Class 5: Equivalence Principle

In this class we will discuss the conceptual foundations of General Relativity, in which gravity may be associated with the reference frames in which perceive events

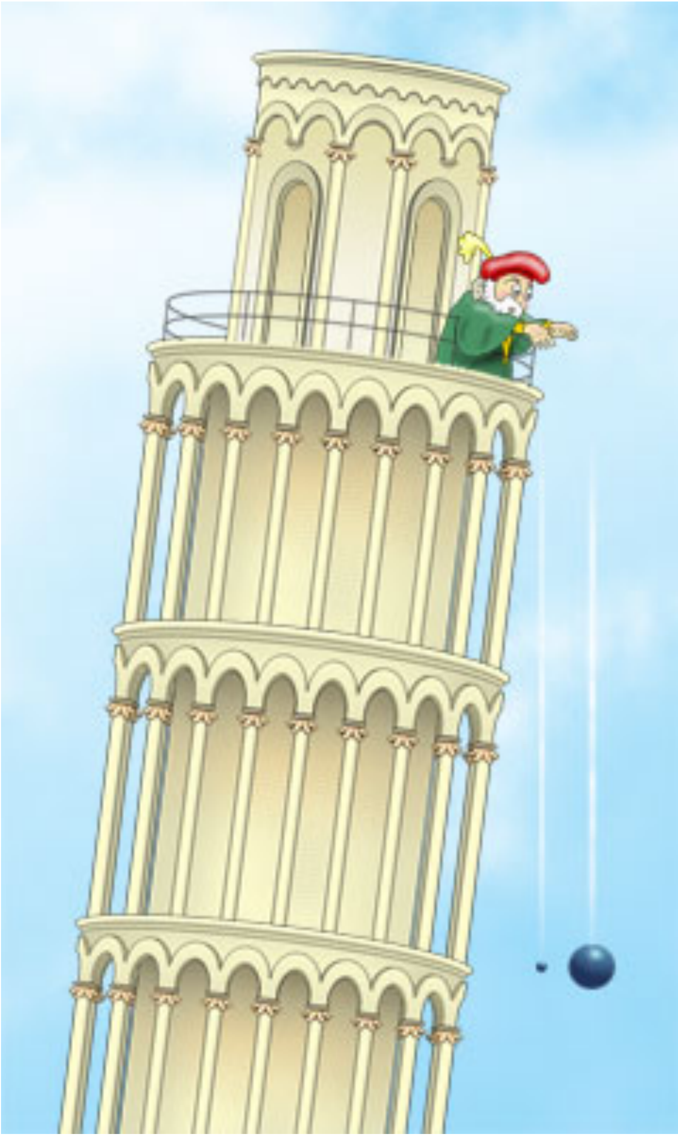
Class 5: Equivalence Principle

At the end of this session you should be able to …

- … state the **Equivalence Principle**: a freely-falling frame is locally equivalent to an inertial frame, and a gravitational field is locally equivalent to an acceleration
- … understand the implications of the Equivalence Principle for **clock rates** and the **curvature of space-time**
- … state **under what conditions Special Relativity applies**, compared to General Relativity
- … discuss the **nature of gravity** according to General Relativity, in comparison with classical physics

Why is gravity special?

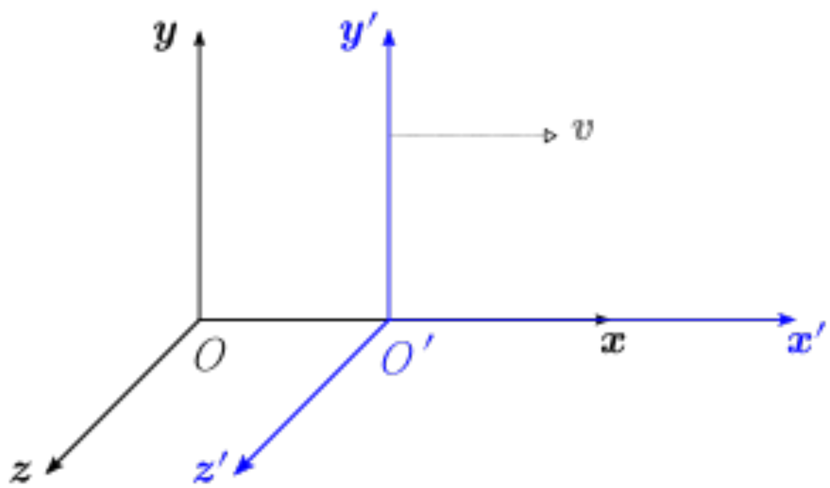
- **All objects fall at the same rate in a gravitational field**
- $F = ma = mg \rightarrow a = g$
- Sometime stated as *"inertial mass is the same as gravitational mass"* (i.e., m's cancel in the equation)
- It's not true for other forces (e.g. electromagnetism)
- ***Is this a coincidence or a deeper truth?***



http://lannyland.blogspot.com.au/2012/12/10-famous-thought-experiments-that-just.html

Freely-falling frames

- In Special Relativity, the laws of physics are the same in all **inertial frames**, which move with constant relative velocity



http://www.miami.com/things-to-do-in-miami/zero-g-weightless-flight-3702/

- The fundamental law of gravity – that all objects fall at the same rate in a gravitational field – suggests that gravity can be understood in terms of **freely-falling frames**

Freely-falling frames

- Einstein was very fond of **thought experiments** – that is, imagining situations which may not be precisely replicated in a laboratory, but which confront the logic of a theory*

* And therefore usually give you a head-ache!

https://www.cartoonstock.com/directory/t/thought_experiment.asp

"SINCE YOU CONDUCT ONLY THOUGHT-EXPERIMENTS, WE WERE HOPING YOU WOULD, FROM TIME TO TIME, COME UP WITH SOME THOUGHT-RESULTS."

Freely-falling frames

Einstein imagined what the laws of physics would look like from the perspective of a freely-falling observer

- If you drop a laboratory in a gravitational field, it acquires the same acceleration as the objects within it

- *Mechanics experiments would yield the same results in a freely-falling laboratory, as a laboratory in empty space*

http://astro.physics.sc.edu/selfpacedunits/Unit57.html

Freely-falling frames

- You can’t tell the difference between being in an inertial frame, and being in free fall!

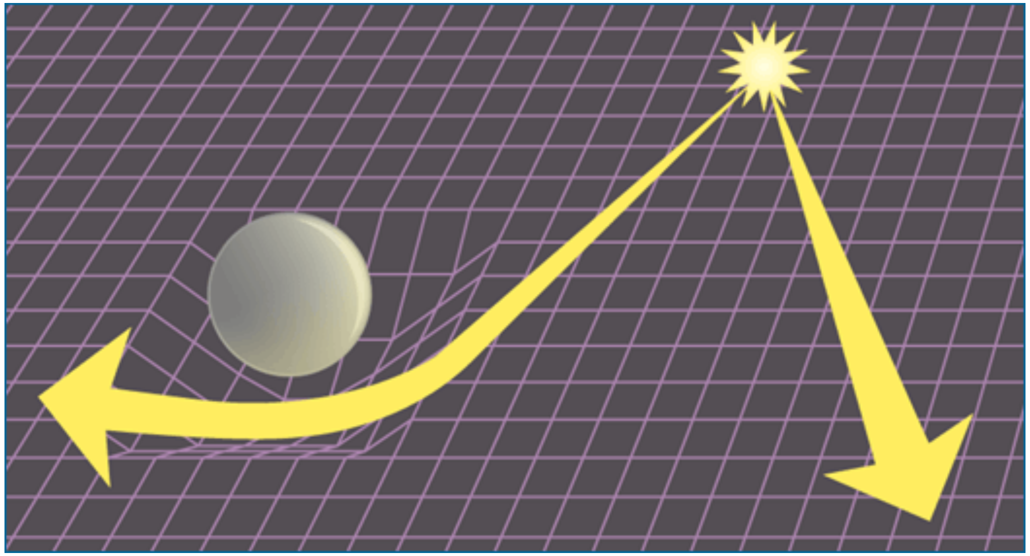
Einstein raised this to the level of a postulate: the Equivalence Principle

- **A freely-falling frame is locally equivalent to an inertial frame**

- You can “turn off” the effect of gravity by setting your lab in free fall!

Does gravity affect light?

- The constancy of the speed of light is fundamental to special relativity – so it's natural to ask, *does gravity affect light?*



https://undsci.berkeley.edu/article/natural_experiments

- **No?** Light has no mass, so cannot be affected by gravity
- **Yes?** All things fall at the same rate under gravity, even light

Does gravity affect light?

- Another of our thought experiments: *suppose a freely-falling observer switches on a torch!*

- Since free-fall is locally equivalent to an inertial frame, the light beam must travel in a straight line, as seen by the freely-falling observer

Does gravity affect light?

- However, the freely-falling observer is dropping in the Earth's frame*, in which the beam must appear to curve!

* Not a freely-falling frame, so no contradiction

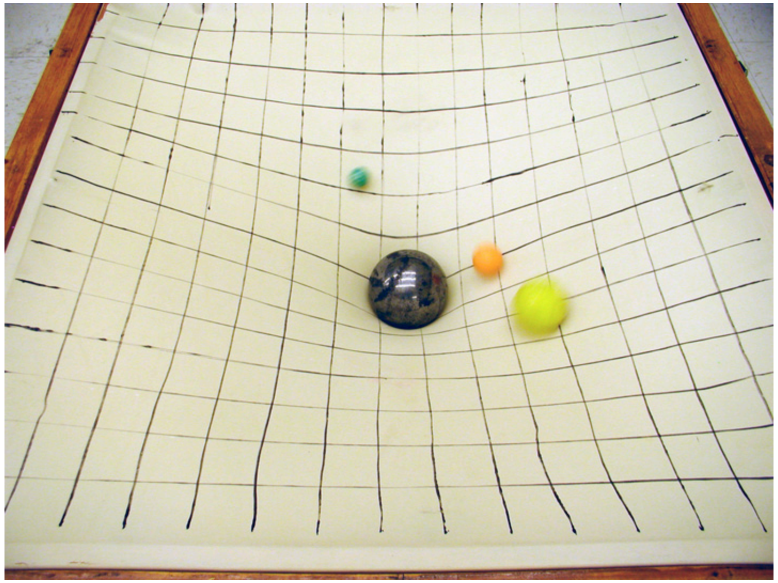
- *Gravity causes naturally straight phenomena to travel on curved rails*

The curving of space-time

- ***Gravity is not a force acting in space, causing the path of an object to curve, but is the curvature of space-time itself!***

- Note that this is the curvature of **space-time**, not just **space** – throwing a ball, and a light ray, across a room look different

- So the “rubber sheet” analogy of gravity is limited!*

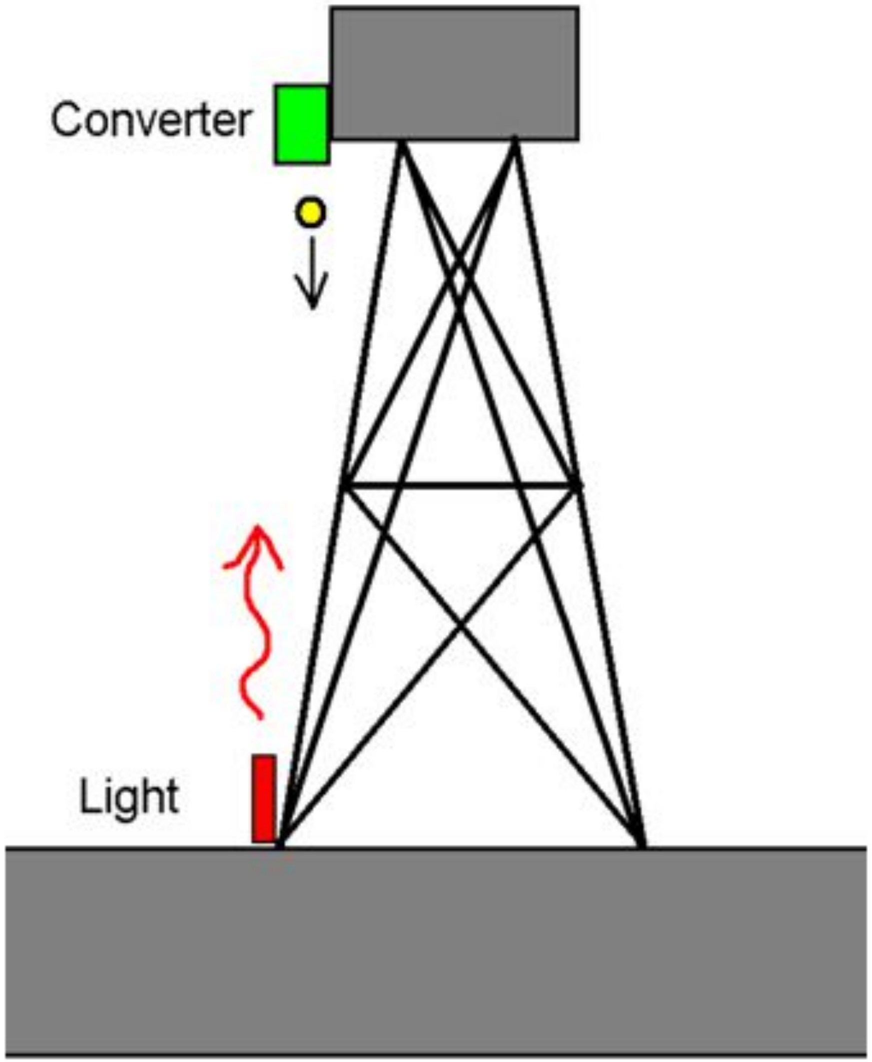


* Not that this will stop me using it. Pictures are nice!

Effect of gravity on clock rates

How does gravity affect time?
Consider a thought experiment based on mass/energy equivalence:

- Imagine a tower on the Earth
- Shine some light from the bottom to the top of the tower
- When the light gets to the top, turn its energy into mass
- Drop the mass back down to the bottom of the tower
- Turn it back into light and repeat



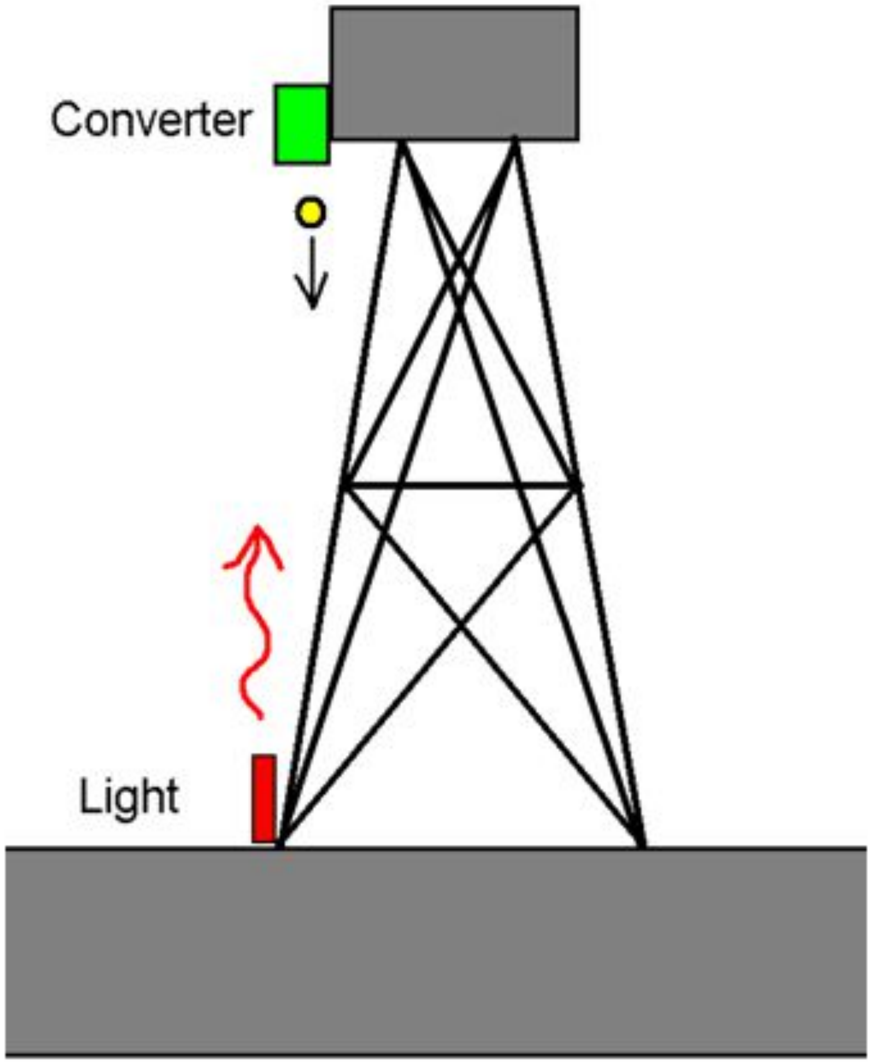
http://slideplayer.com/slide/2803242/

Effect of gravity on clock rates

What is happening here??

- If the light initially contains energy E_{start}, then the mass created at the top is $m = E_{start}/c^2$
- As the mass falls, it picks up speed/energy due to gravity
- So $E_{end} = E_{start} + E_{grav}$
- *Is this a perpetual motion machine??*

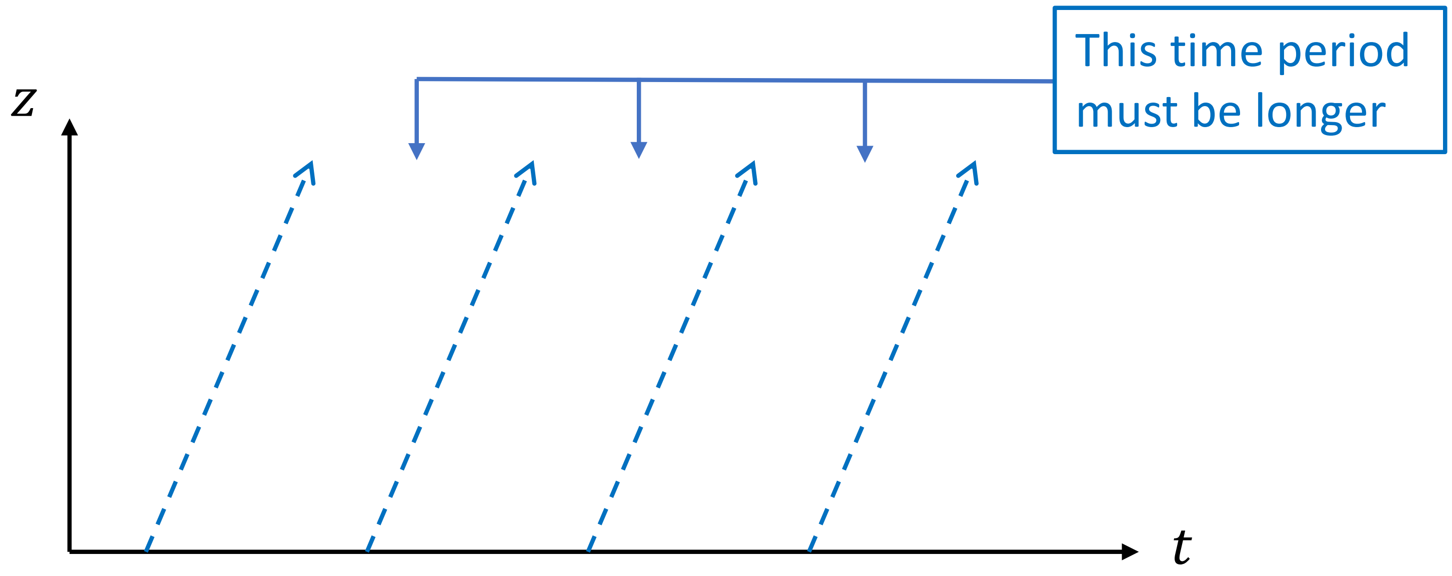
No – light must lose energy as it climbs in a gravitational field



http://slideplayer.com/slide/2803242/

Effect of gravity on clock rates

- If light loses energy as it climbs in a gravitational field, what are the implications?
- Losing energy is the same as reducing frequency ($E = hf$), so the time between wave-crests must be increasing
- Space-time diagram:



- **Time passes more slowly higher up in a gravitational field!**

Effect of gravity on clock rates

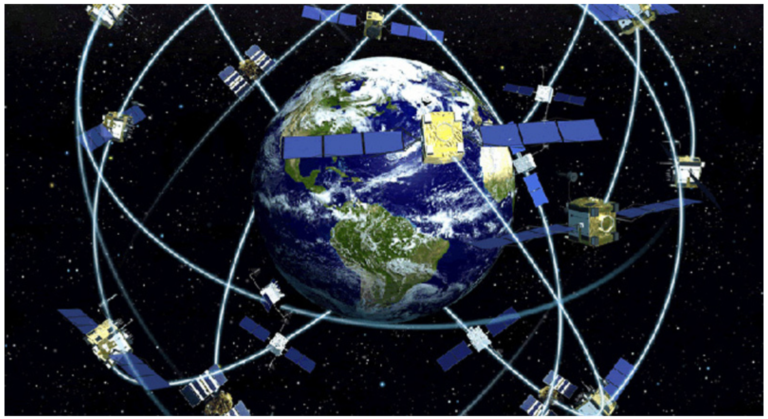
- *Clocks with no relative motion are ticking at different rates!*
- This tells us that, in a gravitational field, **we cannot have an extended inertial frame, only a local inertial frame**
- We can also see this by considering two freely-falling observers on different sides of the Earth

- Both observers are in inertial frames, but are accelerating towards each other! *It's inconsistent with Special Relativity.*

Effect of gravity on clock rates

- These effects have been verified by the *Pound-Rebka experiment* (gamma-ray beam in a tower), *atomic clocks in a plane*, and every day with the *Global Positioning System (GPS)*!

Joseph C. Hafele and Richard E. Keating on board of a jet plane while performing the around-the-word-clock experiment (1971).

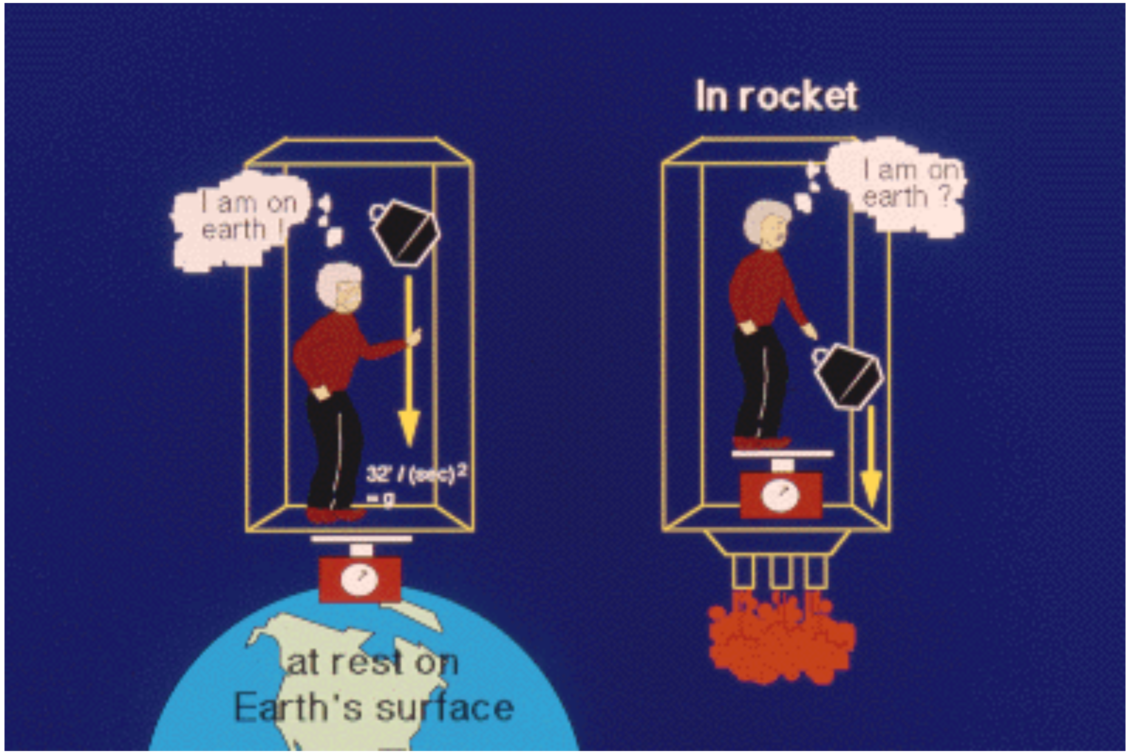


https://hackaday.com/2015/11/12/hackaday-dictionary-the-global-positioning-system-gps/

https://www.relativity.li/en/epstein2/read/i0_en/i4_en

Analogy with accelerating frames

- Another thought experiment – compare the views from an **Earth lab** (with uniform gravitational field g), and a **rocket ship accelerating** at $a = g$ with respect to an inertial frame

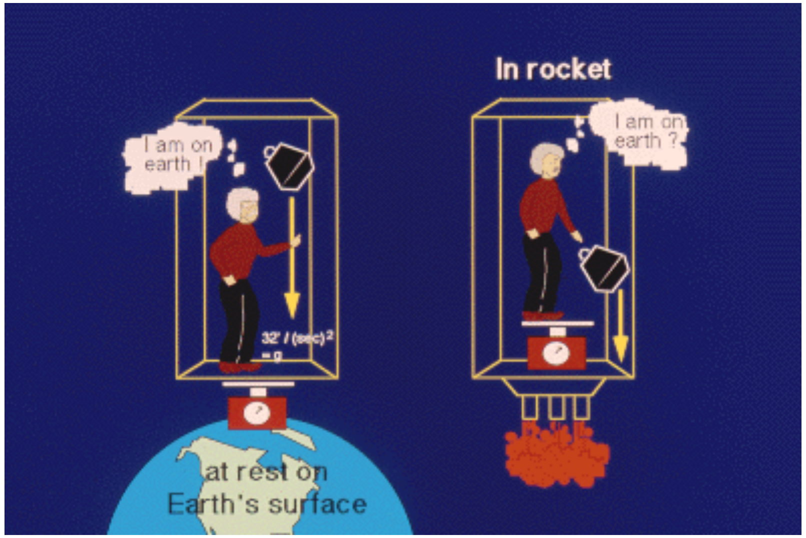


http://astro.physics.sc.edu/selfpacedunits/Unit57.html

Analogy with accelerating frames

Why are these situations equivalent?

- Objects released in each lab fall with the same acceleration
- Standing in each lab, your weight feels the same
- Clocks further up in each lab are running faster *(see our derivation in Class 4 for the accelerating frame)*



Another statement of equivalence: a gravitational field is completely equivalent to an acceleration, for local measurements

Gravity as a “fictitious force”

- This equivalence is interesting, because a rocket observer would attribute their “weight” as a “fictitious force” due to being in a non-inertial co-ordinate system

- In the same sense, **gravity is a fictitious force due to not being in a freely-falling frame**

Gravity as a "fictitious force"

- We are used to standing on the ground, feeling "weight", as being the normal state, and falling being unusual!
- GR says the opposite – *being in free-fall is the "normal state"*, and on the ground we are being accelerated by the floor

Just as objects in a rotating frame feel a fictitious "centrifugal force", objects in a frame that is not freely falling feel a "fictitious weight" due to gravity!

Why is Special Relativity “Special”?

- Where gravity is present, **Special Relativity only applies in small local regions**, in which different inertial observers can compare observations of the same set of events

- In General Relativity **there is no such thing as extended inertial frames**, so we cannot compare separated observations, e.g., the relative velocities of separated objects

- Gravity does not exist in Special Relativity

- GR is “general” because it allows us to describe events from the point of view of accelerated, as well as inertial, observers