Class 1 : Electric Forces

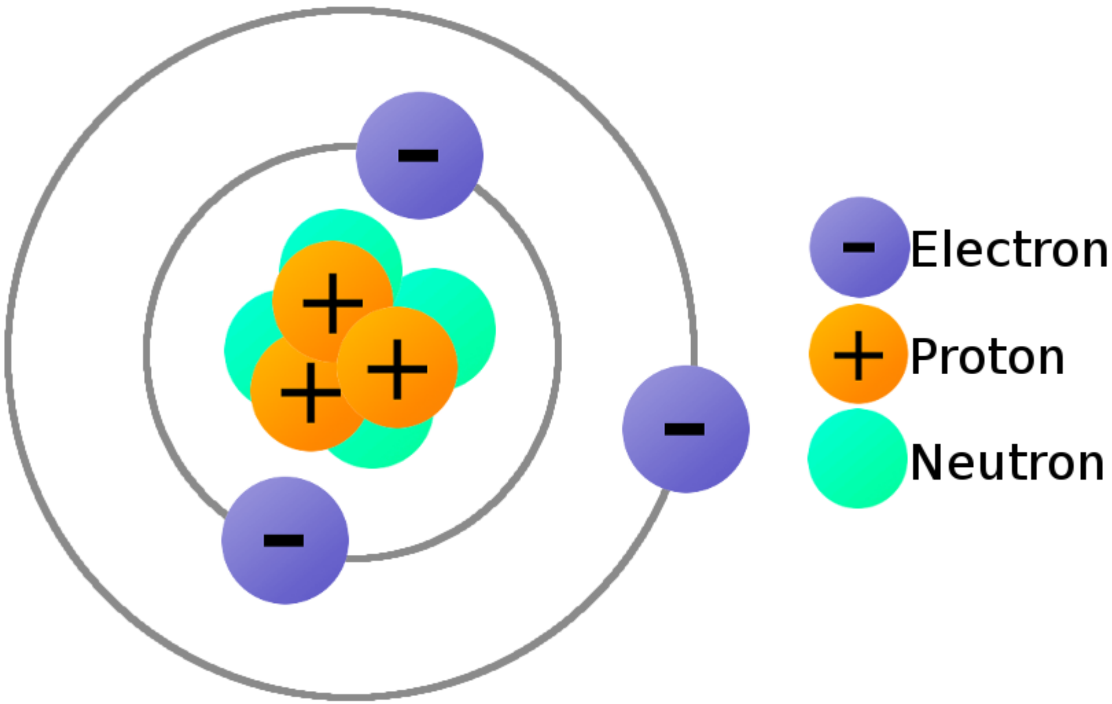
- Electric charge
- Computing electric forces: Coulomb’s Law, superposition
- What is the electric field?
- Properties of field lines

Electric charge

- **Electric charge** is an intrinsic property of the particles which make up matter, which experimentally can be either *positive or negative*

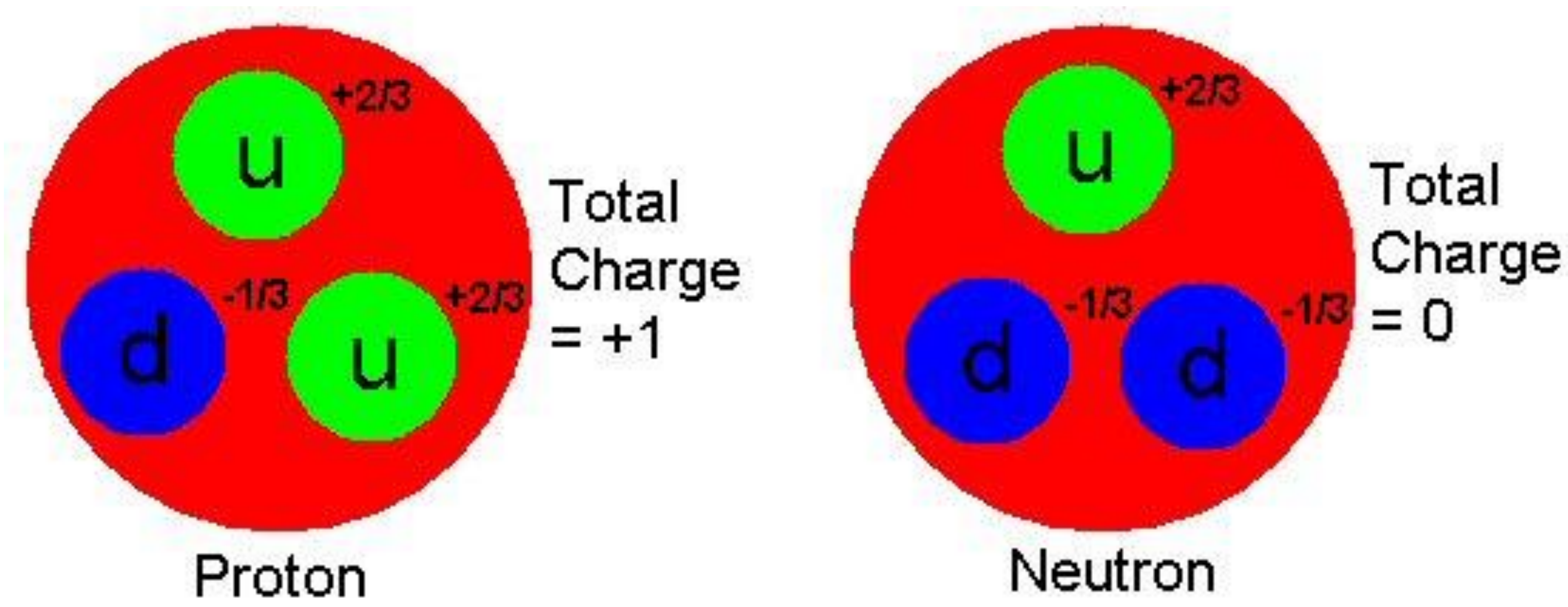
Electric charge

- Electric charge is **quantized** such that protons and electrons have equal and opposite charge $\boldsymbol{e} = \pm \mathbf{1.6} \times \mathbf{10^{-19}}\ \boldsymbol{C}$ [unit C = Coulombs]



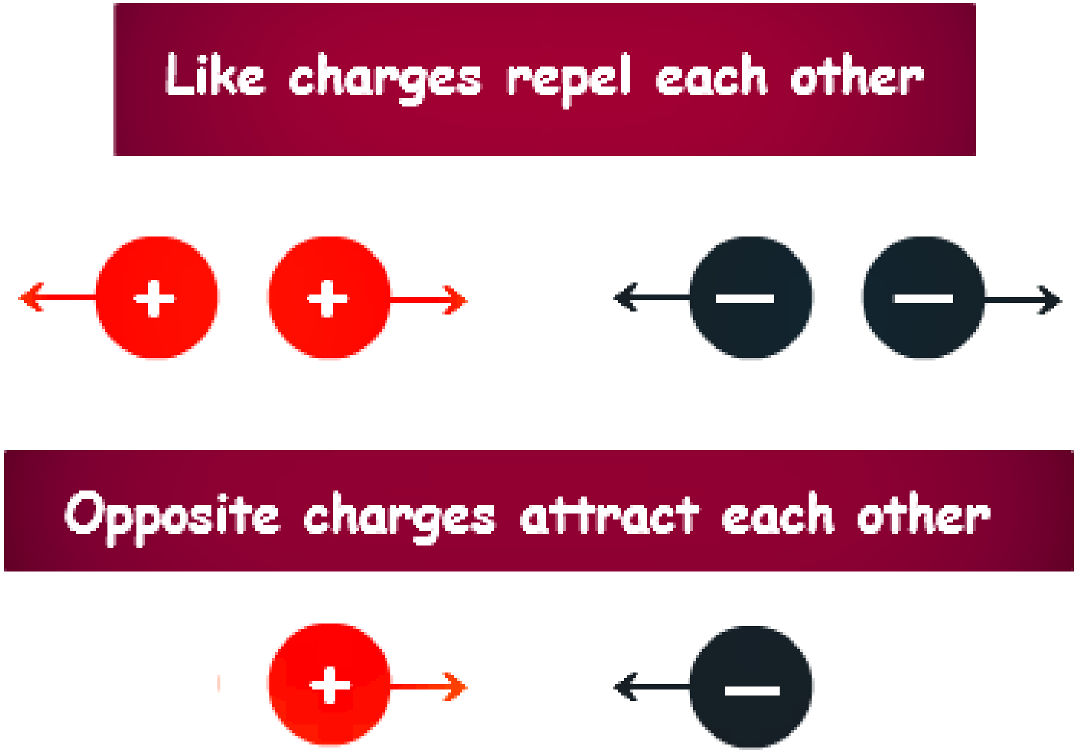
Electric charge

- Electric charge is locally **conserved** and cannot be created or destroyed



Electric charge

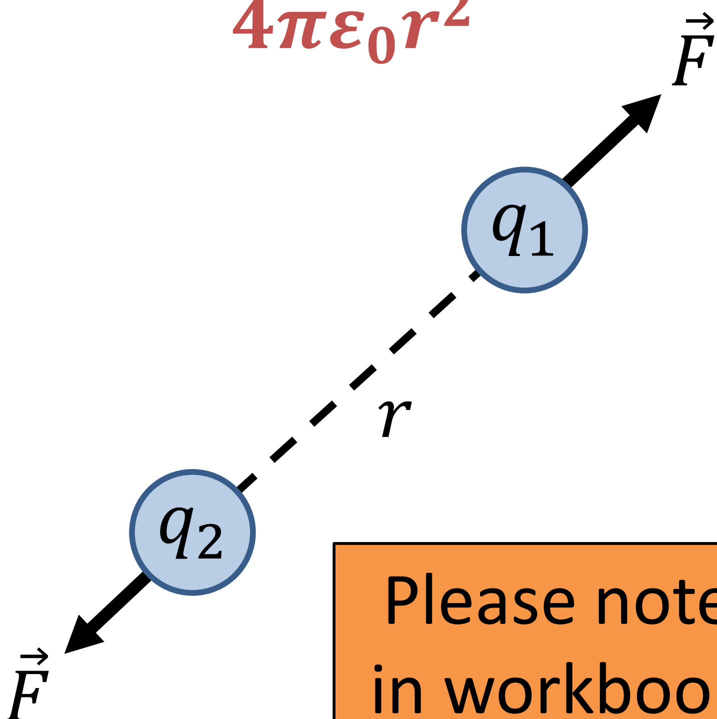
- Two electric charges attract or repel each other with equal and opposite forces



Electric charge

- The strength of the force is given by **Coulomb's Law**

$$\vec{F} = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}\hat{\vec{r}}$$



Please note in workbook

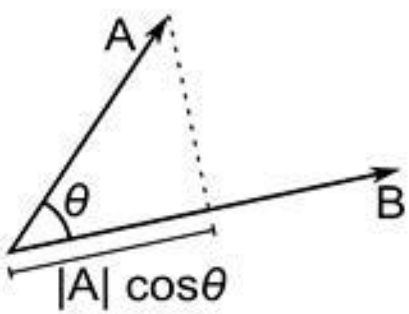
- The force is proportional to the magnitude of the charges q_1, q_2
- The force is inversely proportional to the square of the separation r
- The force acts along the line joining the charges : **by symmetry, no other direction could be singled out**
- The force strength is governed by the **permittivity of free space** ε_0 where $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ Nm^2C^{-2}$

Vectors

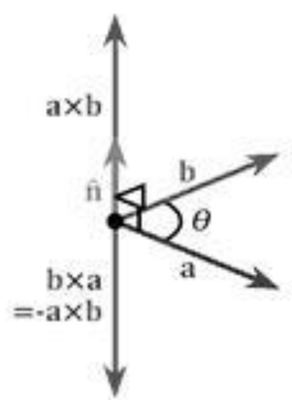
- A **vector** is a quantity – such as a force – which has both a *magnitude* and a *direction*
- It can be indicated by $\vec{F}$ or F̲ or ***F*** (helpful!)
- A vector can be specified by its **components** along co-ordinate axes, such as $\vec{F} = (F_x, F_y, F_z)$
- The **magnitude** of a vector is $|\vec{F}| = \sqrt{{F_x}^2 + {F_y}^2 + {F_z}^2}$
- A **unit vector**, indicated by $\hat{\vec{F}}$, has magnitude $= 1$
- You may also see $\vec{F} = F_x\hat{\imath} + F_y\hat{\jmath} + F_z\hat{k}$, in terms of unit vectors along co-ordinate axes

Vectors

- Vectors may be **added** by summing their components
- The **dot product** of two vectors, $\vec{a}.\vec{b}$, is the projection of one vector along the other, $|\vec{a}||\vec{b}|\cos\theta$ (θ = angle between). It can also be evaluated as $\vec{a}.\vec{b} = a_x b_x + a_y b_y + a_z b_z$
- The **cross product** of two vectors, $\vec{a} \times \vec{b}$, is a vector perpendicular to both with magnitude $|\vec{a}||\vec{b}|\ \sin\theta$



a. Dot Product

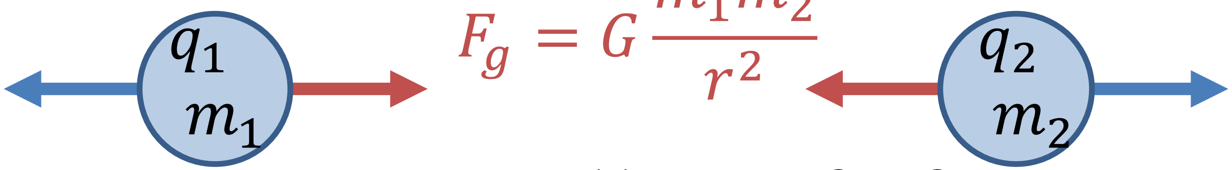


b. Cross Product

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Importance of electromagnetism

- **How strong are electromagnetic forces?**



$$F_g = G\frac{m_1 m_2}{r^2} \qquad F_e = k_e\frac{q_1 q_2}{r^2}$$

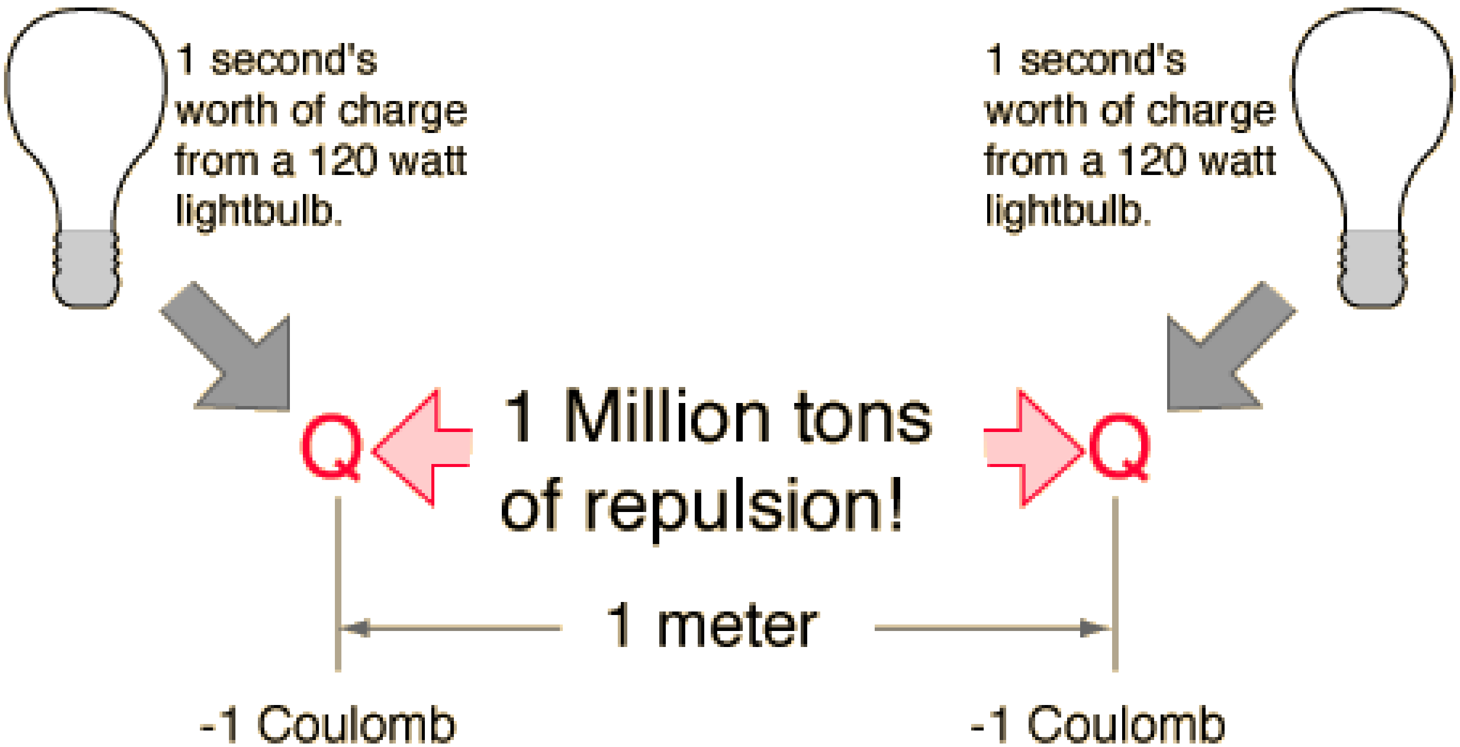
$$G = 7 \times 10^{-11}\ N\ kg^{-2}\ m^2 \qquad k_e = 9 \times 10^{9}\ N\ C^{-2}\ m^2$$

- What is the ratio between the gravitational force F_g and the electrostatic force F_e between two isolated electrons (mass $9 \times 10^{-31}\ kg$, charge $1.6 \times 10^{-19}\ C$) ?

- Does your answer depend on the distance between the electrons?

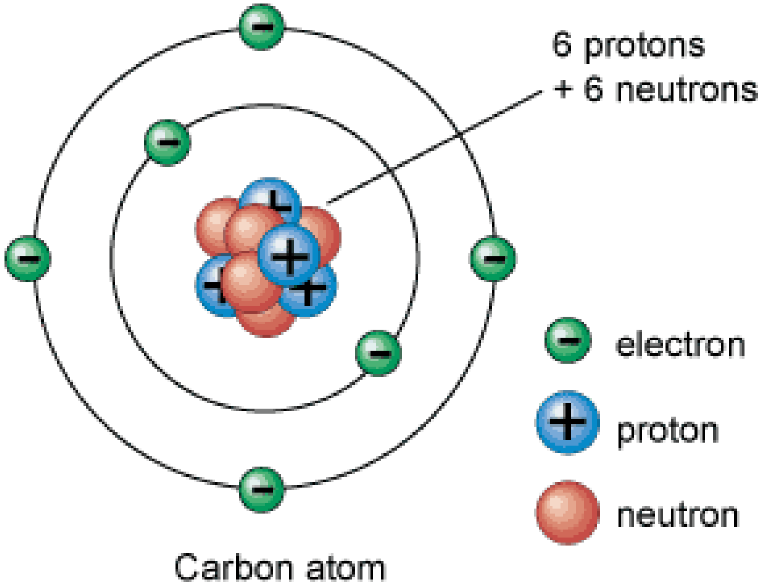
Importance of electromagnetism

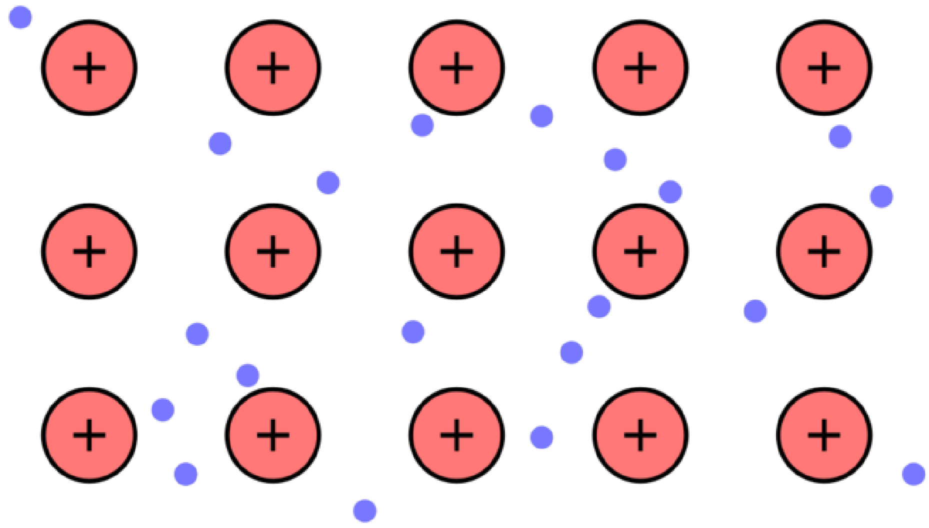
- Electromagnetism is **by far the strongest force** in our everyday experience



Importance of electromagnetism

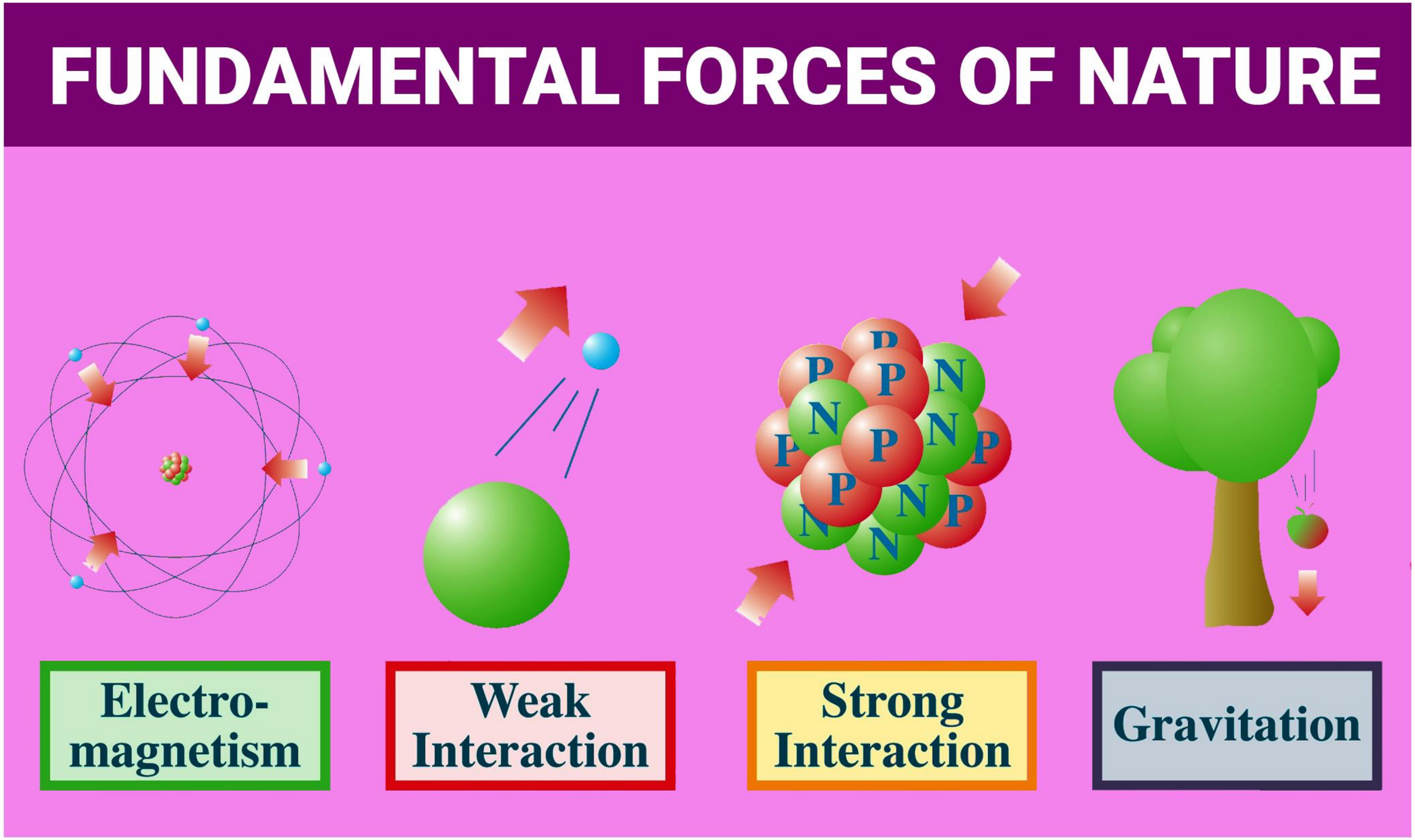
- Electrostatic forces are reduced in practice because matter is **approximately electrically neutral**





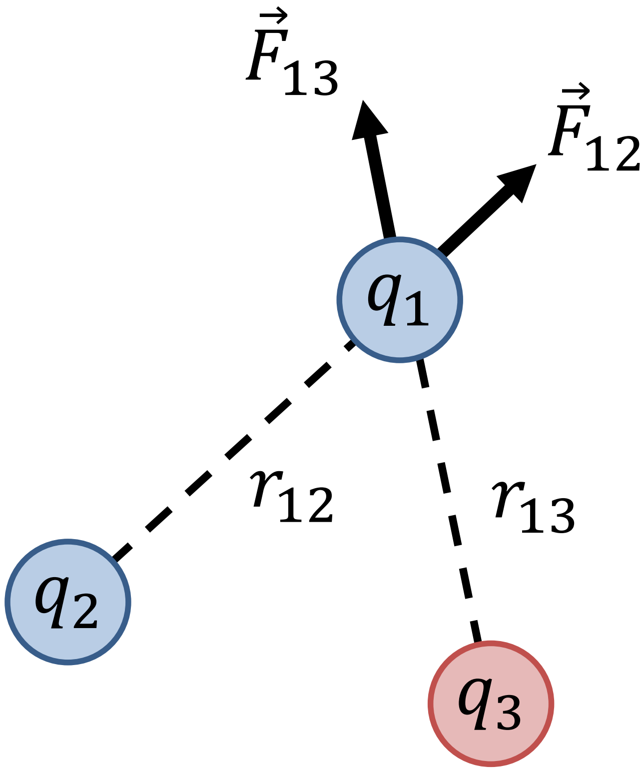
Importance of electromagnetism

- Other forces dominate on different scales …



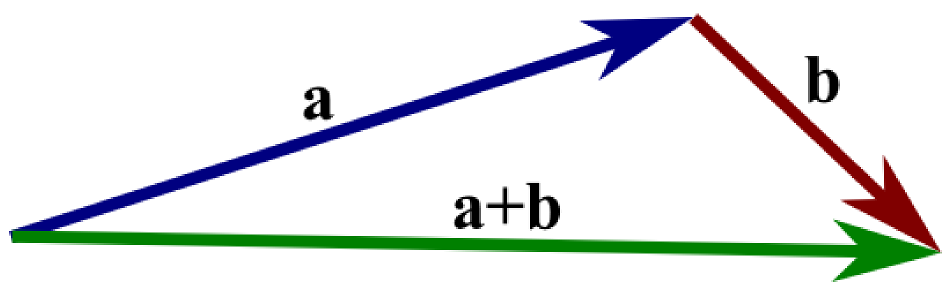
Principle of superposition

- The total force from multiple charges is given by the **principle of superposition**



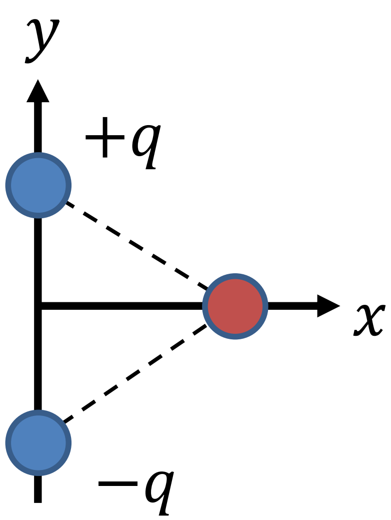
$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13}$$

$$= \frac{q_1 q_2}{4\pi\varepsilon_0 {r_{12}}^2}\hat{\vec{r}}_{12} + \frac{q_1 q_3}{4\pi\varepsilon_0 {r_{13}}^2}\hat{\vec{r}}_{13}$$



Clicker question

Two charges $+q$ and $-q$ are on the y-axis, symmetric about the origin. In what direction is the force on a positive charge on the x-axis?

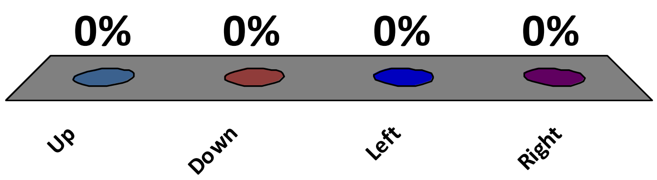


A. Up

B. Down

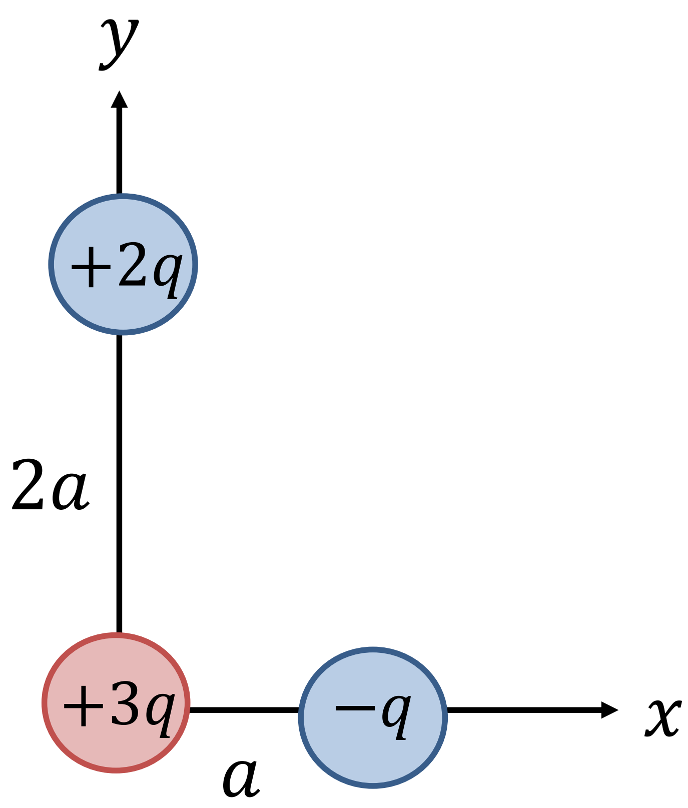
C. Left

D. Right



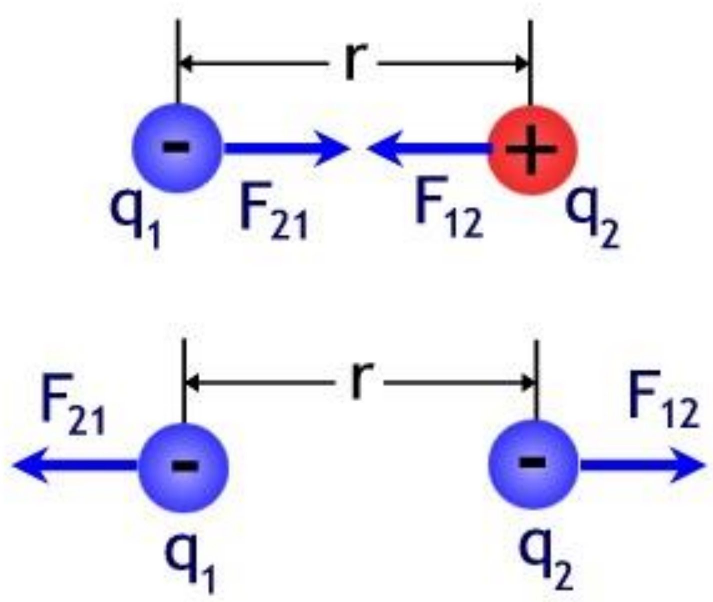
Principle of superposition

- What is the force on the charge at the origin?



Electric field

- Coulomb's Law tells us that *like charges repel and unlike charges attract.* **But what causes one charge to feel the effect of the other**?



- How is a force transmitted across space?
- Is a force transmitted instantaneously and, if so, does that violate special relativity?

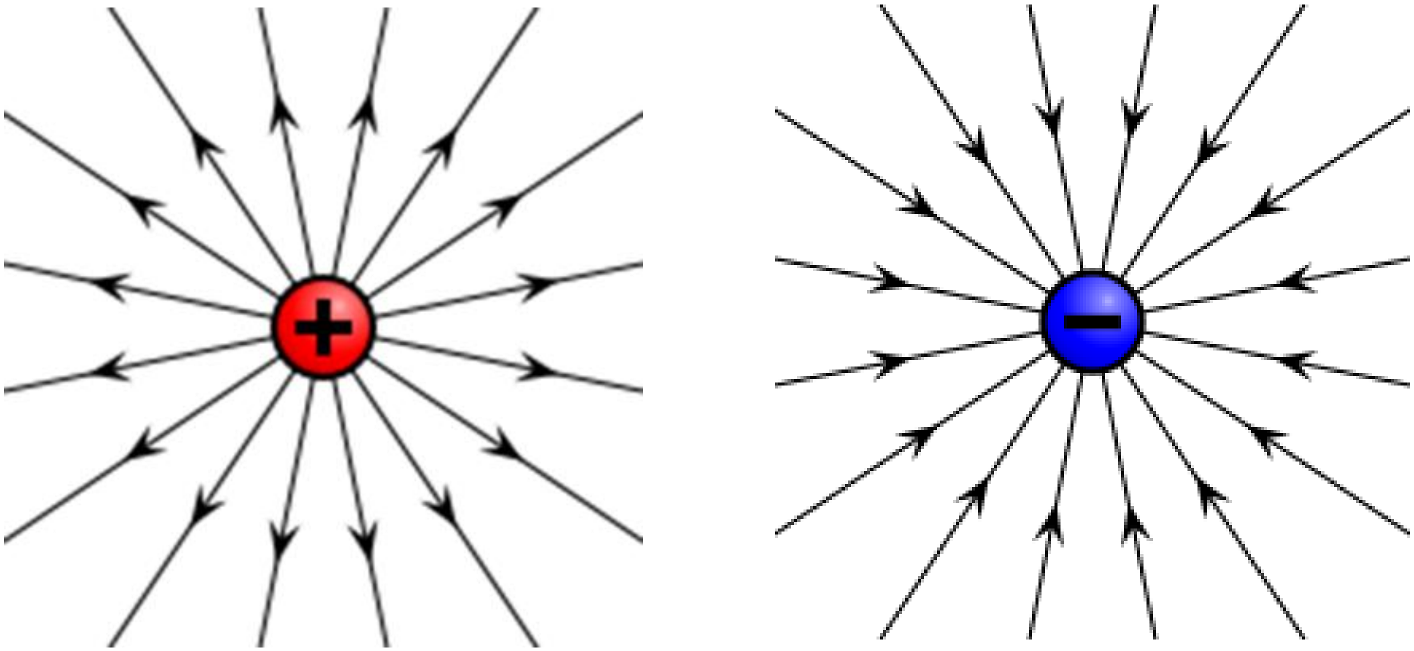
Electric field

- How is a force *communicated* between charges? A useful model is the **electric field**
- We interpret that *electric charges set up an electric field* $\vec{E}$ *in the region of space around them*
- Then, a "test charge" q, placed in the electric field, will feel a force $\vec{F} = q\vec{E}$
- **The electric field is a vector "force field" representing the size/direction of the force per unit charge**

Please note in workbook

Electric field

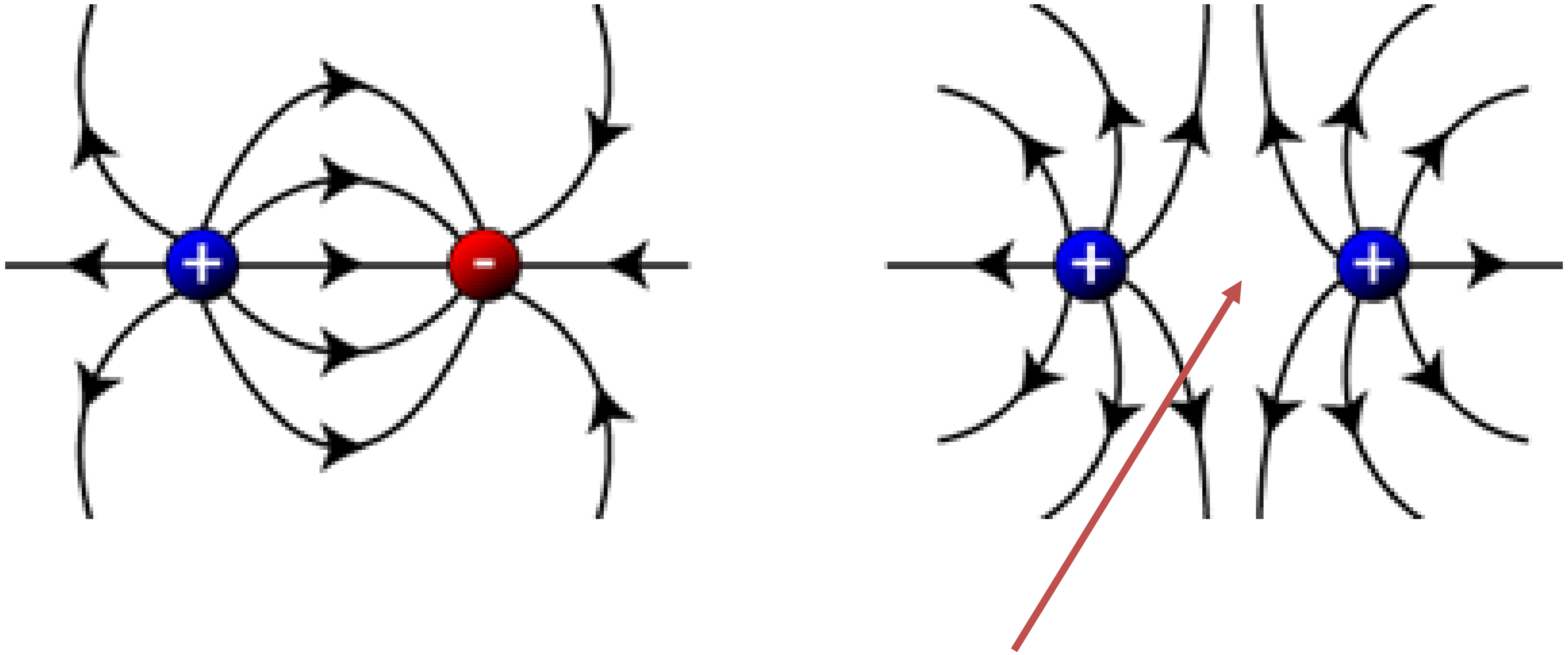
- The electric field can be represented by **field lines**:



- Electric field lines start on positive charges and end on negative charges. Their *direction* shows the force acting on a test charge; their *spacing* indicates the force strength. *They can never cross.*

Electric field

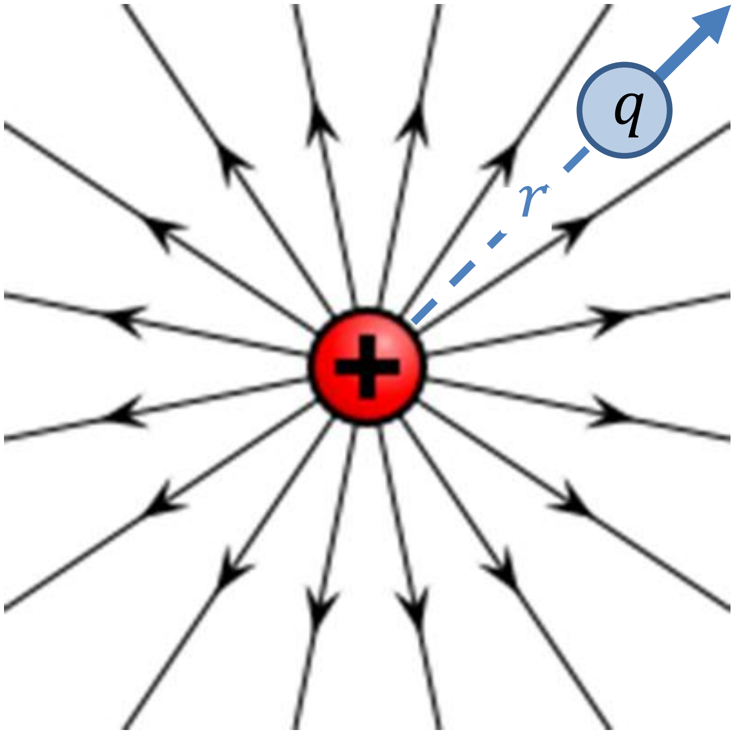
- A more complicated example of an electric field:



There are no electric field lines here, because
a test charge experiences zero net force

Electric field

- The **electric field around a point charge** $+\boldsymbol{Q}$ follows simply from Coulomb's Law

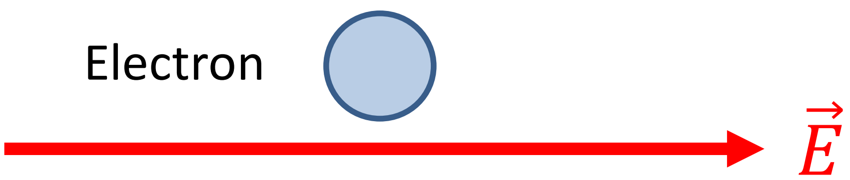


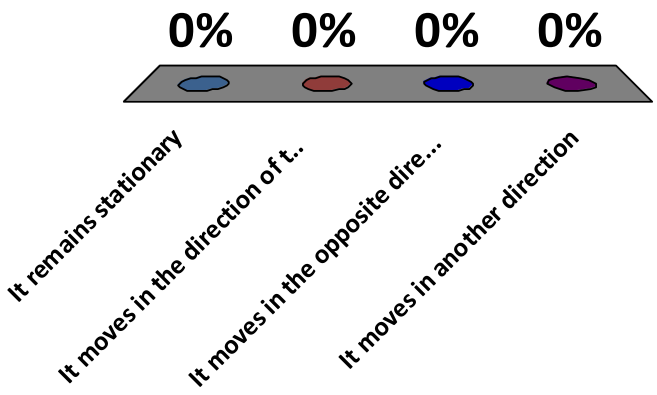
- Place a test charge $+q$ at distance r from $+Q$
- Force $\vec{F} = \frac{Qq}{4\pi\varepsilon_0 r^2}\vec{\hat{r}}$
- Electric field $\boldsymbol{\vec{E}} = \frac{\boldsymbol{\vec{F}}}{\boldsymbol{q}} = \frac{\boldsymbol{Q}}{\boldsymbol{4\pi\varepsilon_0 r^2}}\boldsymbol{\vec{\hat{r}}}$

Try example in workbook

Clicker question

An electron is placed in an electric field. In which direction does it move?

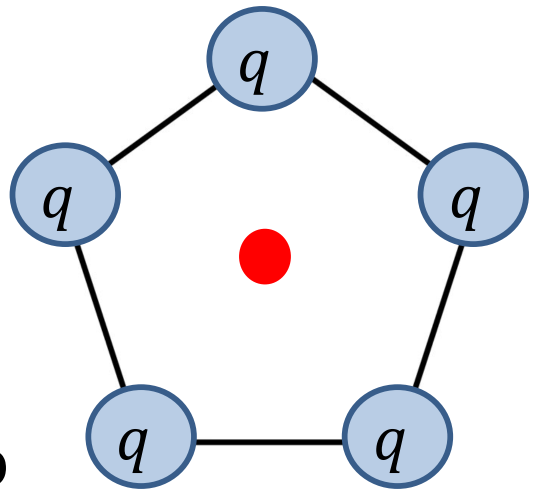




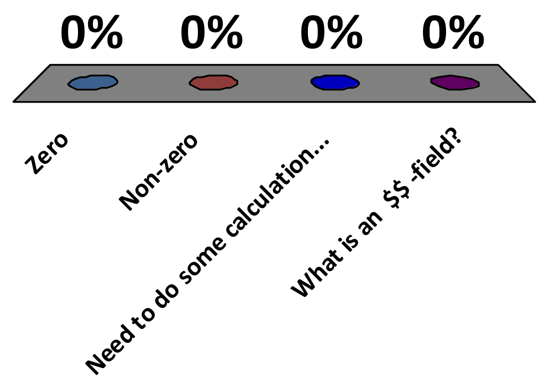
A. It remains stationary
B. It moves in the direction of the electric field lines
C. It moves in the opposite direction to the electric field lines
D. It moves in another direction

Clicker question

5 charges, q, are arranged in a regular pentagon. What is the $\vec{E}$-field at the centre?



A. Zero
B. Non-zero
C. Need to do some calculations to know
D. What is an $\vec{E}$-field?



Summary

- **Electric charge** is a fundamental property of nature
- Electric charges q_1, q_2 at a distance r attract/repel with a **Coulomb force** $\vec{F} = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}\vec{\hat{r}}$
- Forces from different charges may be combined using the **principle of vector superposition**
- Electric charges set up an **electric field** $\vec{E}$ in the region of space around them; a test charge q placed in the field feels a force $\vec{F} = q\vec{E}$