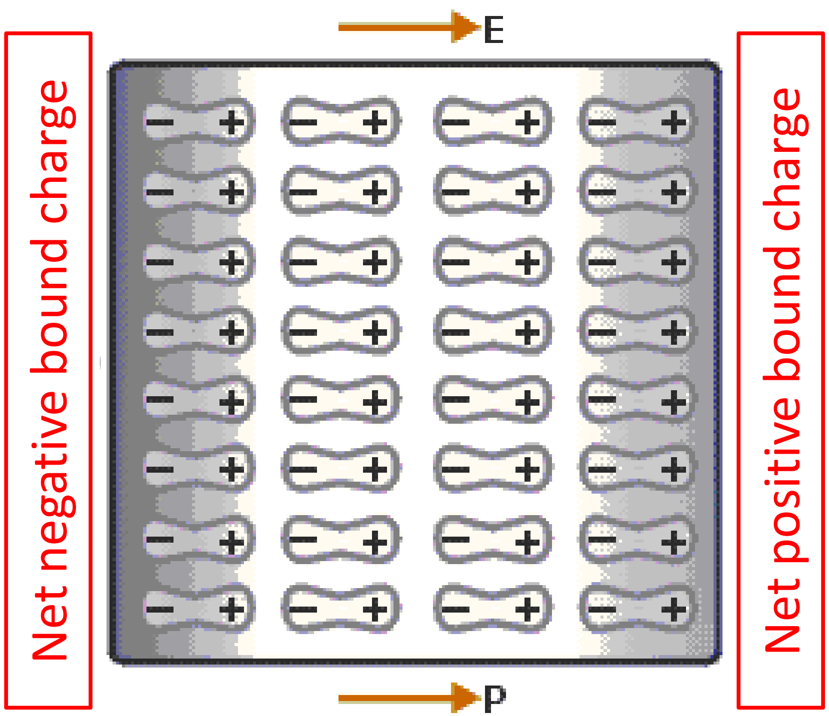
- Now let us define a new quantity called the **electric displacement field, $\vec{\boldsymbol{D}} = \boldsymbol{\varepsilon_0}\vec{\boldsymbol{E}} + \vec{\boldsymbol{P}}$**

- In terms of this quantity, Maxwell's 1st Equation can be written in the general form $\vec{\boldsymbol{\nabla}}.\vec{\boldsymbol{D}} = \boldsymbol{\rho_f}$ (or $\int \vec{D}.d\vec{A} = Q_{enc}$), which *now applies in all materials*

Electric displacement

- In many materials *the polarization is proportional to the applied electric field,* $\vec{P} \propto \vec{E}$, which also implies $\vec{D} \propto \vec{E}$
- We can write $\vec{D} = \varepsilon_r \varepsilon_0 \vec{E}$, where ε_r is the **relative permittivity**, which takes the value $\varepsilon_r = 1$ in a vacuum
- In this case, the polarization $\vec{P} = (\varepsilon_r - 1)\varepsilon_0 \vec{E}$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$



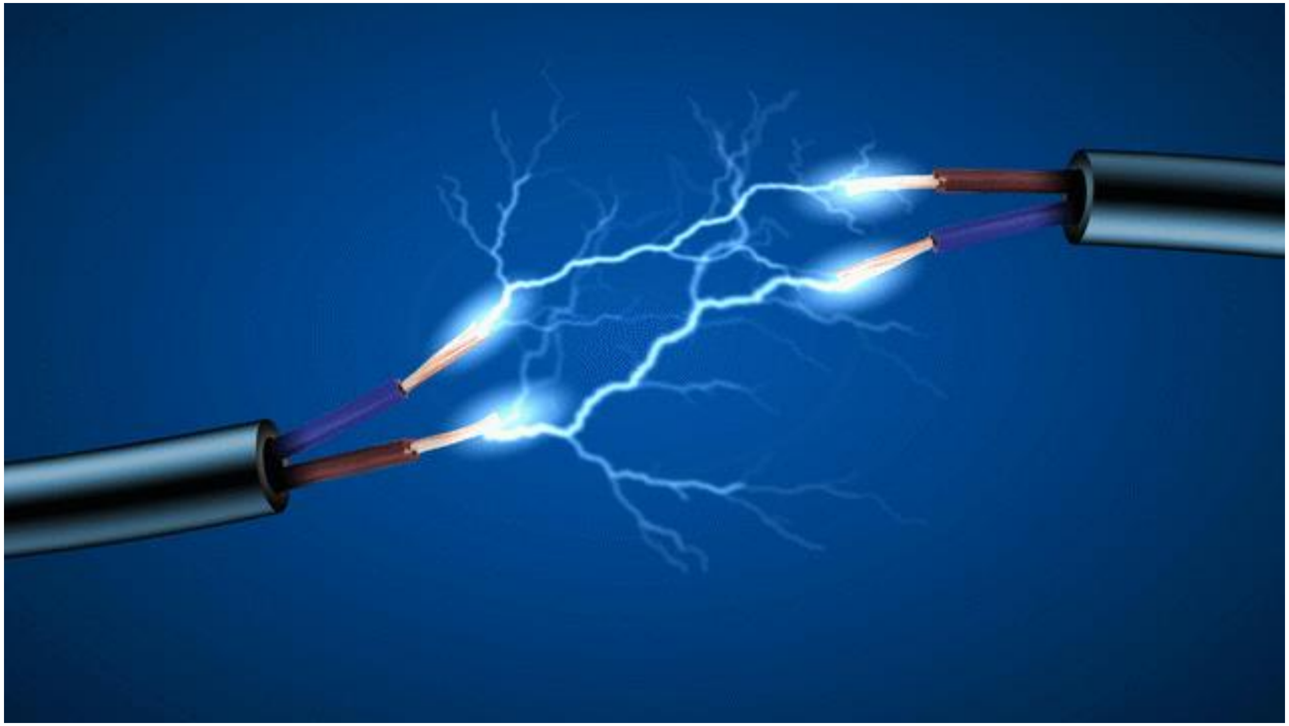
Please note in workbook

Electric displacement

- **To clarify the different roles of $\vec{D}$ and $\vec{E}$:**
- The **electric displacement** $\vec{D}$ is produced by **free charges** with density ρ_f (we can ignore the polarized medium)
- It is computed from Maxwell's 1st Equation $\vec{\nabla}.\vec{D} = \rho_f$ (or the equivalent Gauss's Law, $\int \vec{D}.d\vec{A} = Q_{enc}$)
- The **medium** determines how the electric field $\vec{E}$ is computed from $\vec{D}$: via $\vec{E} = \vec{D}/\varepsilon_r\varepsilon_0$, where ε_r = **relative permittivity**
- $\vec{E}$ determines the force $\vec{F}$ on a test charge q, via $\vec{F} = q\vec{E}$

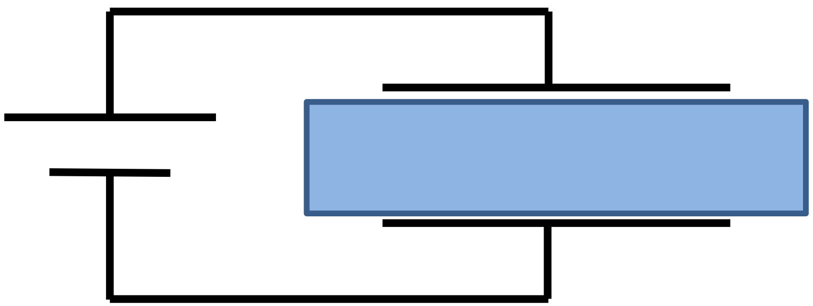
Electric displacement

- Insulators have a **maximum electrical field** they can sustain before “electrical breakdown” occurs, leading to sparking!

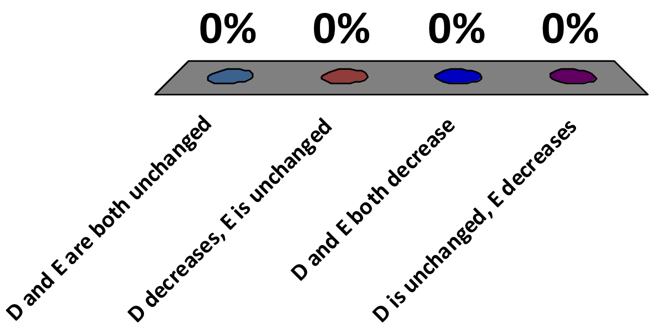


Clicker question

The space inside a charged capacitor is filled by an insulator. What can we say about the $\vec{D}$ and $\vec{E}$-fields?

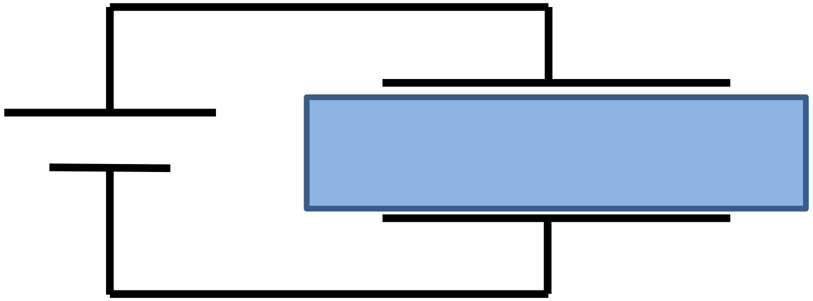


A. D and E are both unchanged
B. D decreases, E is unchanged
C. D and E both decrease
D. D is unchanged, E decreases

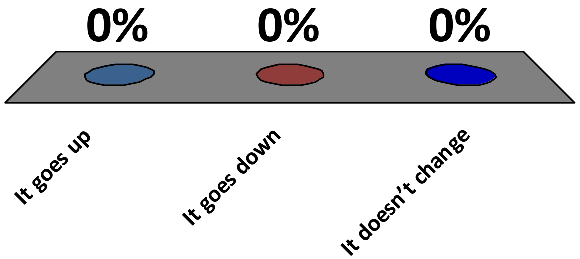


Clicker question

The space inside a charged capacitor is filled by an insulator. What can we say about the capacitance?



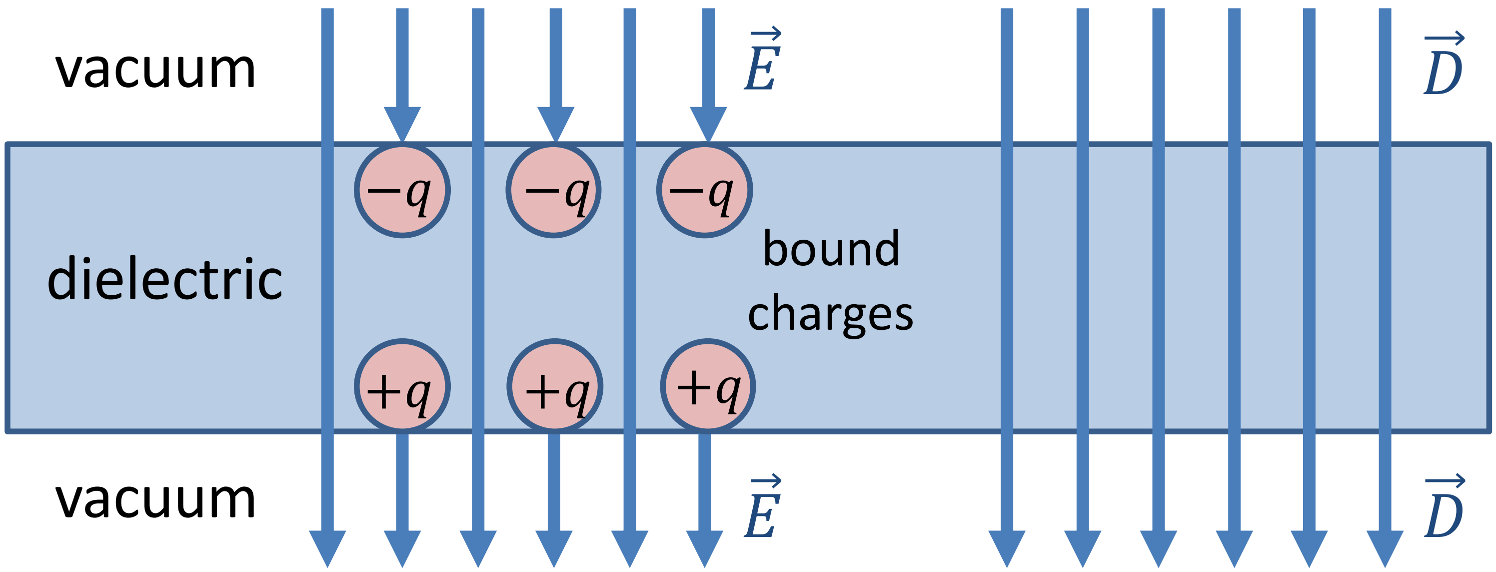
A. It goes up
B. It goes down
C. It doesn't change



Boundary conditions

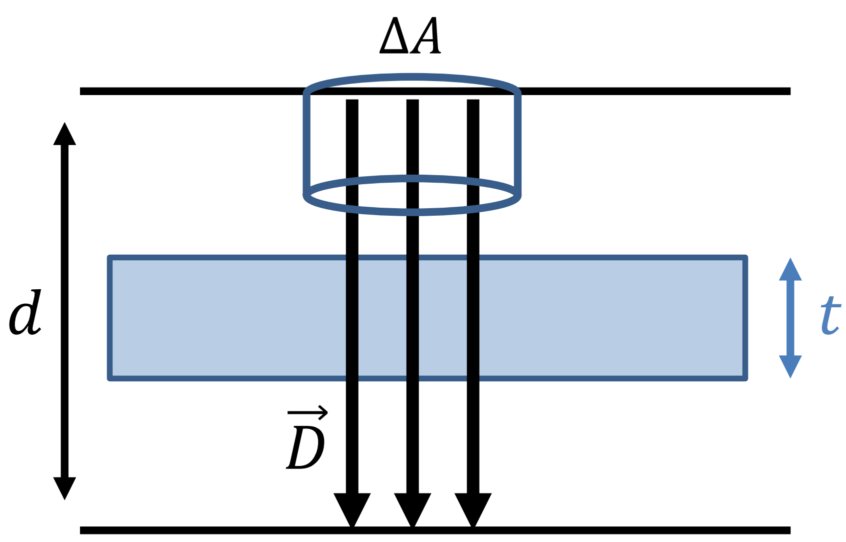
- Lines of $\vec{E}$ end on both free charges and bound charges, so there is a **discontinuity in $\vec{E}$ at the surface of a dielectric**

- Lines of $\vec{D}$ only end on free charges, so **the $\vec{D}$-field is continuous across the surface of a dielectric**



Boundary conditions

- A parallel-plate capacitor of area A and width d has capacitance $C = \frac{\varepsilon_0 A}{d}$. How is C changed if a slab of dielectric of relative permittivity ε_r and width t is inserted?



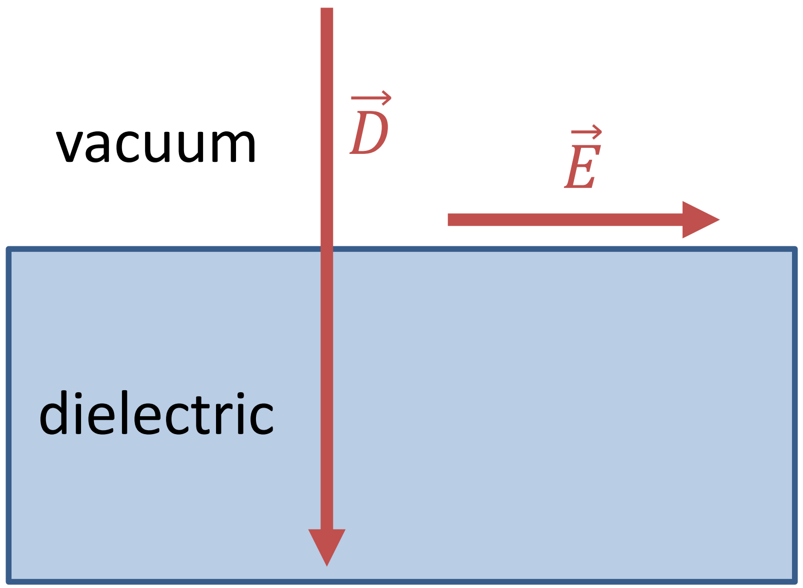
- If σ is the charge density on the plates, then applying Gauss's Law to a cylinder of area ΔA, we find $D \times \Delta A = \sigma \times \Delta A$ or $D = \sigma$
- The electric field is $E = \sigma/\varepsilon_0$ in vacuum, and $E = \sigma/\varepsilon_r \varepsilon_0$ in dielectric
- The potential difference is $V = \frac{\sigma}{\varepsilon_0}(d - t) + \frac{\sigma}{\varepsilon_r \varepsilon_0} t$

- The capacitance $C = \frac{Q}{V} = \frac{\sigma A}{V} = \frac{\varepsilon_0 A}{d - t + t/\varepsilon_r}$

Boundary conditions

- The **normal component of $\vec{D}$ is continuous** across the boundary of a dielectric (in the absence of free charge)

- The **tangential component of $\vec{E}$ is continuous** across the boundary of a dielectric



- The first condition comes from applying Gauss's law $\int \vec{D}.d\vec{A} = 0$ to a short cylinder crossing the surface

- The second condition comes from applying $\oint \vec{E}.d\vec{l} = 0$ to a short loop crossing the surface

Summary

- Two charges $\pm q$ separated by distance $\vec{d}$ form an **electric dipole** $\vec{p} = q\vec{d}$, which produces an electric field $E \propto 1/r^3$
- Insulators are materials in which *charges cannot move freely*
- When an electric field $\vec{E}$ is applied to an insulator, a **polarization** $\vec{P}$ of the molecules is created
- This polarization is the same as a *bound charge density* $-\vec{\nabla}.\vec{P}$
- If we define the **electric displacement** $\vec{D} = \varepsilon_0\vec{E} + \vec{P}$, we can re-write Maxwell's 1st equation as $\vec{\nabla}.\vec{D} = \rho_f$, which now applies in all materials