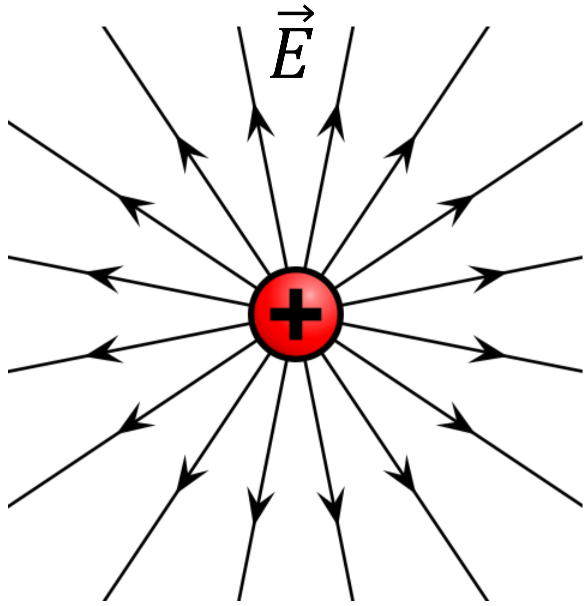
Class 12 : Electromagnetic Forces

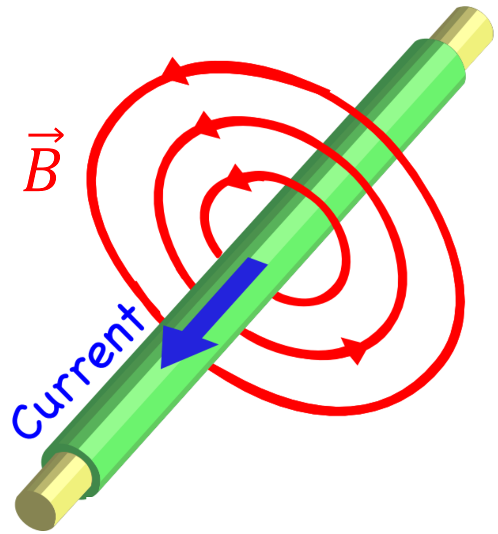
- Forces felt by charges and currents in $\vec{E}$- and $\vec{B}$-fields
- Motion of particle in a $\vec{B}$-field
- Applications

Recap

- Charges set up an *electric field* $\vec{E}$, in which other charges feel an **electric force**



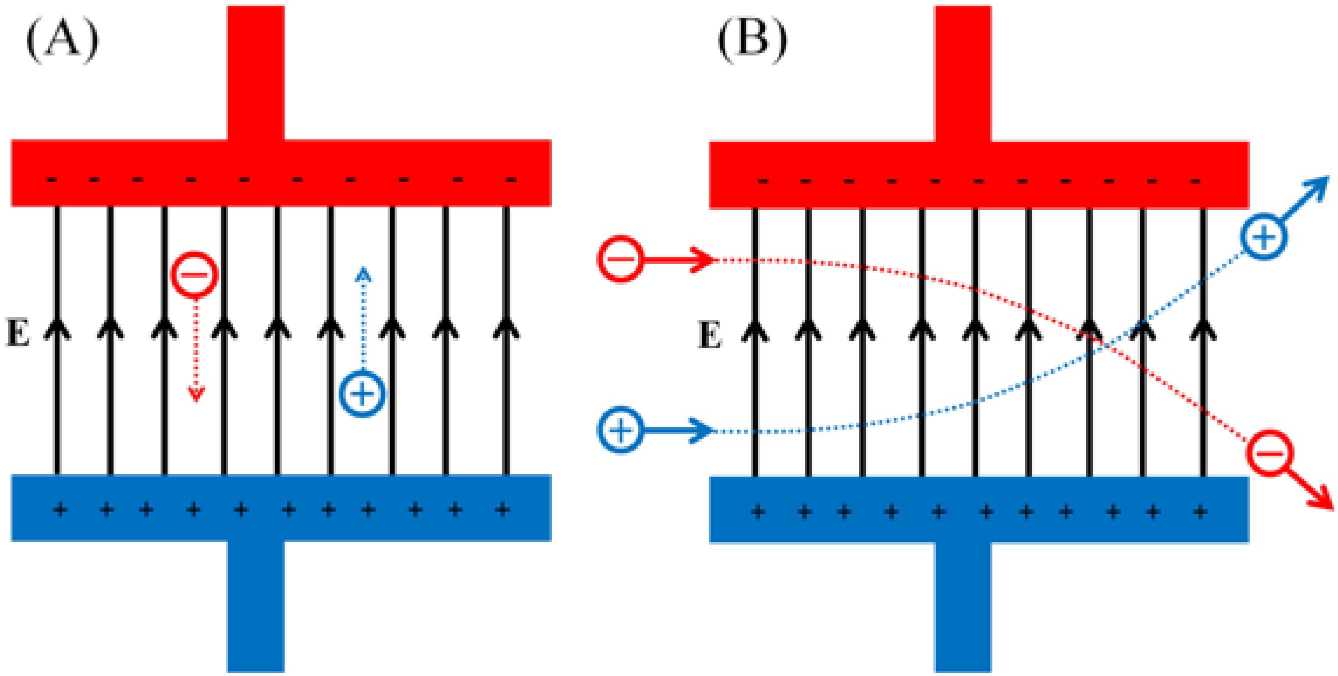
- Currents (or magnets) set up a *magnetic field* $\vec{B}$, in which other currents (or magnets) feel a **magnetic force**



- In this lecture we will examine these *electromagnetic forces* in more detail

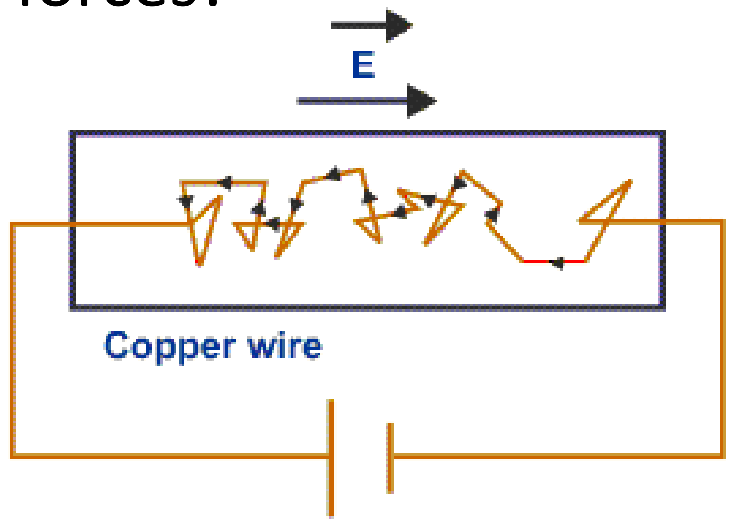
Forces felt in $\vec{E}$- and $\vec{B}$-fields

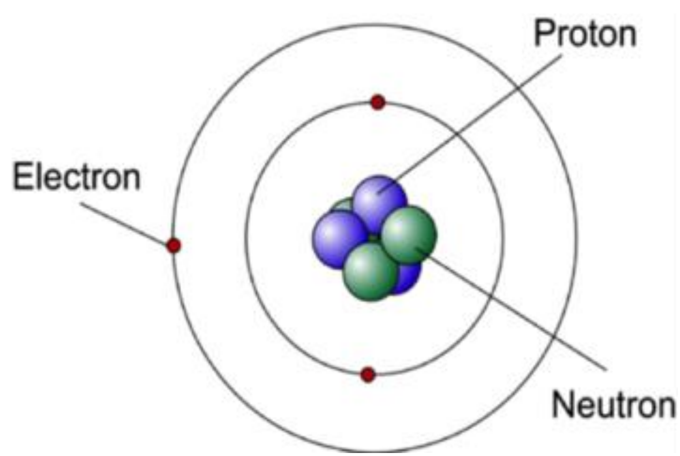
- A charge q in electric field $\vec{E}$ feels force $\vec{F}_E = q\vec{E}$
- This causes *acceleration parallel to the* $\vec{E}$*-field*



Forces felt in $\vec{E}$- and $\vec{B}$-fields

- Other examples of electric forces!





Forces felt in $\vec{E}$- and $\vec{B}$-fields

- A **current $\boldsymbol{I}$** or a **charge $\boldsymbol{q}$ moving with velocity $\vec{\boldsymbol{v}}$** in a magnetic field $\vec{B}$ feels a force *perpendicular to both the direction of current/motion and* $\vec{B}$

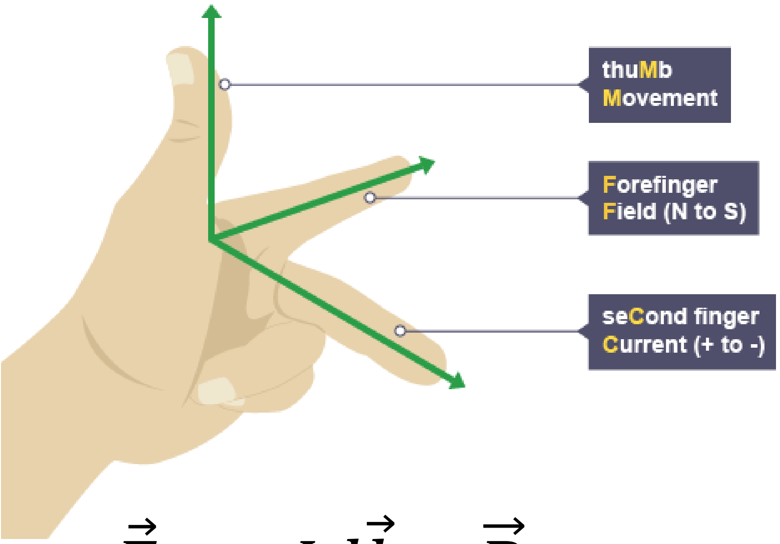
- Force on a current element : $\vec{F}_B = I\, d\vec{l} \times \vec{B}$

- Force on a charge : $\vec{F}_B = q\vec{v} \times \vec{B}$

- (Recall, the cross product of two vectors is perpendicular to both)

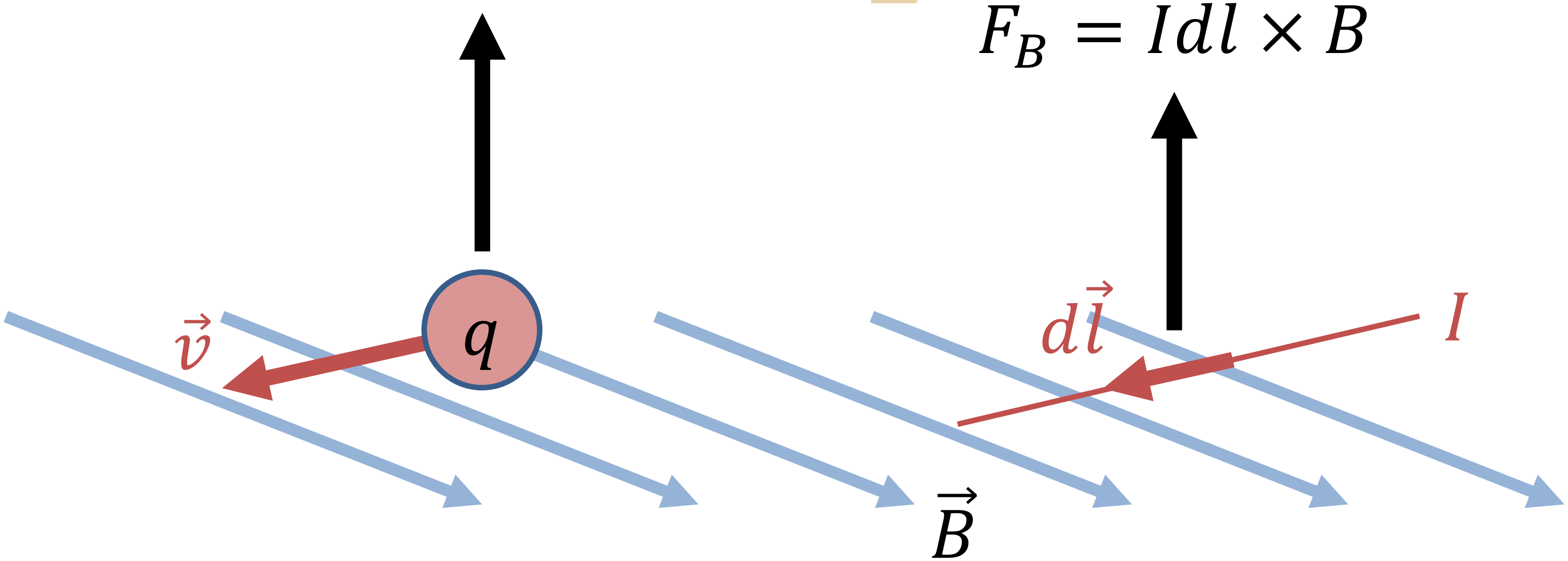
Forces felt in $\vec{E}$- and $\vec{B}$-fields

- Direction of the force from "Fleming's left-hand rule"



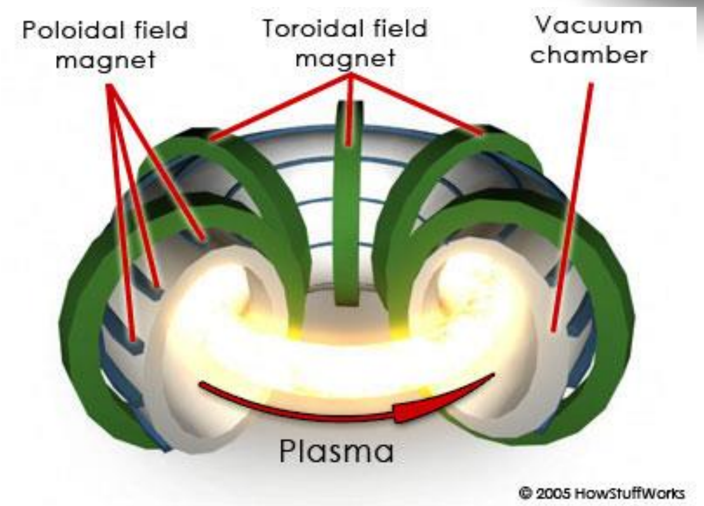
$$\vec{F}_B = q\vec{v} \times \vec{B}$$

$$\vec{F}_B = I d\vec{l} \times \vec{B}$$



Forces felt in $\vec{E}$- and $\vec{B}$-fields

- Other examples of magnetic forces!

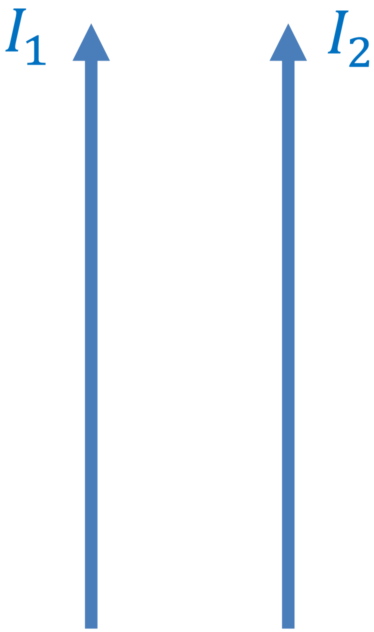


Forces felt in $\vec{E}$- and $\vec{B}$-fields

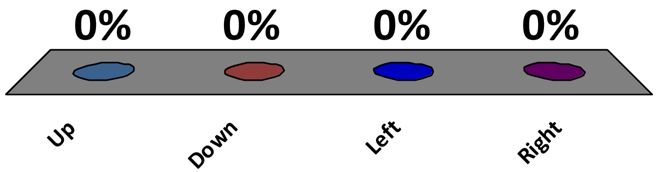
- What is the force $\vec{F}$ on a charge $q = 1\ \mu C$ moving with velocity $\vec{v} = (1,2,3) \times 10^5\ km/s$ in a region with electric field $\vec{E} = (0,100,200)\ V/mm$ and magnetic field $\vec{B} = (0.5,0,0)\ T$?

Clicker question

Two long parallel wires carry currents I_1 and I_2 in the same direction. In which direction is the force on I_2?



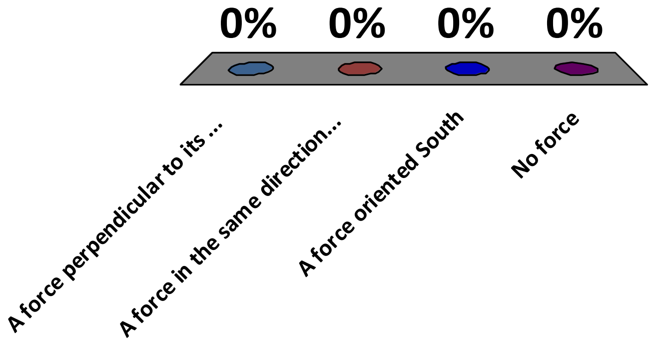
A. Up
B. Down
C. Left
D. Right



Clicker question

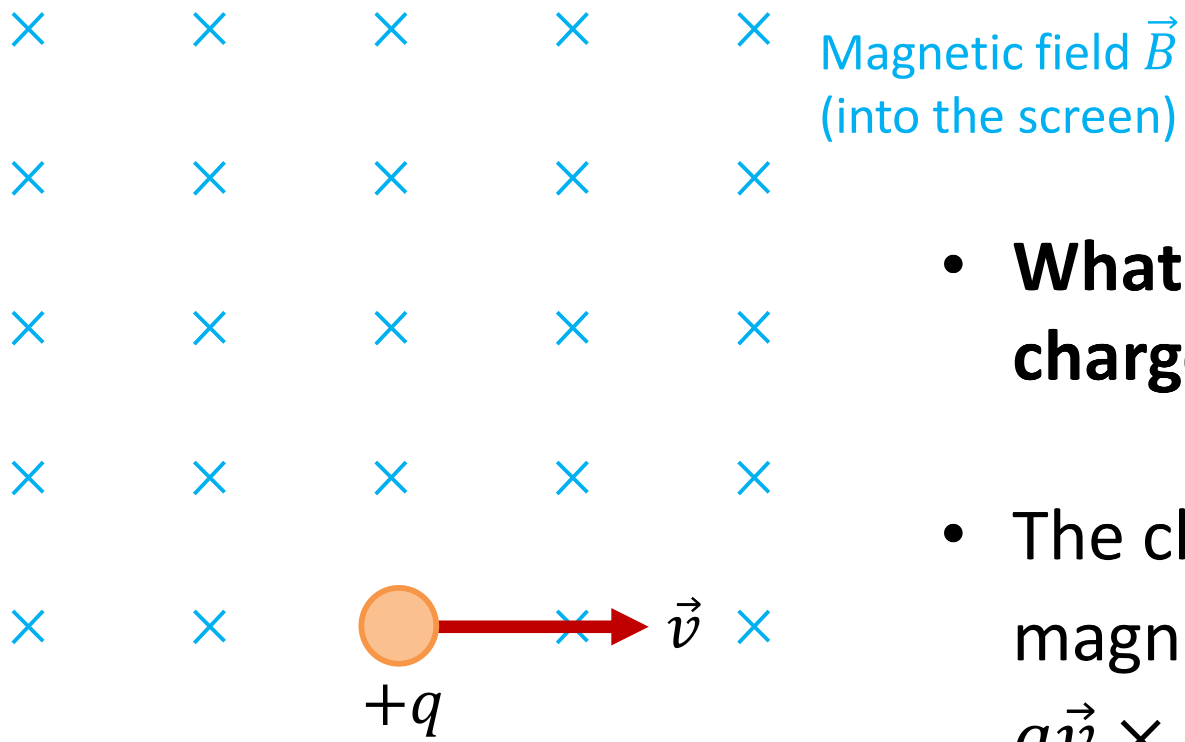
An electron moving West through a uniform magnetic field pointing East experiences:

A. A force perpendicular to its velocity
B. A force in the same direction as its velocity
C. A force oriented South
D. No force



Motion of particle in a $\vec{B}$-field

- Consider a charge $+q$ injected into a magnetic field B (into the screen) with velocity v (to the right)

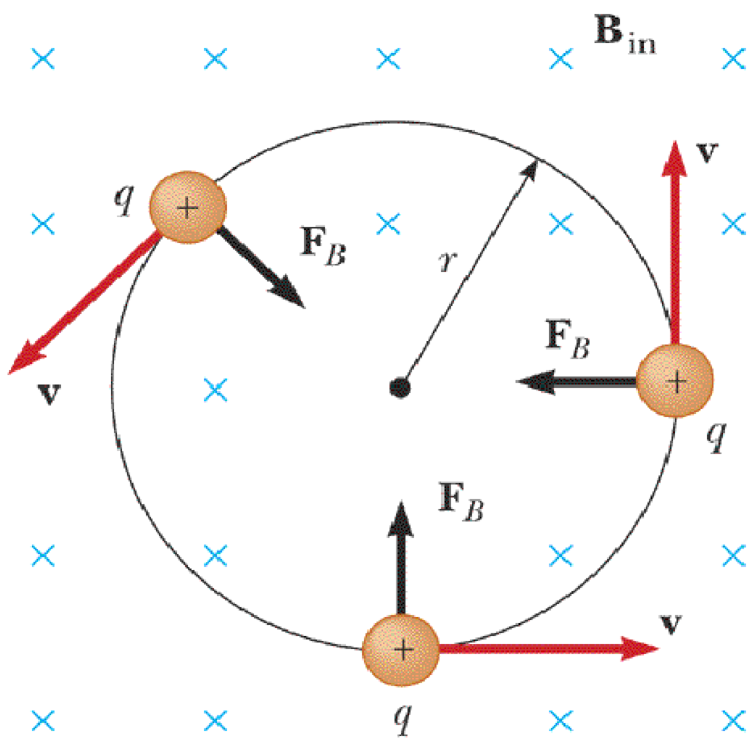


- **What path will the charge follow?**

- The charge will feel a magnetic force $\vec{F}_B = q\vec{v} \times \vec{B}$ (i.e., upward)

Motion of particle in a $\vec{B}$-field

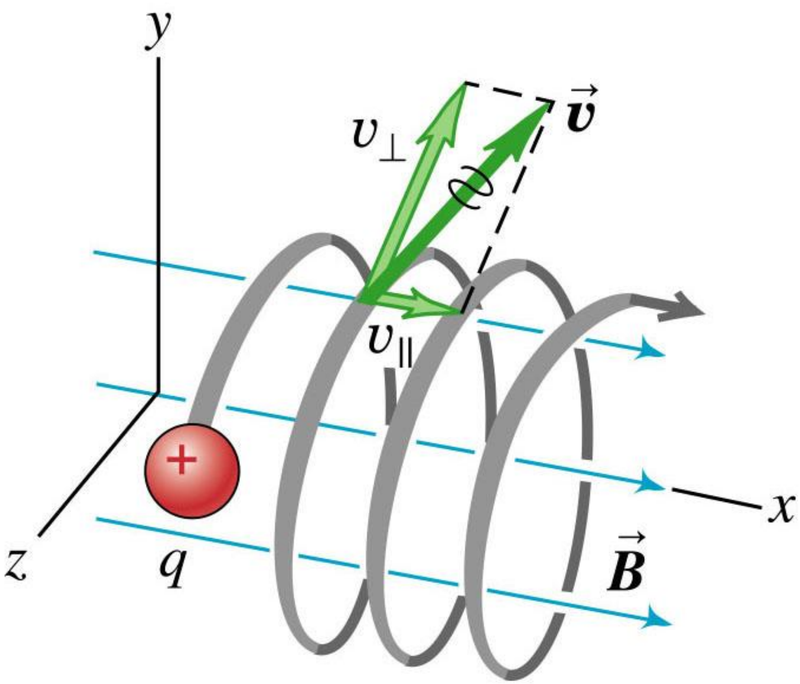
- Because the magnetic force is always perpendicular to $\vec{v}$, it will drive the charge in a **circular orbit**



- This is a *centripetal force*
- Hence $F_B = qvB = \frac{mv^2}{r}$, where m is the mass of the charge q, and r is the radius
- **Radius of orbit** $r = \frac{mv}{qB}$
- **Time period** $= \frac{2\pi r}{v} = \frac{2\pi m}{qB}$

Motion of particle in a $\vec{B}$-field

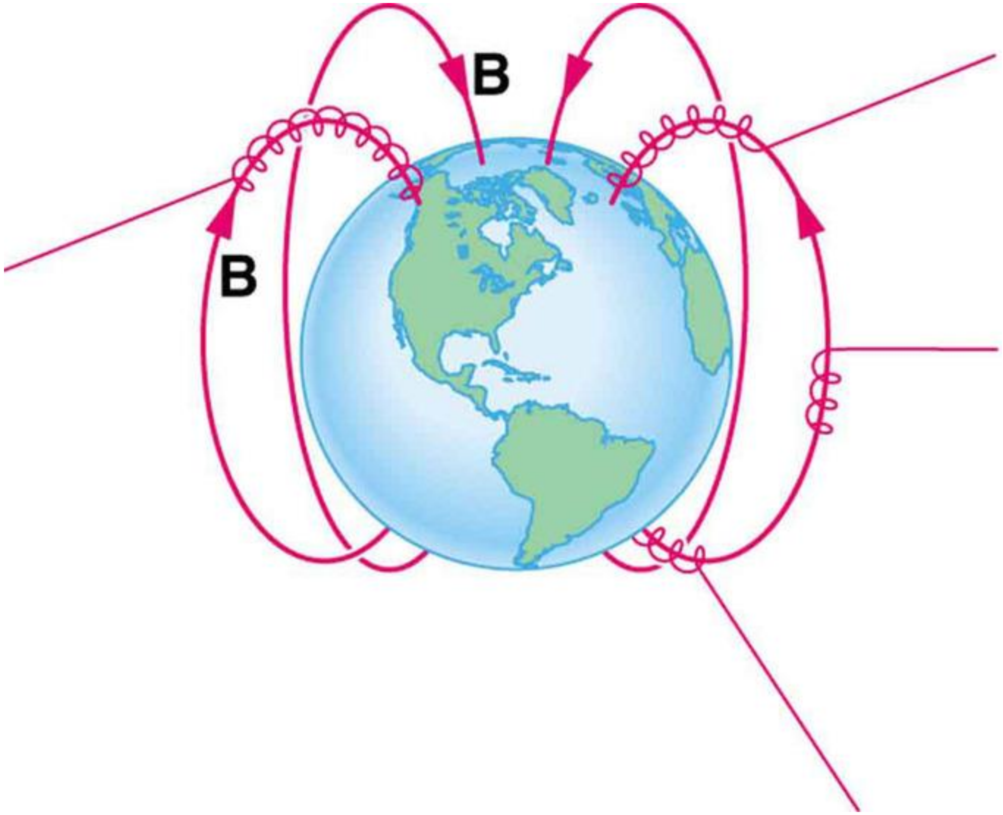
- What is the path if the charge also has a velocity component $v_{\parallel}$ parallel to $\vec{B}$?



- The motion will be a **helix**
- $\boldsymbol{v_{\perp}}$ **drives a circular orbit** with radius $\frac{mv_{\perp}}{qB}$ and time period $t = \frac{2\pi m}{qB}$
- $\boldsymbol{v_{\parallel}}$ **drives a translational motion** with helix separation $v_{\parallel}t = \frac{2\pi m v_{\parallel}}{qB}$

Motion of particle in a $\vec{B}$-field

- A consequence is that charged particles will tend to **follow magnetic field lines** (spiralling around them)

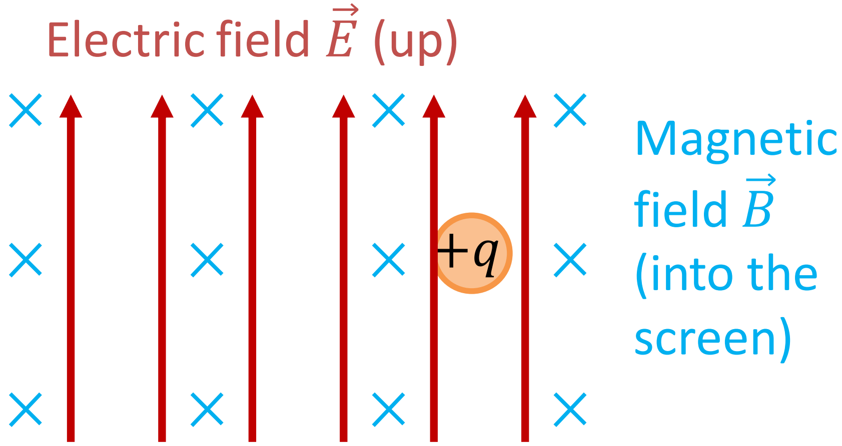


Motion of particle in a $\vec{B}$-field

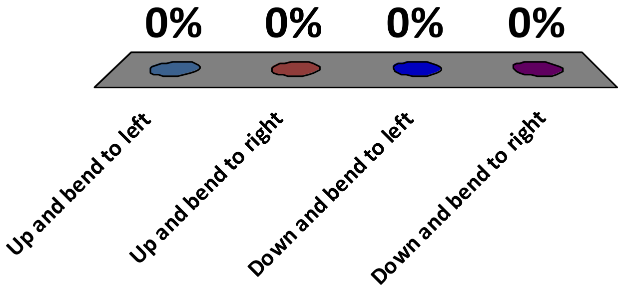
- ***How does the energy of the charge moving in a $\vec{B}$-field change?***
- A charge moving in a $\vec{B}$-field *feels a force* but *never increases its energy*
- This is because the force is always perpendicular to the motion and so *never does any work*
- Mathematically, the force does work at a rate $\vec{F}.\vec{v} = q(\vec{v} \times \vec{B}).\vec{v} = 0$ because $\vec{v} \times \vec{B}$ is always perpendicular to $\vec{v}$

Clicker question

A proton is released from rest in uniform $\vec{E}$ and $\vec{B}$-fields. Which path will it follow?



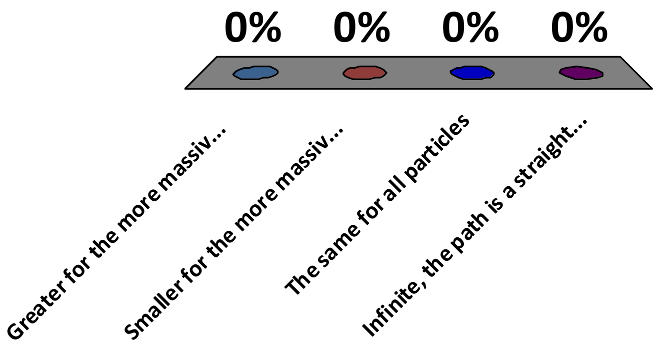
A. Up and bend to left
B. Up and bend to right
C. Down and bend to left
D. Down and bend to right



Clicker question

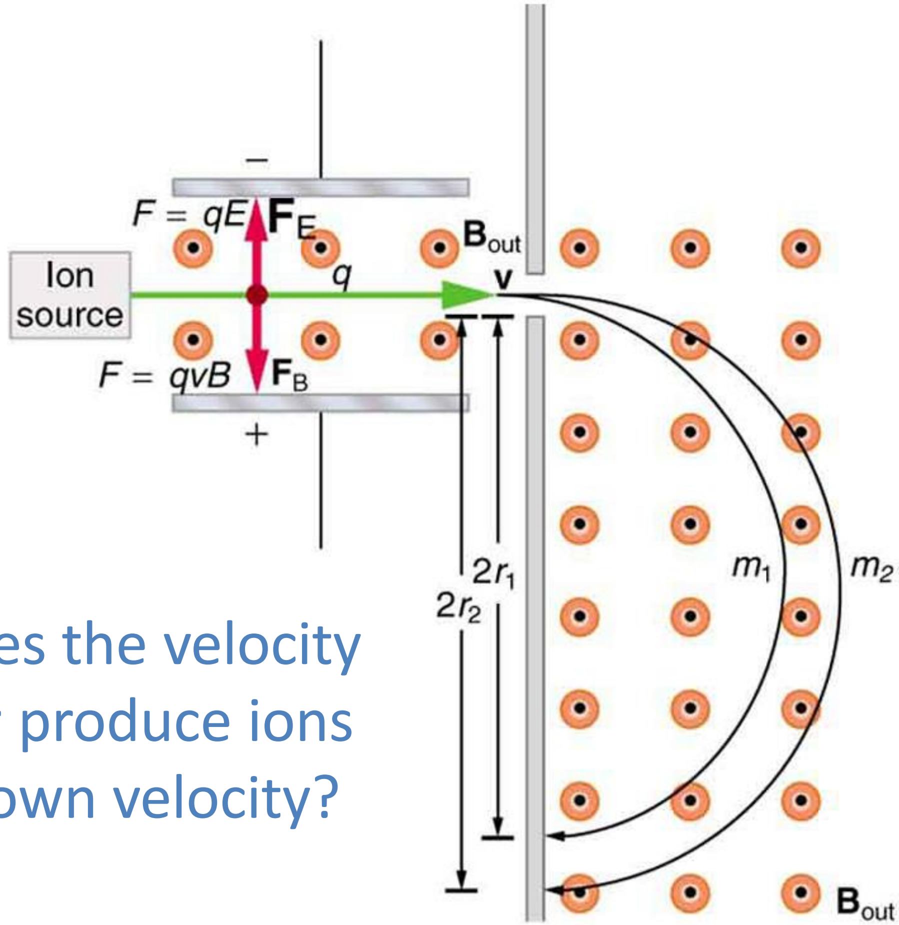
Particles with the same charge and speed, but different masses, enter a uniform magnetic field. The radius of the path they follow is:

A. Greater for the more massive particles
B. Smaller for the more massive particles
C. The same for all particles
D. Infinite, the path is a straight line



Applications

- How does a **mass spectrometer** work?

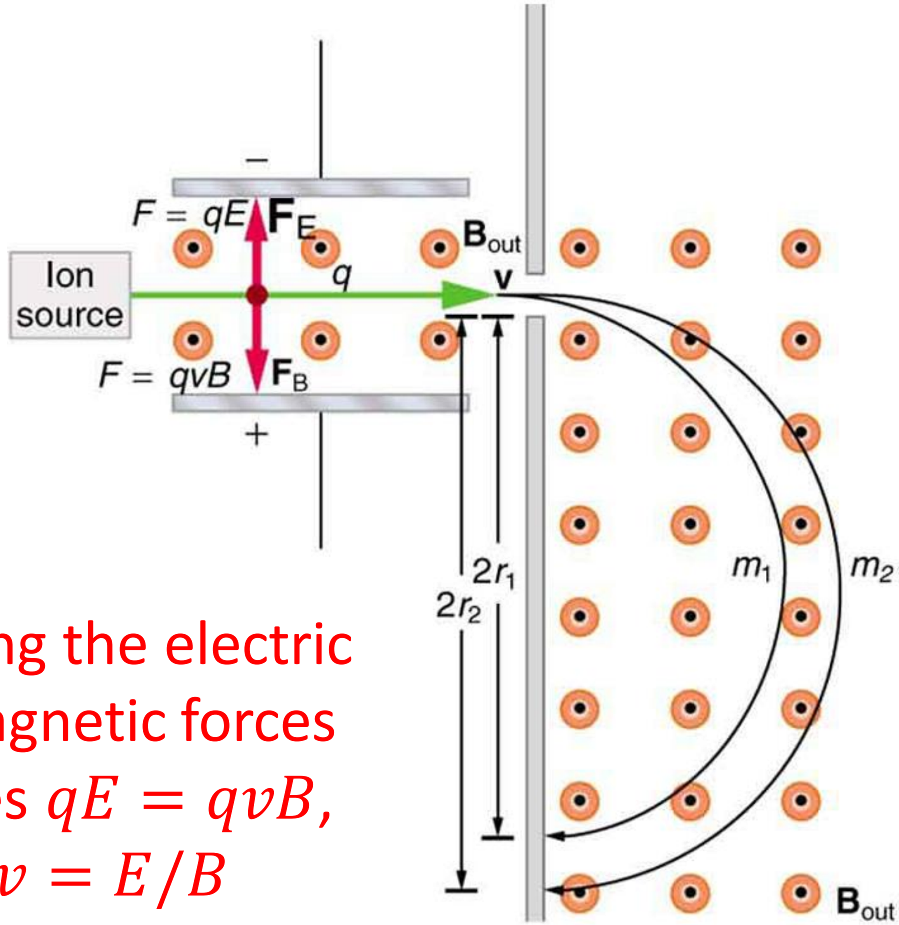


How does the motion in the magnetic field allow us to determine the mass of the ions?

How does the velocity selector produce ions of a known velocity?

Applications

- How does a **mass spectrometer** work?



The radius of orbit depends on the mass, $r = mv/qB$

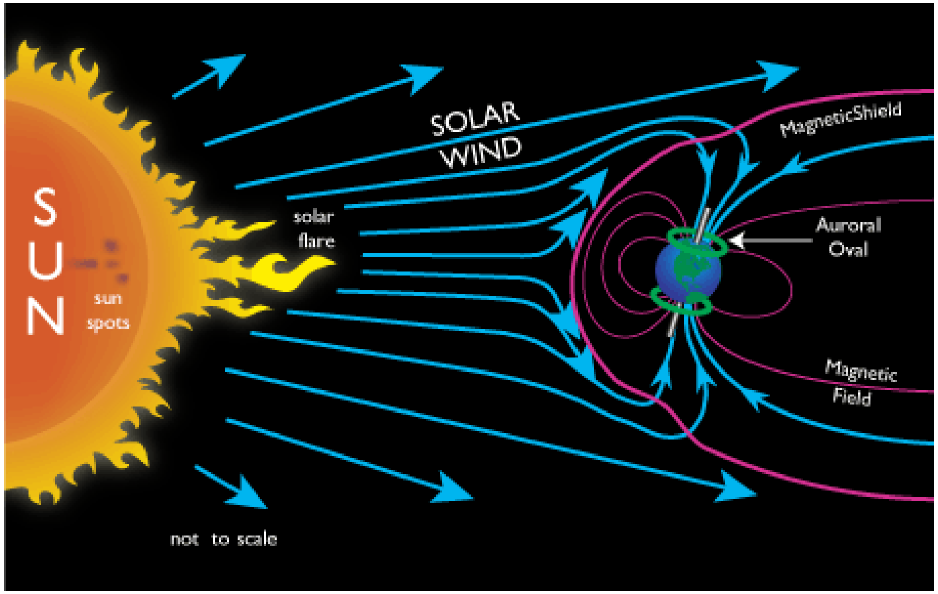
Balancing the electric and magnetic forces requires $qE = qvB$, or $v = E/B$

Applications

- What produces the **aurorae**?

What causes this emission?

Why does it occur at the poles?

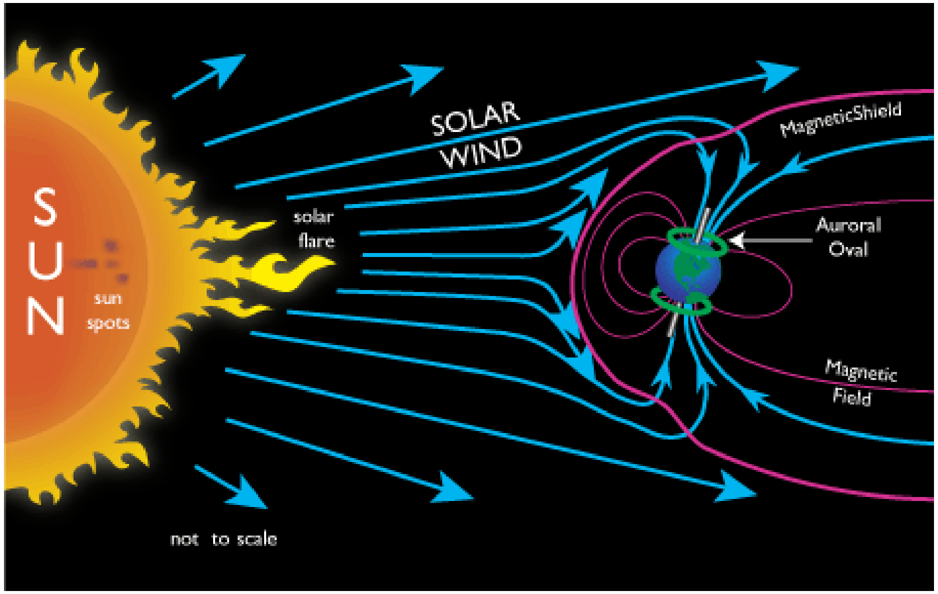


Applications

- What produces the **aurorae**?

Charged particles in the solar wind collide with atmospheric atoms, exciting them to energy levels from where they decay, emitting radiation

The Earth's magnetic field funnels the charged particles towards the poles



Applications

- How does the **Large Hadron Collider** create a circular beam of particles?

Applications

- How does the **Large Hadron Collider** create a circular beam of particles?

Superconducting electromagnets are used to produce a magnetic field which guides charged particles (e.g. protons, nuclei) around the accelerator ring

Summary

- **Magnetic forces** : A charge q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ feels force $\vec{F}_B = q\vec{v} \times \vec{B}$; a current element $I\, d\vec{l}$ feels force $\vec{F}_B = I\, d\vec{l} \times \vec{B}$

- A charged particle moving in a $\vec{B}$-field will trace out a *helical path*

- For *general motion* in crossed $\vec{E}$- and $\vec{B}$-fields, $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$