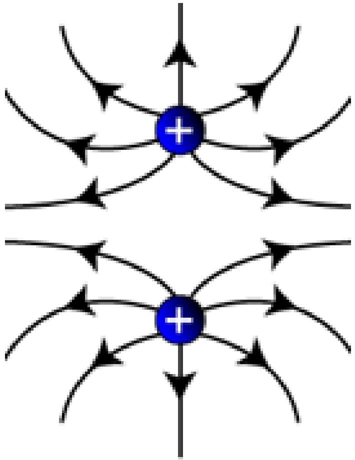
Class 5 : Conductors and Capacitors

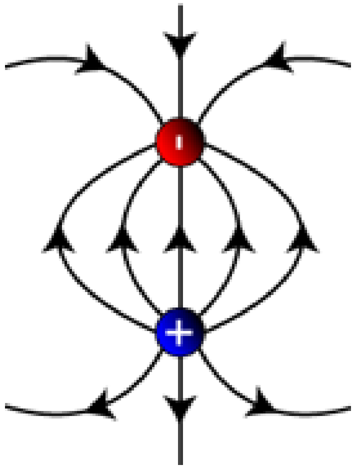
- What is a conductor?
- Field and potential around conductors
- Defining and evaluating capacitance
- Potential energy of a capacitor

Recap

- **Gauss's Law** $\int \vec{E}.\,d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$ and **Maxwell's 1st equation** $\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0}$ are equivalent *integral* and *differential* formulations for the electric field $\vec{E}$ produced by charge density ρ



- The electric field $\vec{E}$ must also satisfy **Maxwell's 2nd equation** $\vec{\nabla} \times \vec{E} = \vec{0}$. This implies that the electric field can be generated by the gradient of an electrostatic potential V, $\vec{E} = -\vec{\nabla}V$

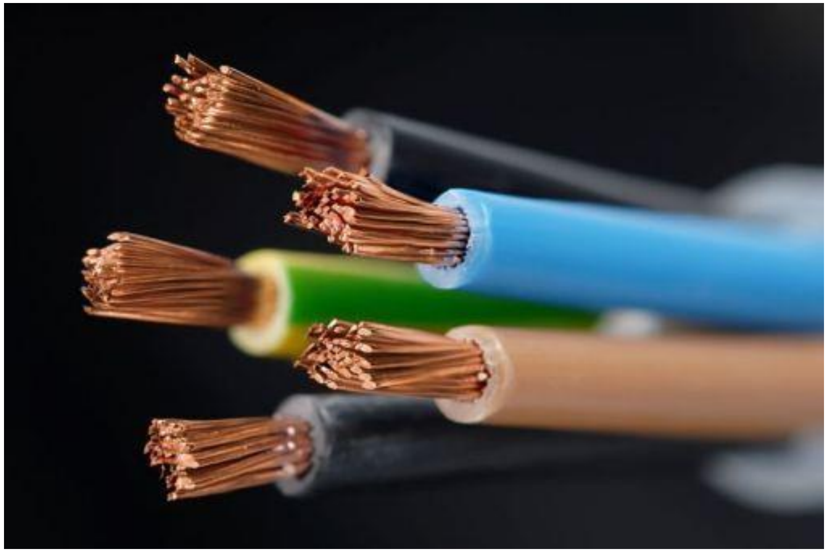


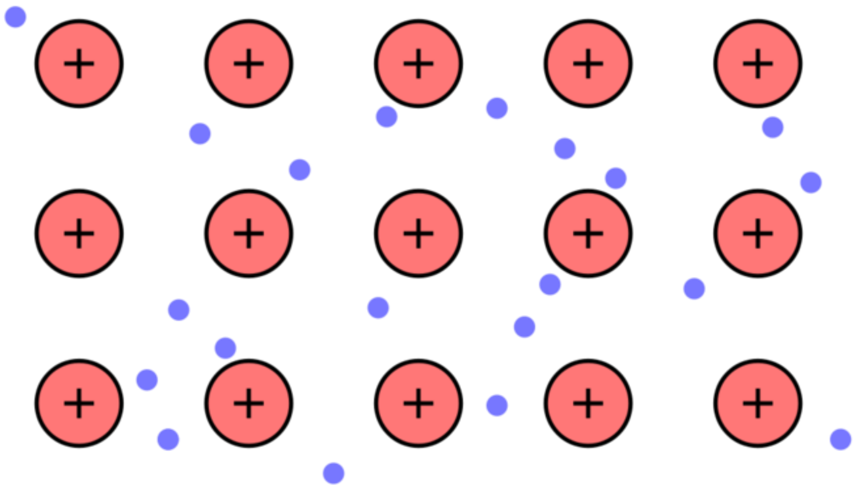
Conductors

- We'll now consider the behaviour of $\vec{E}$ in materials, which we divide into **conductors** and **insulators**

Conductors

- In a **conductor**, charges can flow freely
- In practice, this usually involves **free electrons** moving within an ionic lattice

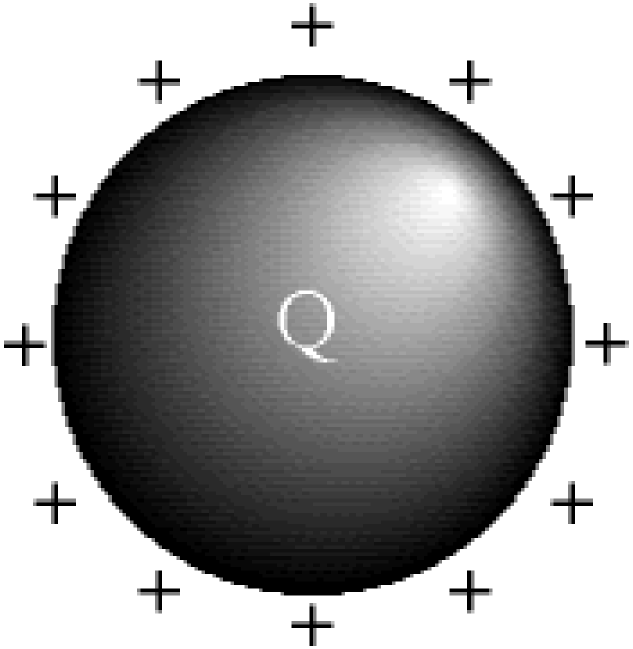




Conductors

- What can we say about the electric field in and around a charge-carrying conductor in equilibrium?

Place charge Q on a conducting sphere



Conductors

- What can we say about the electric field in and around a charge-carrying conductor in equilibrium?

- First, all **charge must be located on the surface** (otherwise it would move due to forces from other charges)

- Hence from Gauss's Law, $\vec{\boldsymbol{E}} = \vec{\boldsymbol{0}}$ **inside a conductor**

- Hence, because $\vec{E} = -\vec{\nabla} V$, all points of the conductor are at **constant electrostatic potential**

Please note in workbook

Conductors

- An application of this effect is **electrostatic shielding**

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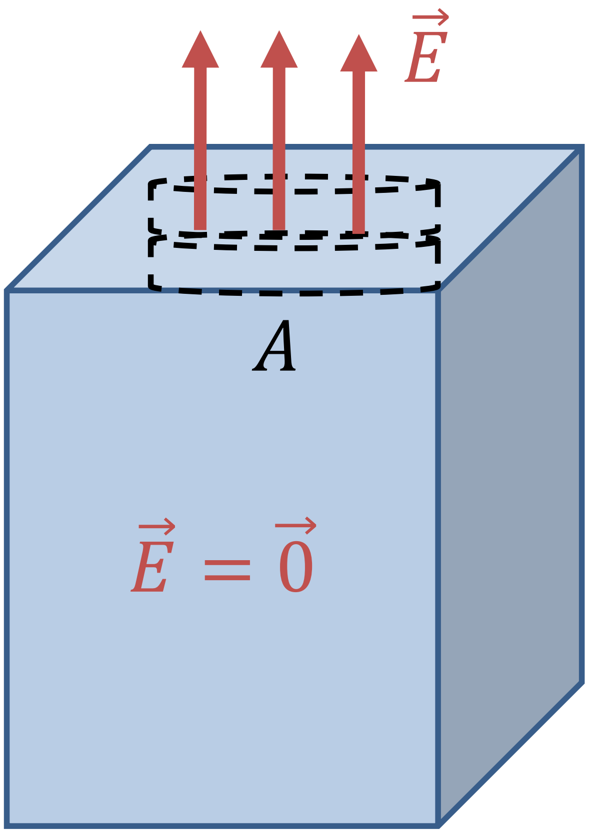
Conductors

- What about the electric field just outside the conductor?



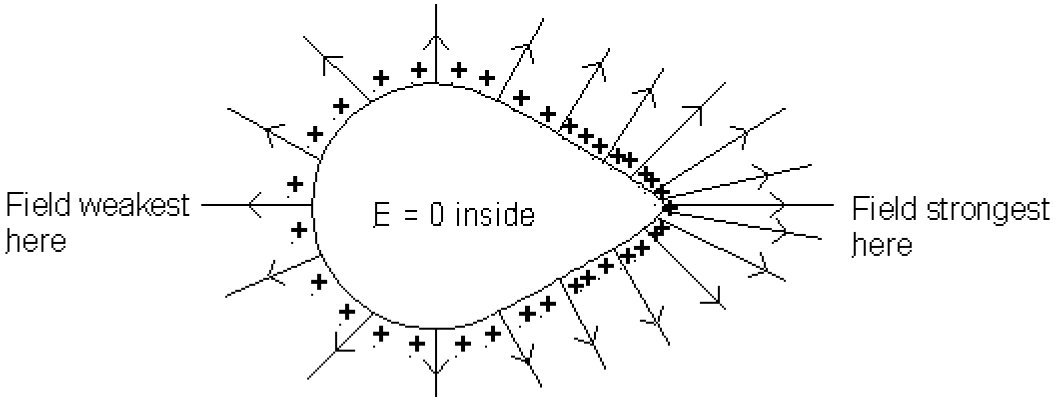
- **There can be no component of $\vec{E}$ parallel to the surface, otherwise charges would move**
- Consider a Gaussian cylinder of cross-sectional area A crossing the surface
- $\vec{E}$ is perpendicular to the surface and zero inside, such that $\int \vec{E}.d\vec{A} = E \times A$
- Let the charge per unit area at the surface be σ, then $Q_{enclosed} = \sigma A$
- Applying Gauss's Law: $\boldsymbol{E = \frac{\sigma}{\varepsilon_0}}$

Conductors

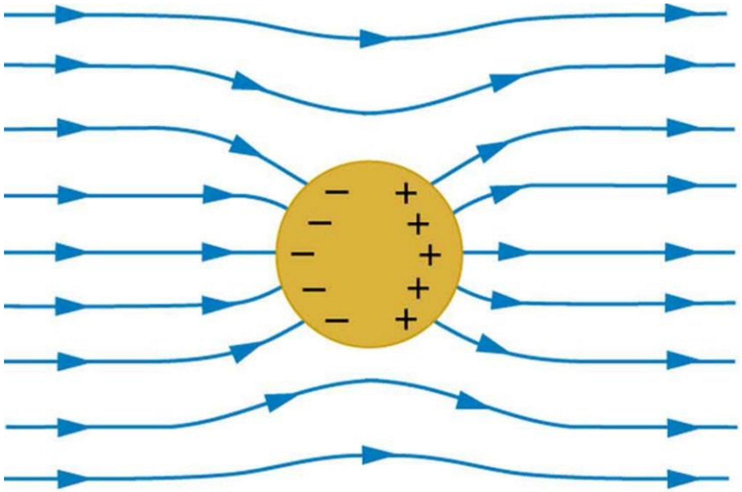
- The electric field just outside a conductor is **perpendicular to the surface** and **proportional to the charge density**

Please note in workbook

Electric field around charged conductor

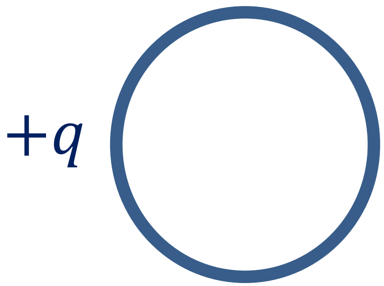


Uncharged conductor in applied field



Clicker question

If a hollow sphere is coated with charge, what can you say about the potential inside the sphere?

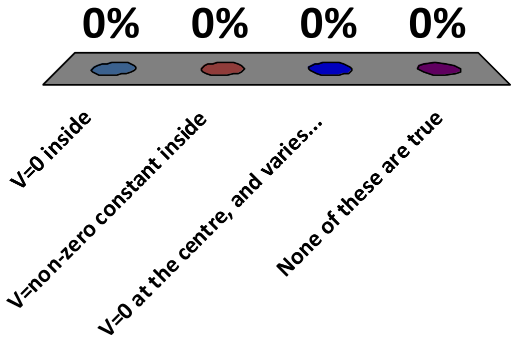


A. V=0 inside

B. V=non-zero constant inside

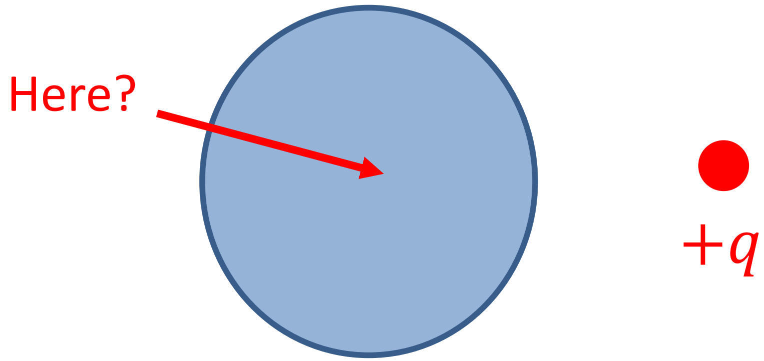
C. V=0 at the centre, and varies with position

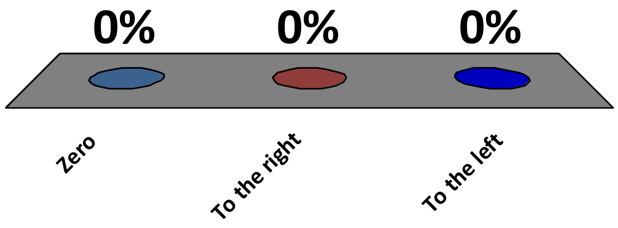
D. None of these are true



Clicker question

A point charge $+q$ is placed near a neutral solid copper sphere. What is the electric field inside the sphere?





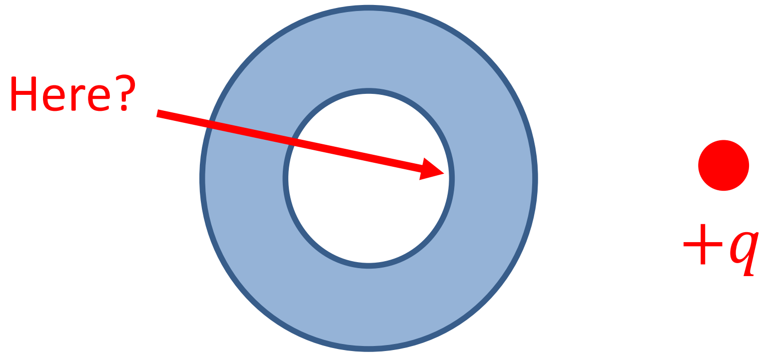
A. Zero

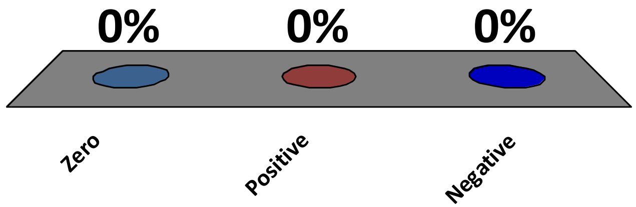
B. To the right

C. To the left

Clicker question

A point charge $+q$ is placed near a neutral hollow copper sphere. What is the charge density on the inside surface of the sphere?





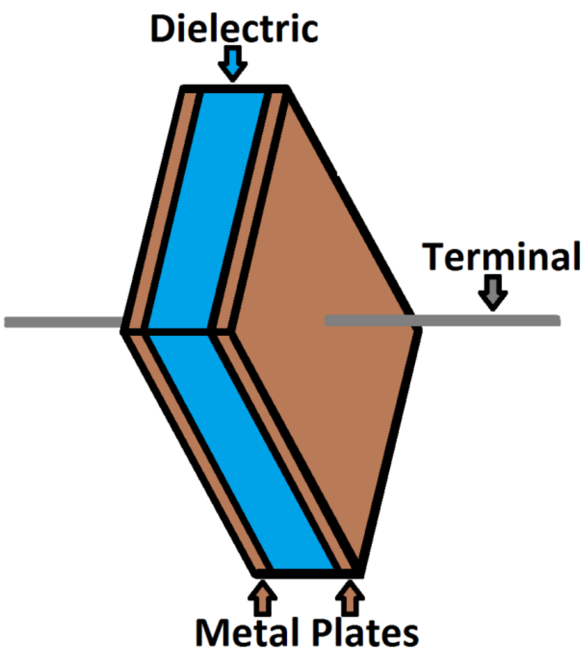
A. Zero

B. Positive

C. Negative

Capacitors

- A **capacitor** is a very useful circuit component formed by two parallel conductors separated by an insulator (or "dielectric")
- When connected to a battery at potential V, charge $\pm Q$ flows onto the plates. The **capacitance** is $C = Q/V$ [unit: Farads, F]



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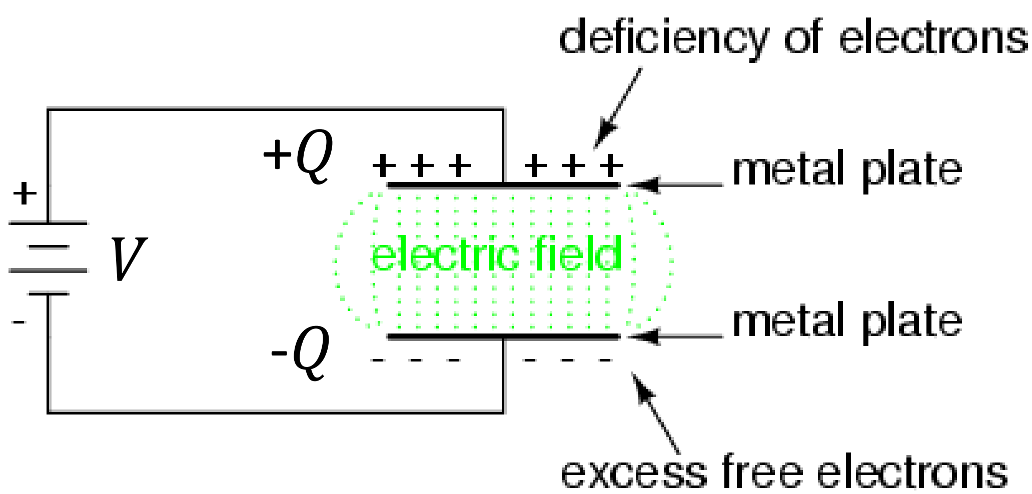
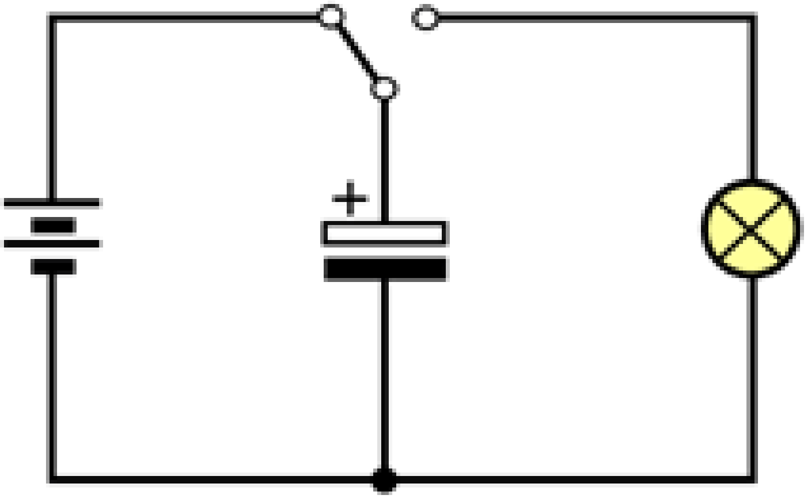


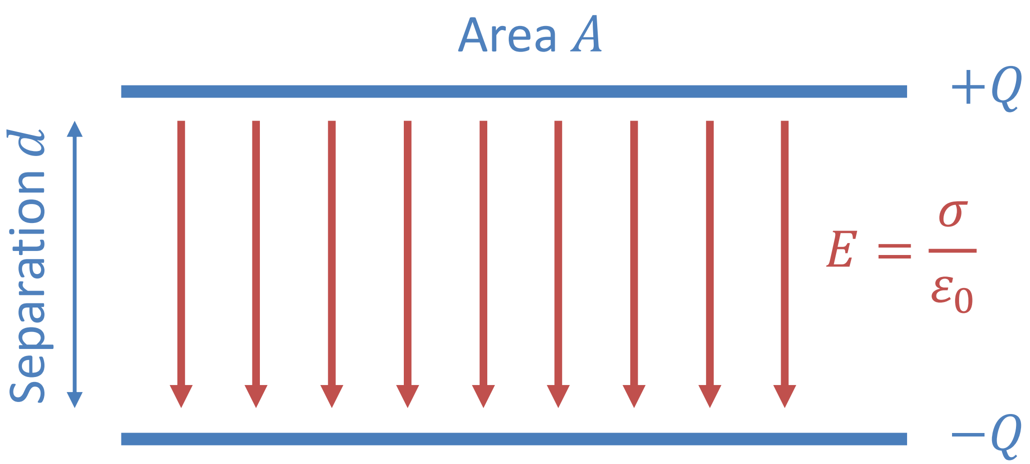
Capacitors

- Capacitors are useful for **storing charge** (or, potential energy) and then releasing it



Capacitors

- What is the capacitance of a parallel-plate capacitor, where the plates have area A and separation d?

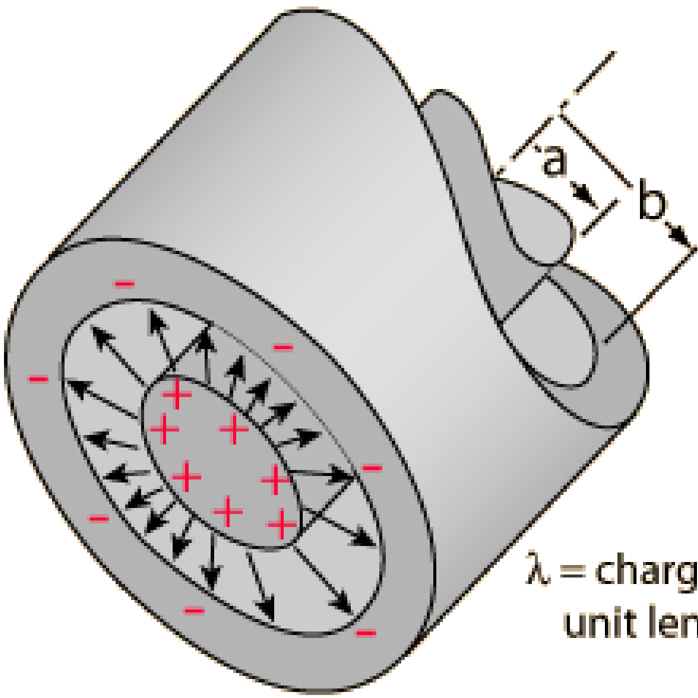


- From Gauss's Law (previous slides), electric field $E = \frac{\sigma}{\varepsilon_0}$

- Capacitance $C = \frac{Q}{V} = \frac{\sigma \times A}{E \times d} = \frac{\varepsilon_0 \, A}{d}$

Capacitors

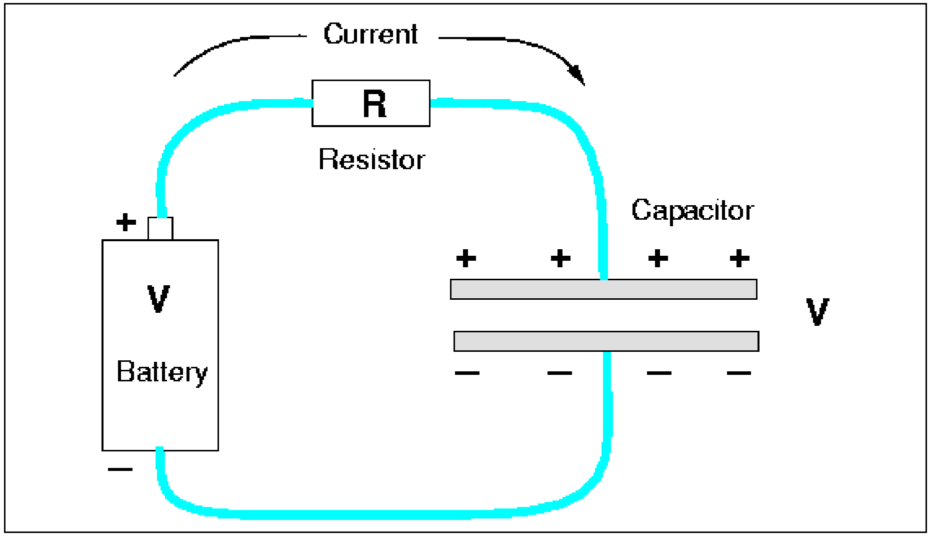
- What is the capacitance per unit length of a pair of concentric cylinders of radii a and $b > a$?



- Suppose the charge per unit length on the cylinders is $\pm\lambda$
- Applying Gauss's Law to a cylinder of radius r and length L, we find $E \times 2\pi r L = \lambda L/\varepsilon_0$ or $E = \lambda/2\pi\varepsilon_0 r$
- Potential difference between the cylinders is $V = \int_a^b E\, dr = \frac{\lambda}{2\pi\varepsilon_0}\int_a^b \frac{1}{r}\, dr = \frac{\lambda}{2\pi\varepsilon_0} log_e\left(\frac{b}{a}\right)$
- Capacitance $C = \frac{\lambda}{V} = \frac{2\pi\varepsilon_0}{log_e\left(\frac{b}{a}\right)}$

Potential energy

- Another example is the **charging of a capacitor**. The capacitor C reaches potential V storing charge $Q = CV$



- Transporting additional charge dQ through potential $V = \frac{Q}{C}$ requires work $dW = V\, dQ$

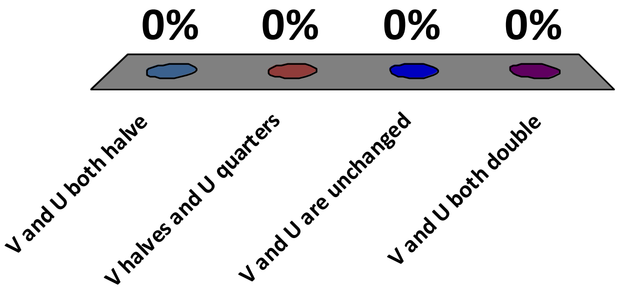
- Total work done $W = \int V\, dQ = \frac{1}{C}\int Q\, dQ = \frac{Q^2}{2C} = \frac{1}{2}CV^2$

- The **potential energy** stored in the capacitor is $U = \frac{1}{2}CV^2$

Clicker question

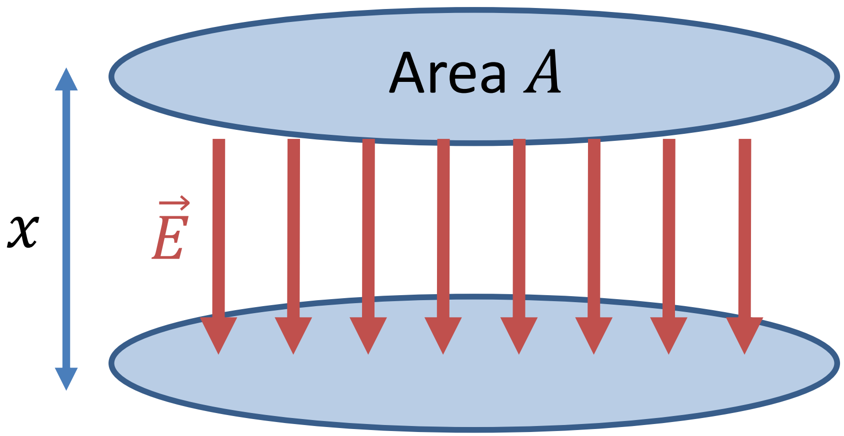
A capacitor holds charge $\pm Q$ and is disconnected from the battery. If half the charge is drained away, how does the potential difference V between the plates, and potential energy U, change?

A. V and U both halve
B. V halves and U quarters
C. V and U are unchanged
D. V and U both double



Potential energy

- It can be useful to think of this energy $U = \frac{1}{2}CV^2$ as being **stored in the electric field**

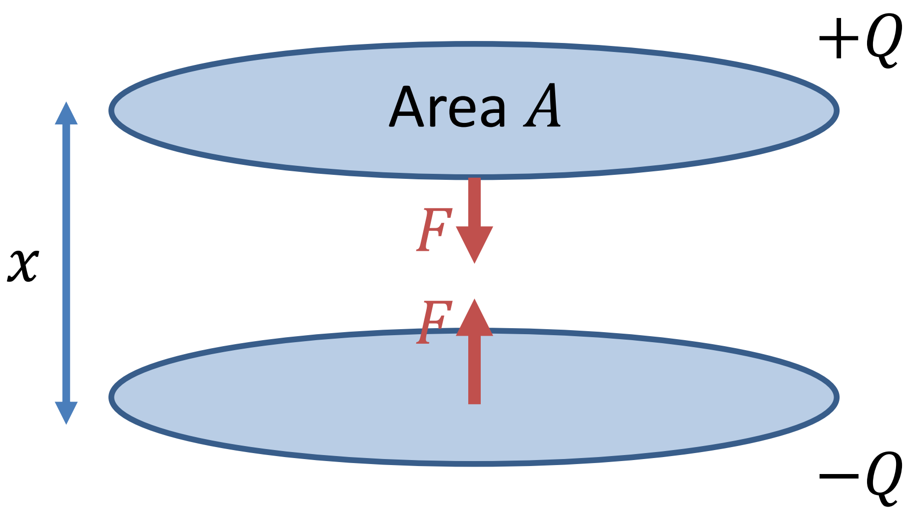


- Capacitance $C = \frac{\varepsilon_0 A}{x}$
- Potential difference $V = E \times x$
- $U = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)(Ex)^2 = \frac{1}{2}\varepsilon_0 E^2 \times Ax$, where Ax is the volume

- **We can think of the $\vec{E}$-field as storing potential energy with density $\frac{1}{2}\varepsilon_0 E^2$**

Potential energy

- One application of this relation is to derive the *force between two capacitor plates* (for fixed charge Q)



- Force $F = -\frac{dU}{dx}$
- $U = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$
- Capacitance $C = \frac{\varepsilon_0 A}{x}$
- Force $F = -\frac{d}{dx}\left(\frac{Q^2 x}{2\varepsilon_0 A}\right) = -\frac{Q^2}{2\varepsilon_0 A}$

- How would this calculation change if the capacitor were connected to a fixed source of potential V?

Summary

- **Conductors** are materials in which charges can flow freely. All charge will reside on the surface, and $\vec{E} = \vec{0}$ inside

- Two separated conductors storing charge $\pm Q$ form a **capacitor** C. If the potential difference is V, then $C = \frac{Q}{V}$

- We can consider this energy *stored in the $\vec{E}$-field with density* $\frac{1}{2}\varepsilon_0 E^2$