Class 13 : Induction

- Phenomenon of induction and Faraday's Law
- How does a generator and transformer work?
- Self- and mutual inductance
- Energy stored in $\vec{B}$-field

Recap (1)

- Maxwell's Equations describe the electric field $\vec{E}$ and magnetic field $\vec{B}$ generated by **stationary** charge density ρ and current density $\vec{J}$:

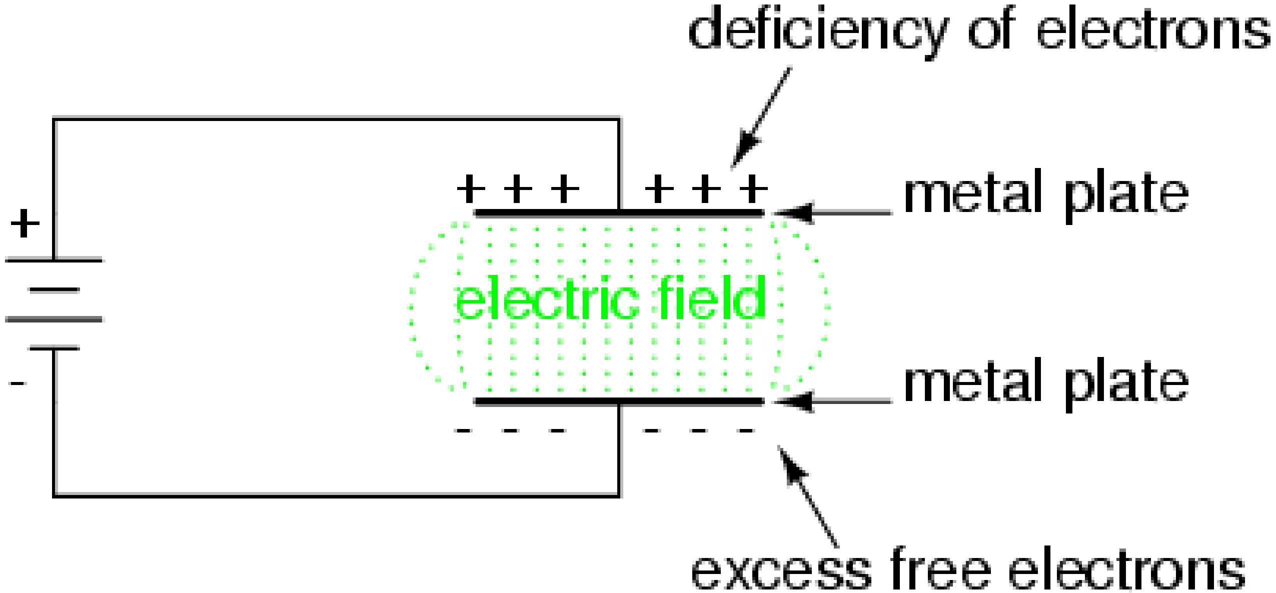
$$\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0} \qquad \vec{\nabla} \times \vec{E} = \vec{0}$$

$$\vec{\nabla}.\vec{B} = 0 \qquad \vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

Recap (2)

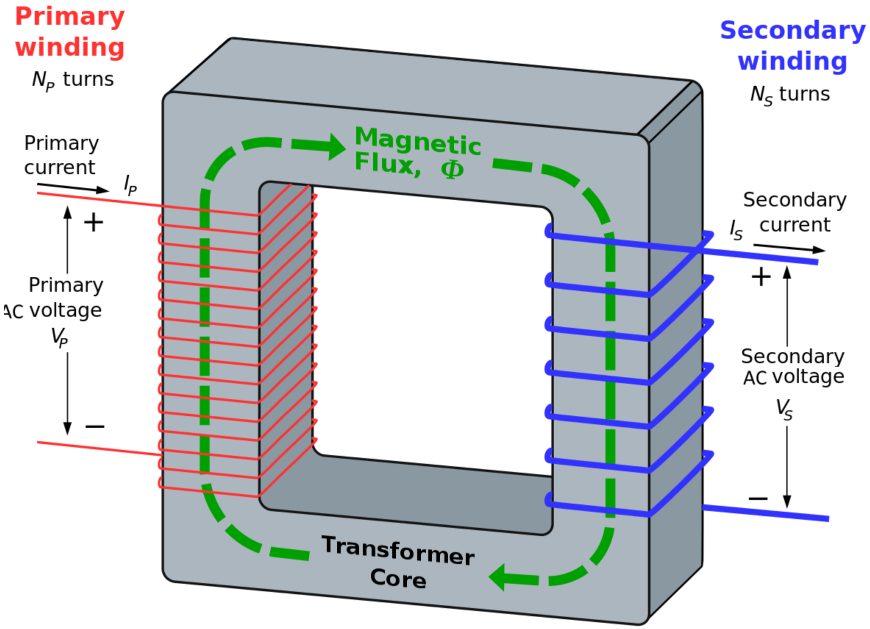
- It requires work to assemble a distribution of electric charges against the electric forces

- This work creates **potential energy** which we can think of as stored in the electric field $\vec{E}$ with **density** $\frac{1}{2}\varepsilon_0 E^2$



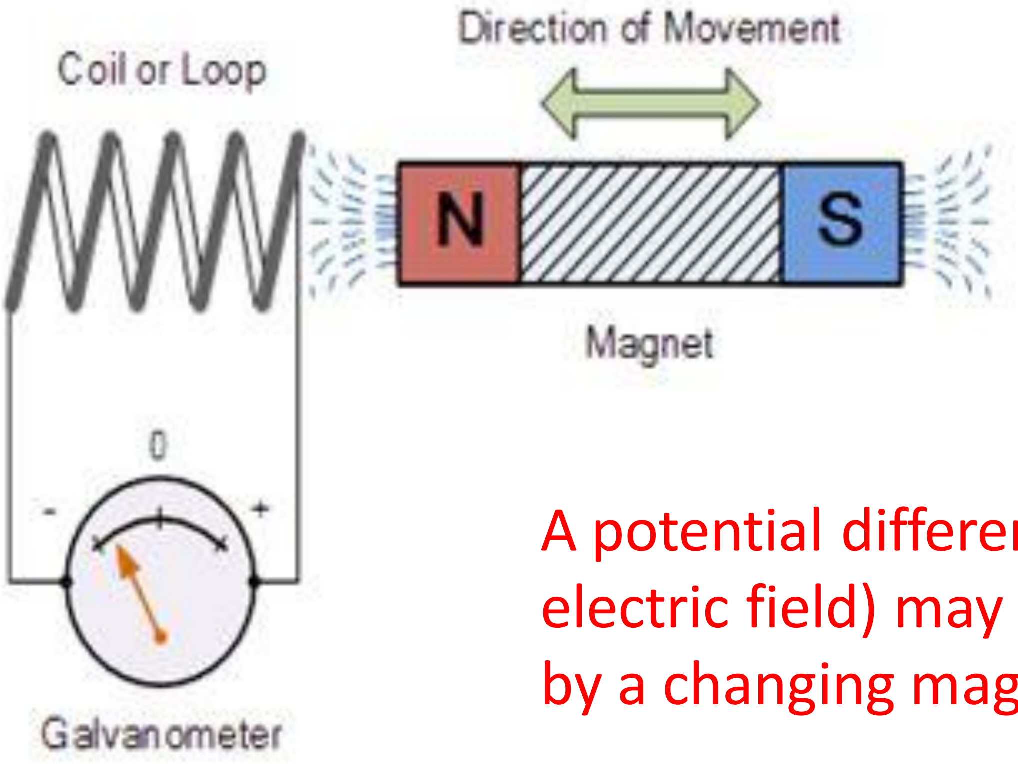
Time-varying fields

- New and important phenomena and applications are produced when the electric or magnetic fields are not stationary, but **time-varying**



Faraday's Law

- Faraday's law says that **an electric current is set up if the magnetic field through a circuit is changed**



A potential difference (i.e. an electric field) may be produced by a changing magnetic field!

Faraday's Law

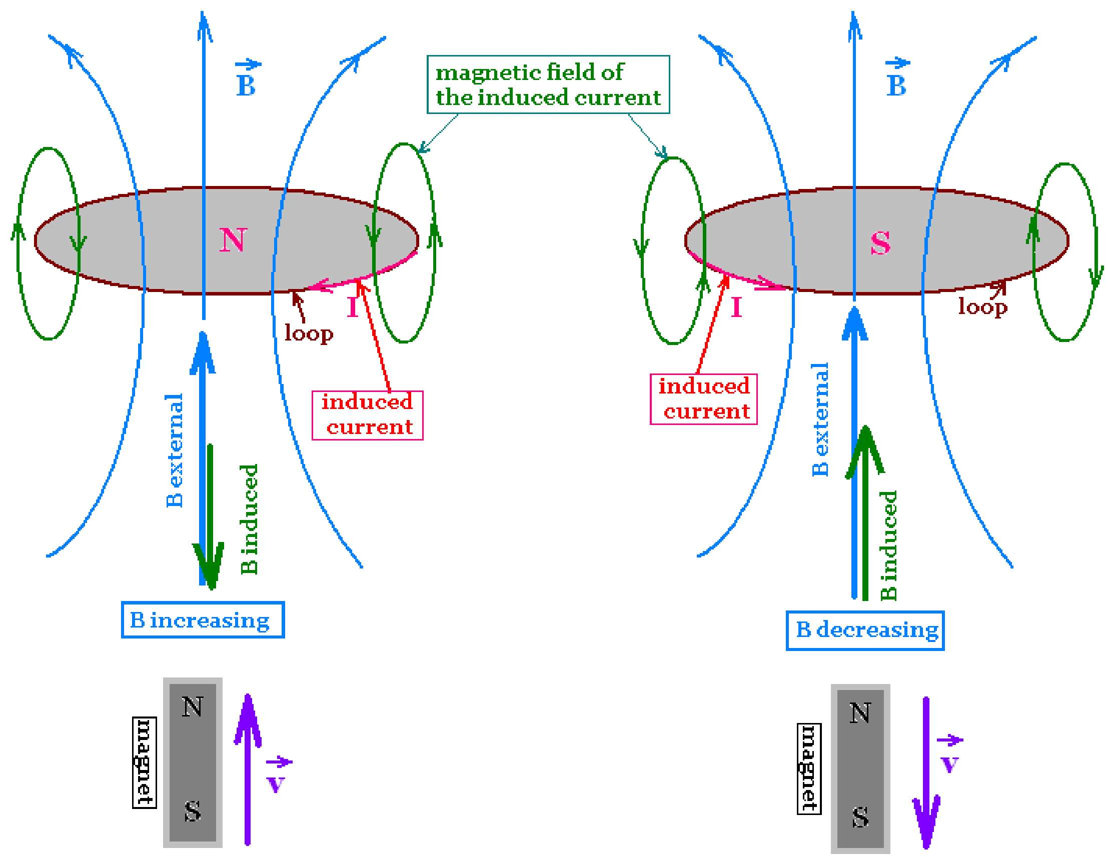
- In mathematical terms, Faraday's Law can be written in the form $\boldsymbol{V = -\frac{\partial \Phi}{\partial t}}$
- V is the potential difference (or voltage, or **electromotive force**) created around the circuit
- Φ is the magnetic flux through the circuit, given by $\Phi = \int \vec{B}.d\vec{A}$ (where the integral is over the area of the circuit)
- The minus sign indicates that the voltage causes a current to flow which **opposes the change** (this bit is known as **Lenz's Law**)

Please note in workbook

Faraday's Law

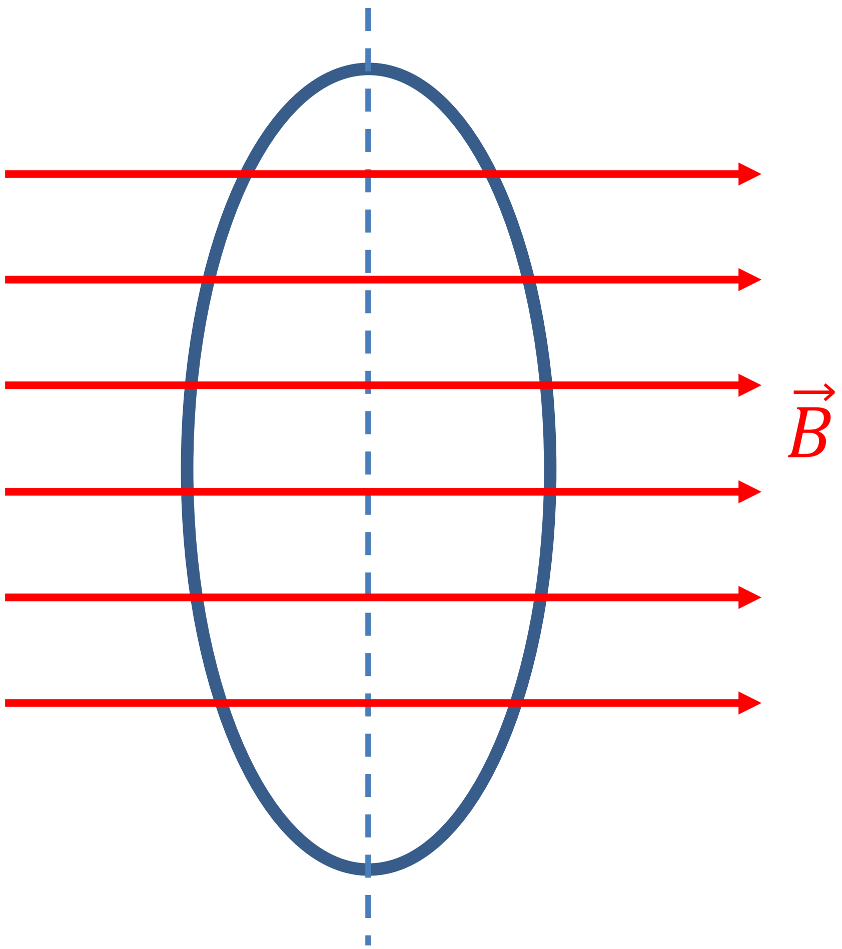
- The fact that the **induced voltage opposes the change that produced it** is needed to satisfy conservation of energy

Examples bringing a magnet towards and away from a coil of wire:



Faraday's Law

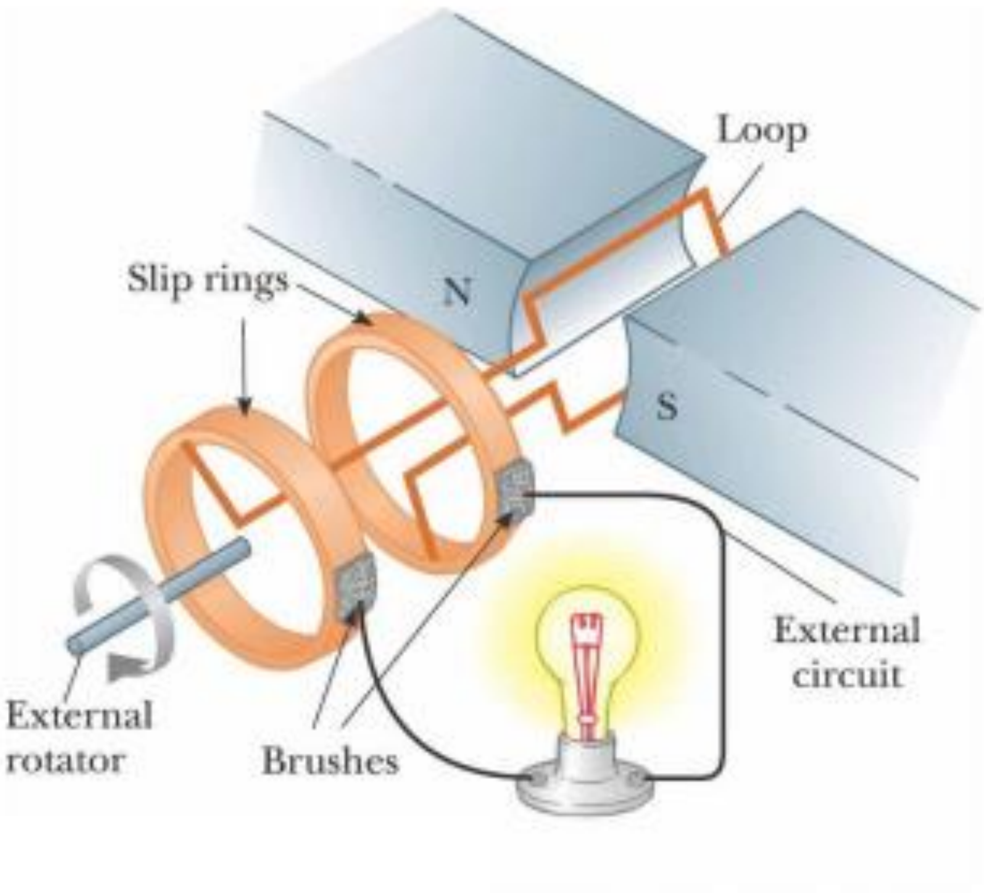
- An example is provided by a **simple generator**



- Consider a conducting loop being rotated about its axis with angular frequency ω in a uniform B-field
- How does the magnetic flux Φ threading the loop vary with time?

Faraday's Law

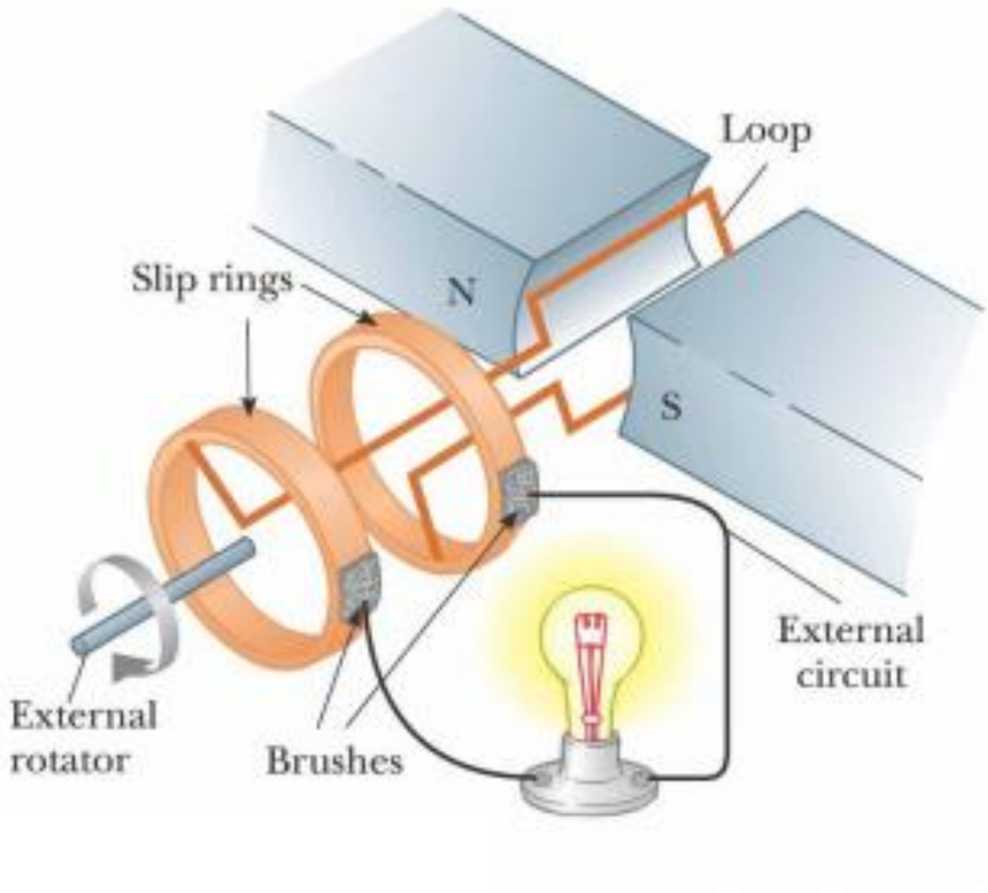
- An example is provided by a **simple generator**



- Why does this work?
- As the loop rotates, the *area of the loop threaded by the magnetic field changes*, hence so does the flux $\Phi = B.A$
- This creates a voltage in the circuit $V = -\partial\Phi/\partial t$, which powers the lamp

Faraday's Law

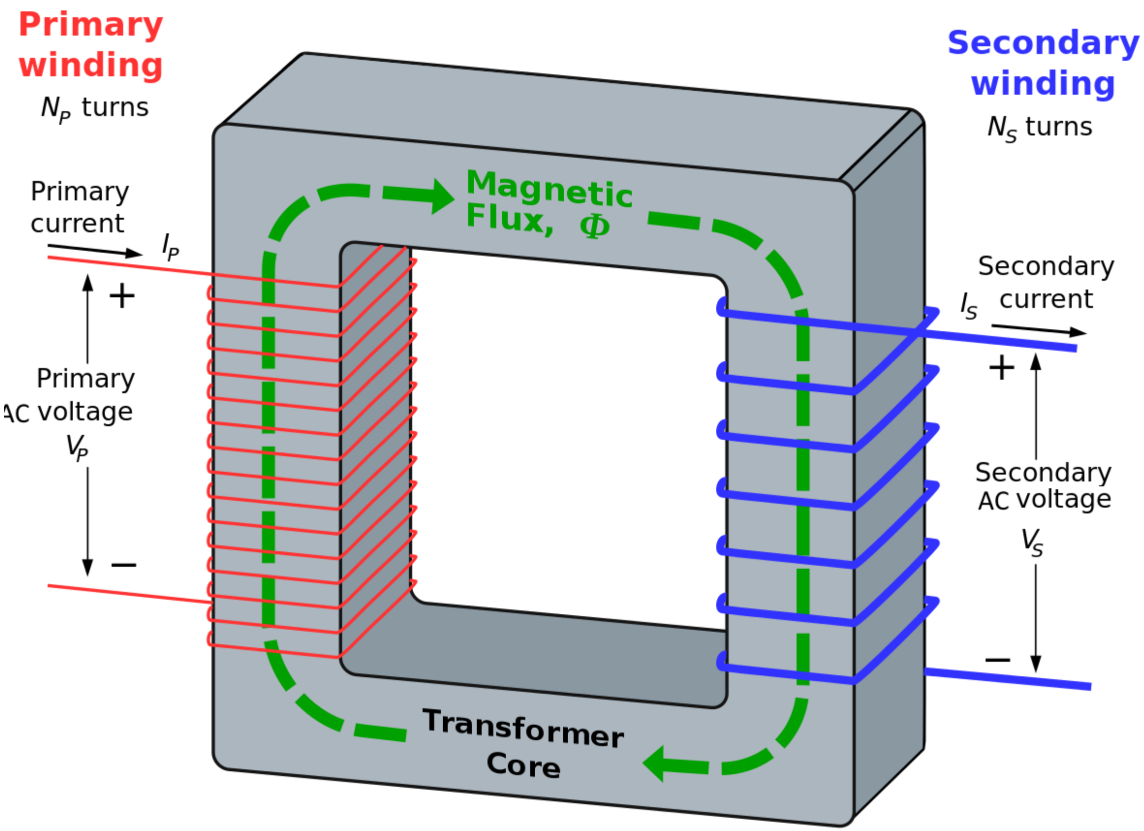
- An example is provided by a **simple generator**



- If the lamp has resistance R, what is the average power dissipated?

Faraday’s Law

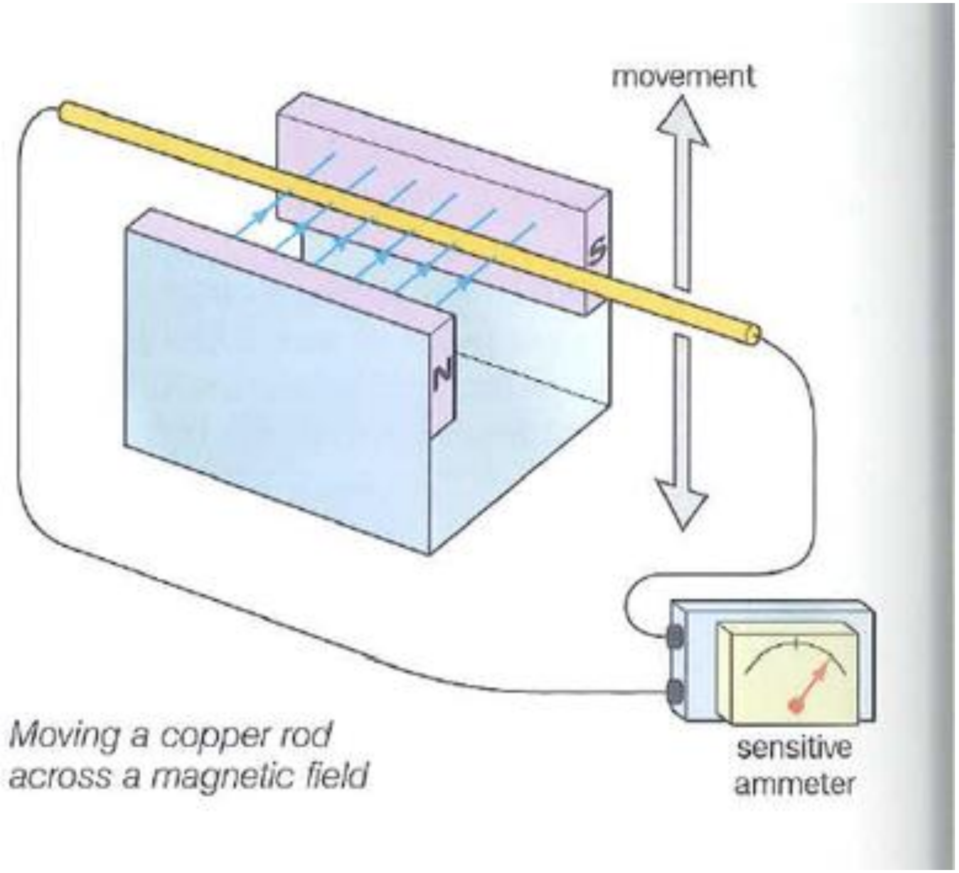
- Faraday’s Law is very useful for stepping a voltage up or down via an **electrical transformer**



Why does this work?

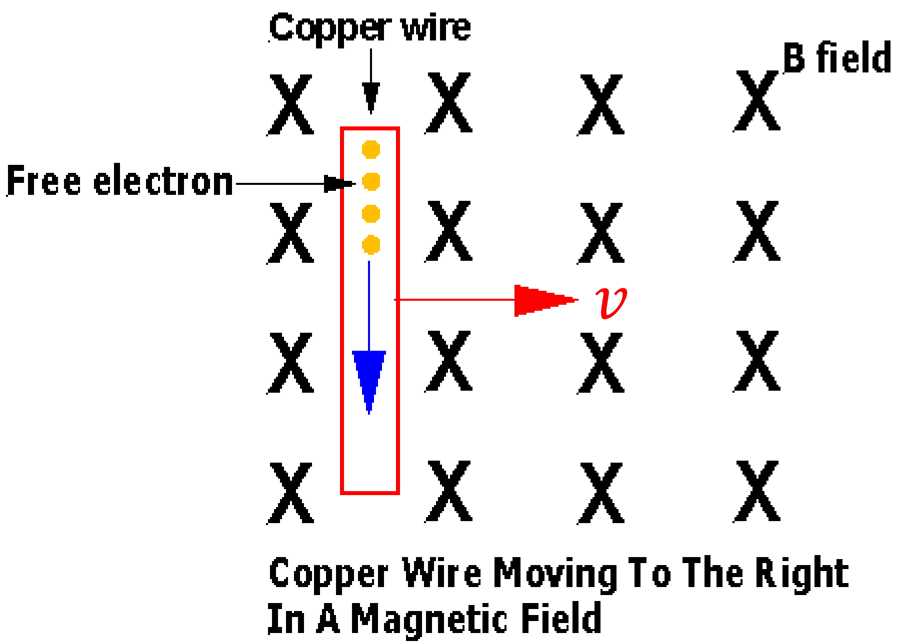
Faraday's Law

- Faraday's Law also applies if **a wire is moving through a magnetic field**



Faraday's Law

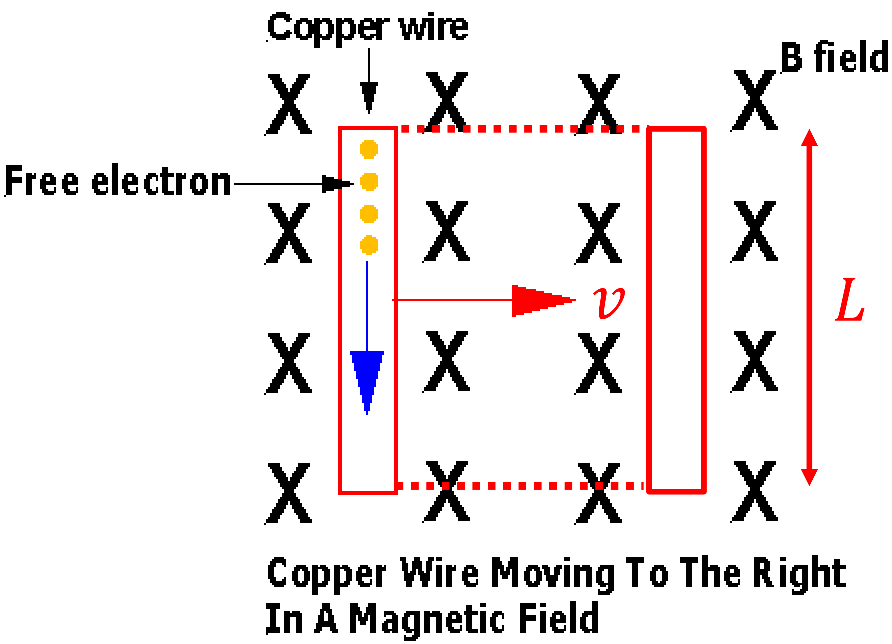
- A voltage is induced between the ends of the wire which is **equal to the rate of cutting of magnetic flux,** $V = -\frac{\partial \Phi}{\partial t}$



- The free electrons in the wire feel a force $F_B = qvB$
- This force causes them to move along the wire as if there was an applied voltage V
- The electrons separate until they set up an electric field E which exerts a force $F_E = qE$ which cancels F_B

Faraday's Law

- A voltage is induced between the ends of the wire which is **equal to the rate of cutting of magnetic flux,** $V = -\frac{\partial \Phi}{\partial t}$

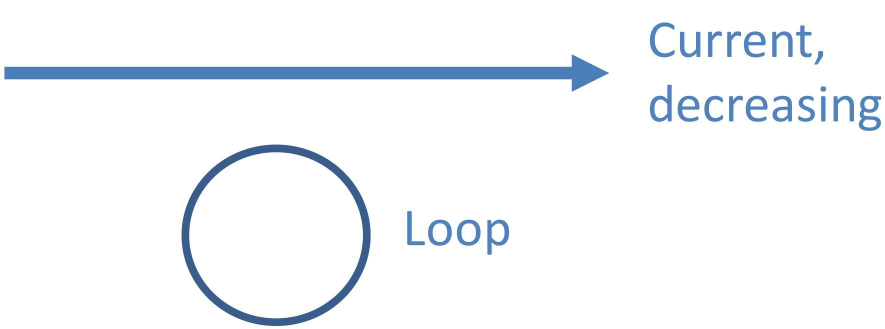


- If the wire has length L, then the magnetic flux cut in time t is contained in an area Lvt. Hence, $\Phi = B.A = BLvt$

- The voltage induced is hence $V = -\frac{\partial \Phi}{\partial t} = BLv$

Clicker question

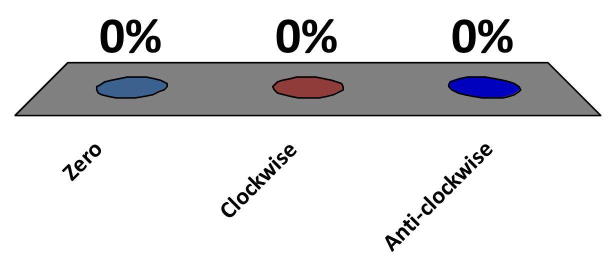
A loop of wire is near a long straight wire which is *carrying a current which is decreasing*. The current induced in the loop is …



A. Zero

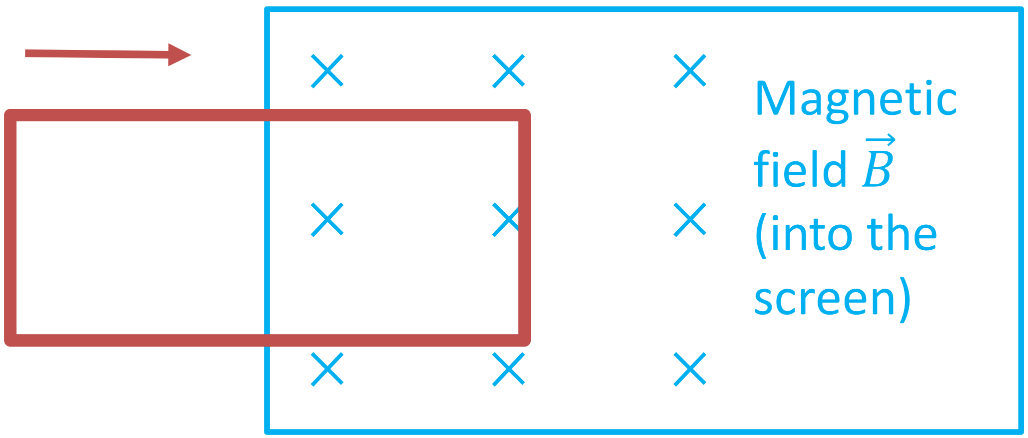
B. Clockwise

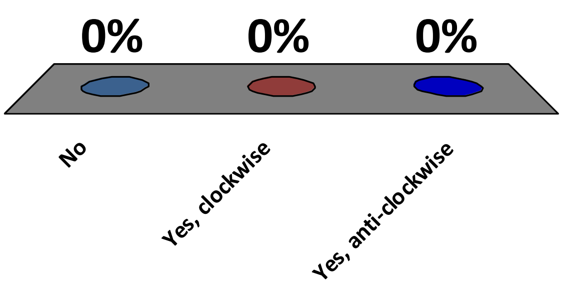
C. Anti-clockwise



Clicker question

One end of a rectangular metal loop enters a region of uniform magnetic field (into the screen) with constant speed. Will current flow in the loop?

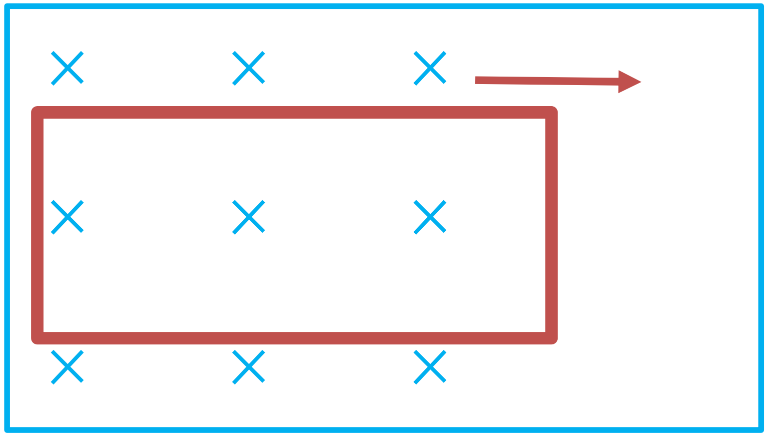


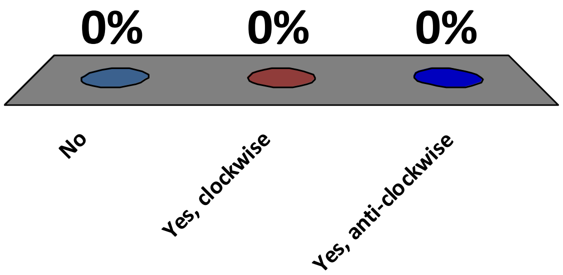


A. No
B. Yes, clockwise
C. Yes, anti-clockwise

Clicker question

At a slightly later time, is current flowing in the loop?





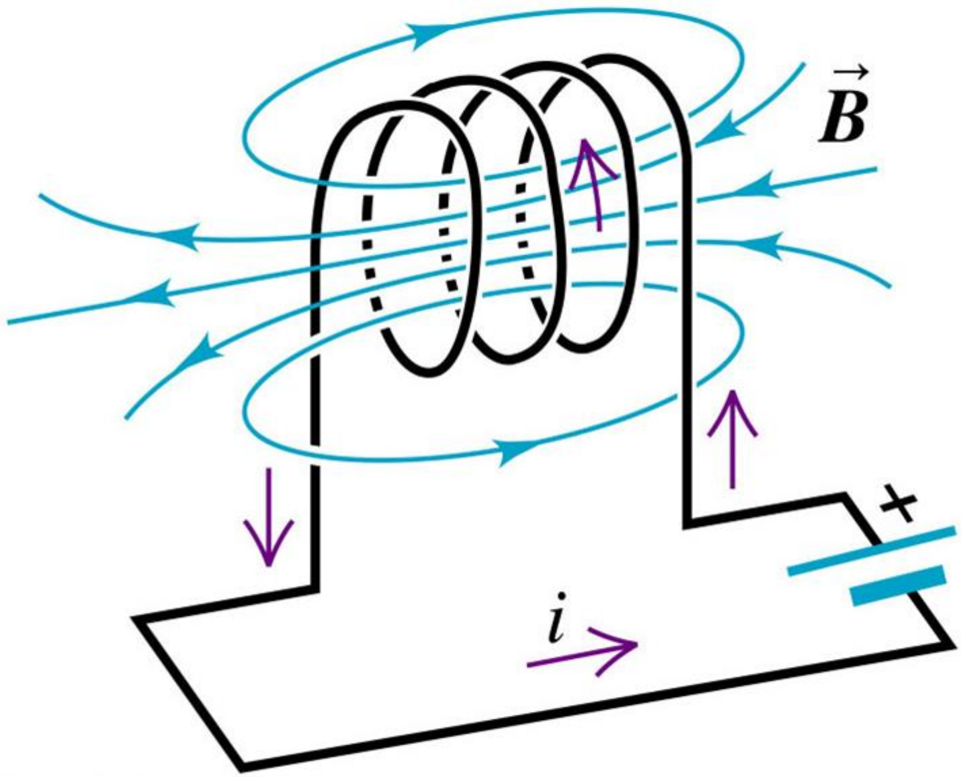
A. No

B. Yes, clockwise

C. Yes, anti-clockwise

Inductance

- *Current flowing in a circuit produces a magnetic field which threads the circuit itself!*

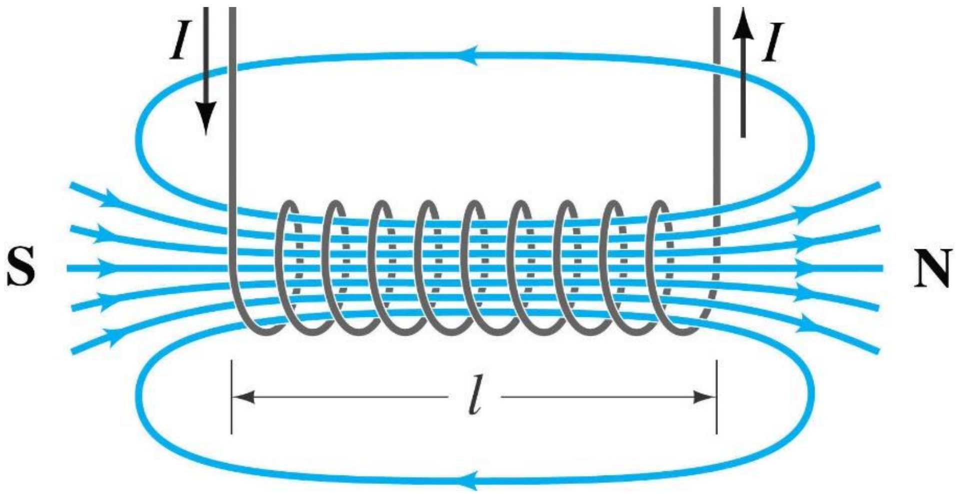


- By Faraday's Law, this will **cause an extra voltage to be induced in the circuit if the current is changed**

- This kind of circuit element is known as an *inductor*

Inductance

- If current I flows in a circuit, inducing a magnetic field which threads flux Φ through the circuit, then the **self-inductance** L of the circuit is defined by $\boldsymbol{\Phi = L\, I}$

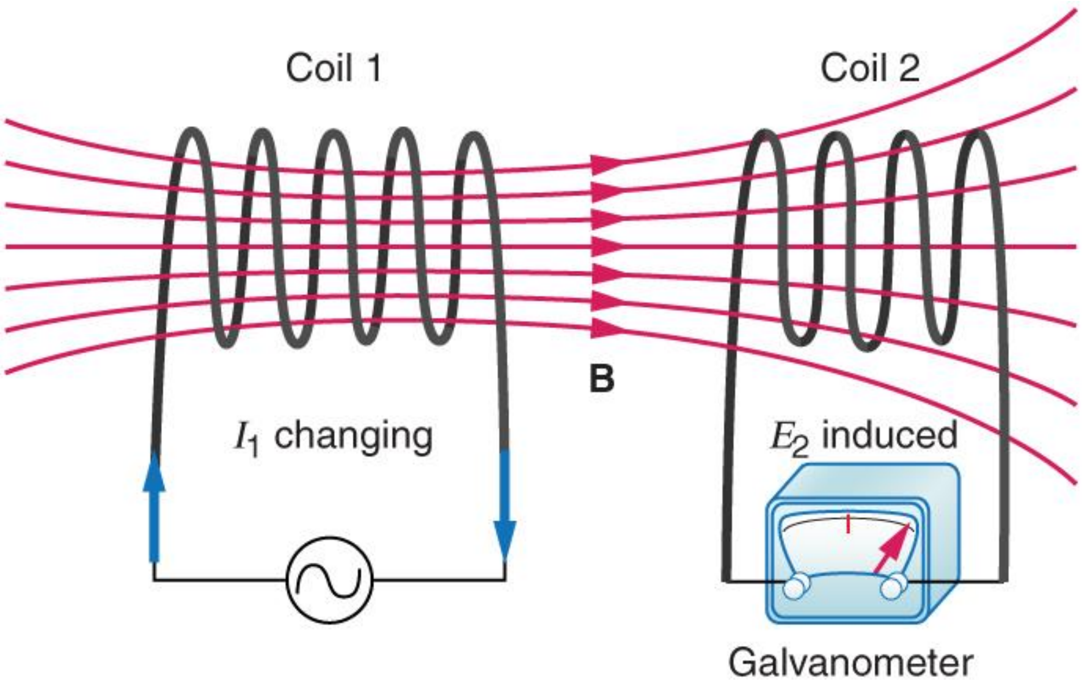


- Consider a solenoid of length l, consisting of N turns of cross-sectional area A
- From previous lectures, the magnetic field is $B = \frac{\mu_o NI}{l}$
- The magnetic flux is $\Phi = NBA$ (note: as it threads N loops)
- The self-inductance is hence $\boldsymbol{L = \frac{\Phi}{I} = \frac{\mu_o\, N^2\, A}{l}}$

Inductance is measured in "Henrys" (H) – honestly!

Inductance

- Similarly, if current I_1 flows in one circuit, and produces a magnetic field which causes magnetic flux Φ_2 to thread a second circuit, then the **mutual inductance** $M = \Phi_2 / I_1$



Energy stored in $\vec{B}$-field

- In electrostatics, we can think of the work done in assembling a configuration of charges as *stored in the electric field* $\vec{E}$ **with energy density** $\boldsymbol{\frac{1}{2}\varepsilon_0 E^2}$

- Work is also done to set up a current in a circuit, to *drive that current against the induced voltage* $V = -\frac{d\Phi}{dt} = -L\frac{dI}{dt}$

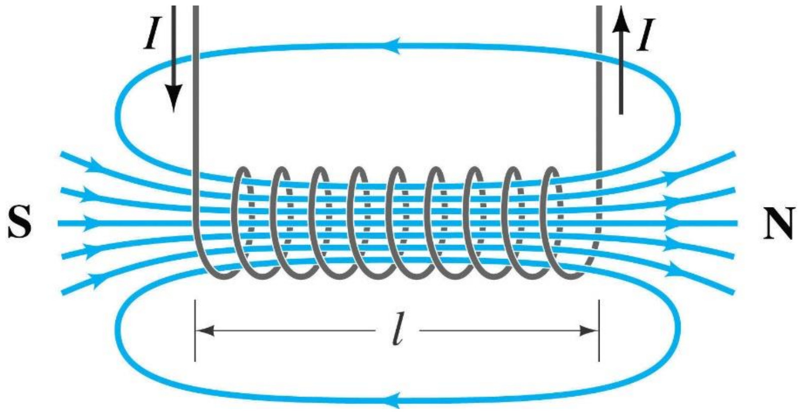
- The work done in a small time interval dt is given by $dW = -V\,dq = -V\,I\,dt = L\,I\,dI$

- Integrating this work between current $I = 0$ and $I = I_{final}$, we find that the total work is $\boldsymbol{W = \int_0^{I_{final}} LI\,dI = \frac{1}{2}LI_{final}{}^2}$

Energy stored in $\vec{B}$-field

- We can think of this work **being transformed to potential energy which is stored in the magnetic field**

- For a coil, substituting $B = \frac{\mu_0 N I}{l}$ and $L = \frac{\mu_0 N^2 A}{l}$ into the work done $W = \frac{1}{2} L I^2$, we find $W = \frac{B^2}{2\mu_0} \times A\, l$

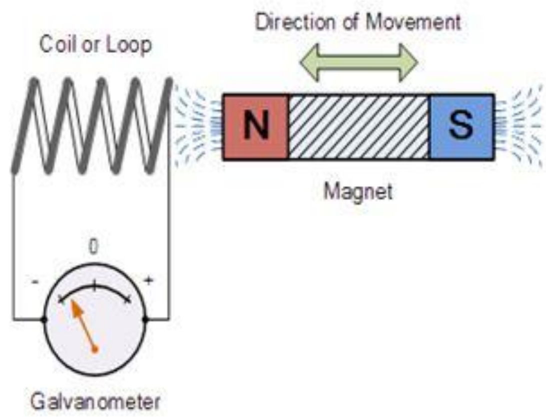


Volume of coil $= A\, l$

- We can think of **storing energy in a $\vec{B}$-field with density $\frac{B^2}{2\mu_0}$**

Summary

- A changing magnetic flux Φ through a circuit induces a voltage $V = -\frac{d\Phi}{dt}$ (**Faraday's Law**), which opposes the change which produced it (**Lenz's Law**)



- The **inductance** L of a circuit relates the flux to the current I flowing : $\Phi = L\, I$

- Work is required to set up a current in a circuit; this can be considered stored in the magnetic field $\vec{B}$ with density $\frac{B^2}{2\mu_0}$