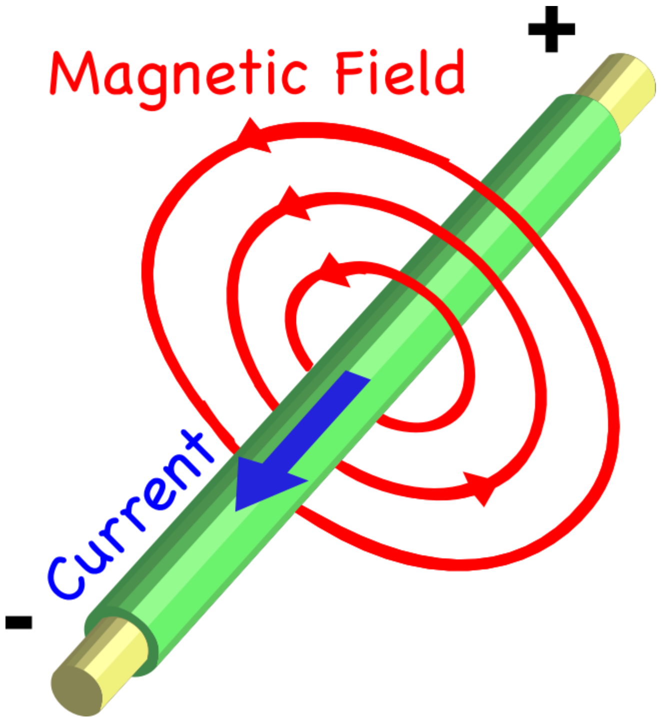
Class 11 : Magnetic materials

- Magnetic dipoles
- Magnetization of a medium, and how it modifies magnetic field
- Magnetic intensity
- How does an electromagnet work?
- Boundary conditions for $\vec{B}$

Recap (1)

- Electric currents give rise to **magnetic fields**
- The magnetic field $\vec{B}$ generated by current density $\vec{J}$ satisfies the two Maxwell equations $\vec{\boldsymbol{\nabla}}.\vec{\boldsymbol{B}} = \mathbf{0}$ and $\vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{B}} = \boldsymbol{\mu_0}\, \vec{\boldsymbol{J}}$

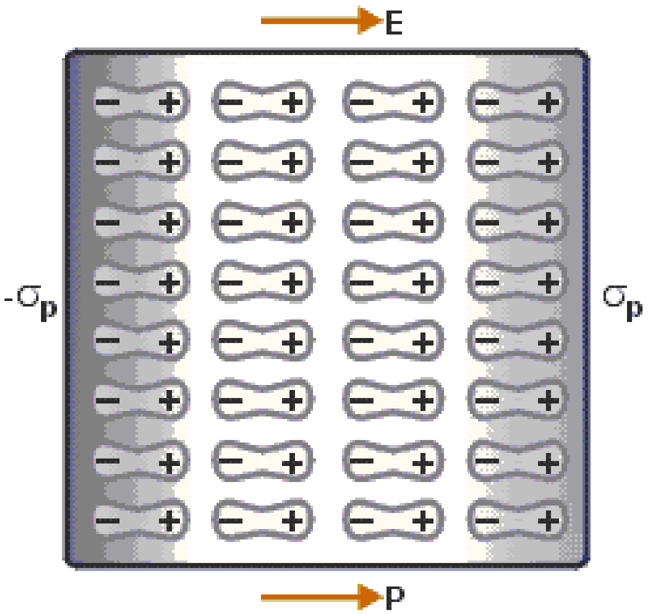


Recap (2)

- An external electric field $\vec{E}$ causes **polarization** $\vec{P}$ of insulators into electric dipoles, altering the electric field inside the material

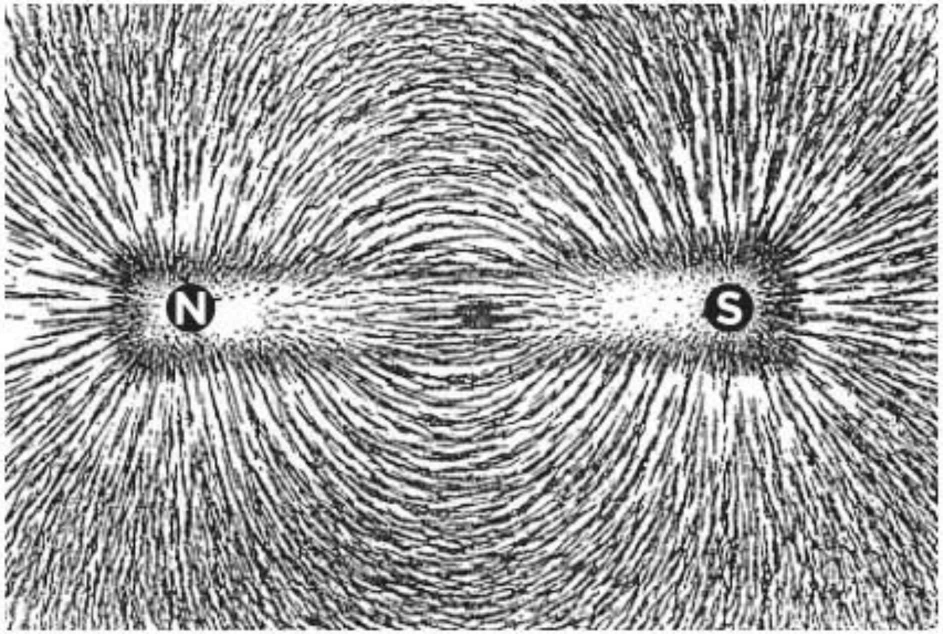
- The combined field is described by the **electric displacement** $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$, which can be used to express Maxwell's 1st Equation as $\vec{\nabla} . \vec{D} = \rho_f$

- In many materials $\vec{D} = \varepsilon_r \varepsilon_0 \vec{E}$, in terms of the **relative permittivity** $\varepsilon_r \gg 1$



Magnetization

- We often think of **magnetism** as a intrinsic property of certain materials. How can we model this phenomenon?

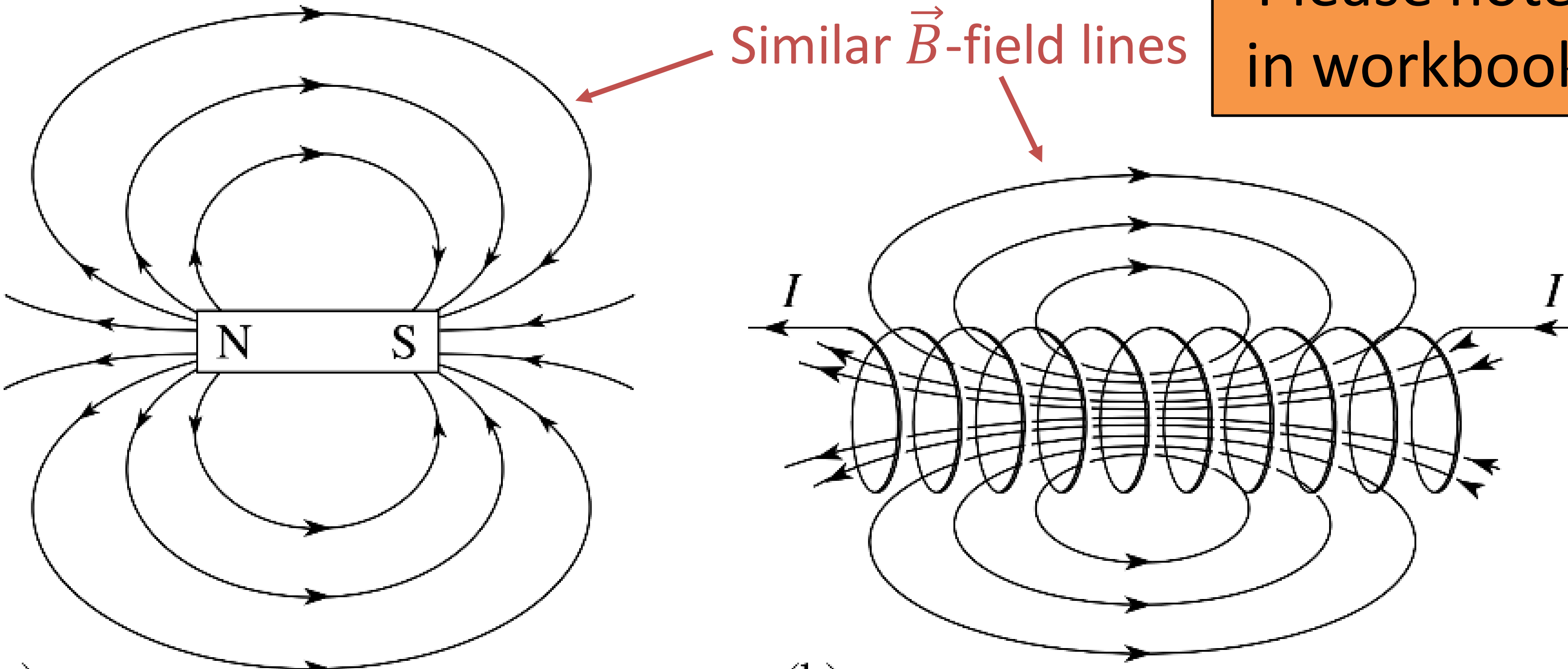


Magnetization

- The $\vec{B}$-field from a **bar magnet** (= **magnetic dipole**) is analogous to the $\vec{B}$-field from a **current loop**

Please note in workbook

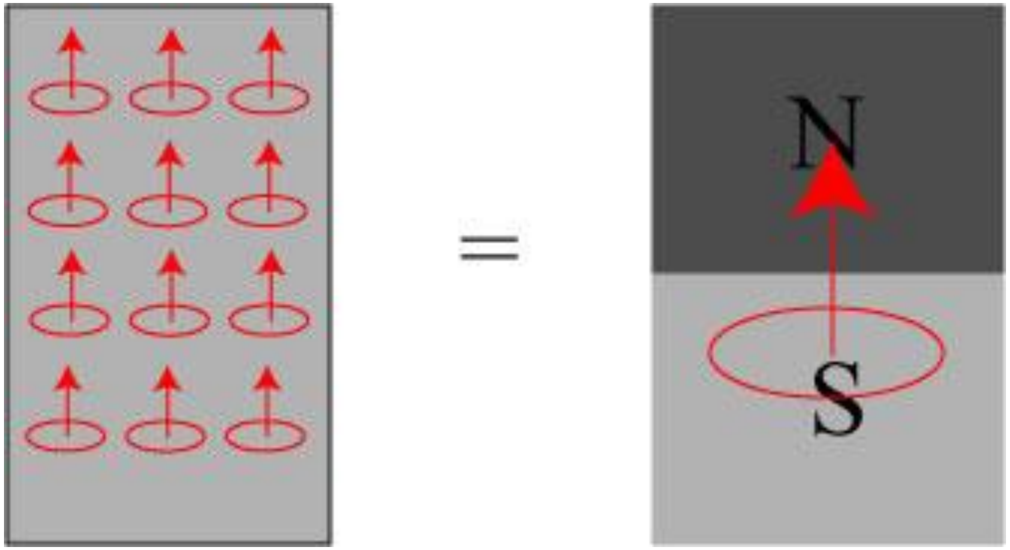
Similar $\vec{B}$-field lines



(a) (b)

Magnetization

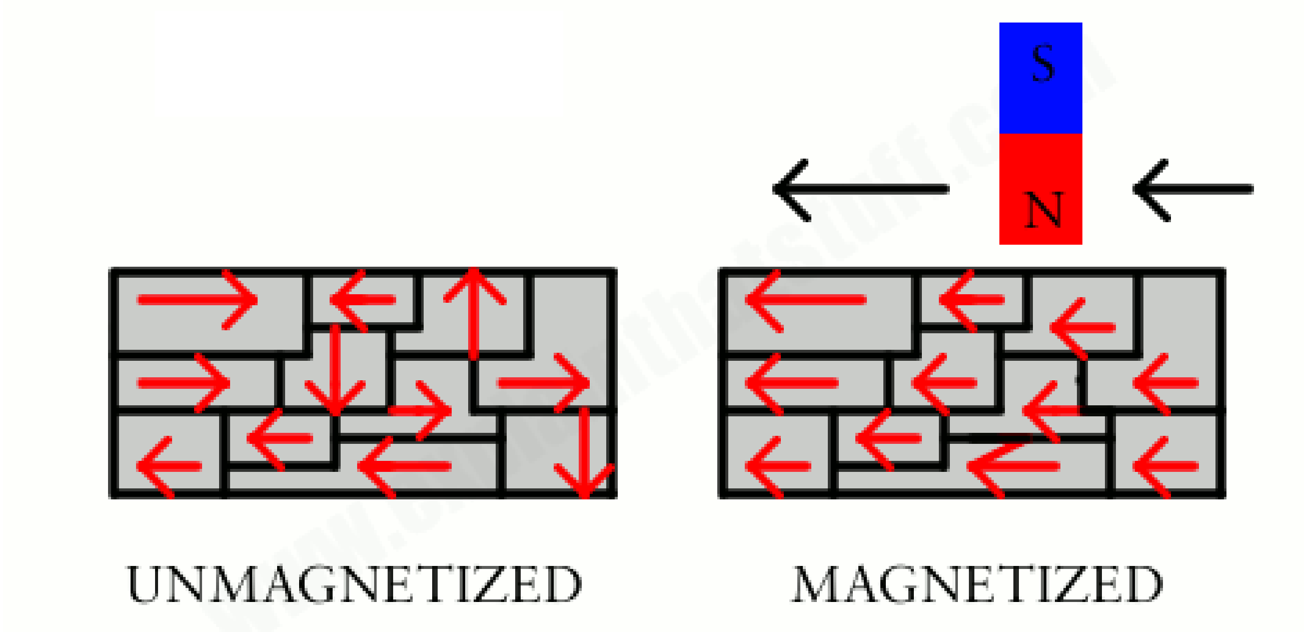
- In this sense, we can model a magnetic material via the **combined action of many tiny current loops**



- We can think of these current loops as *"electrons circulating within each atom"* or *"atomic spin"* or *"magnetic domains"*

Magnetization

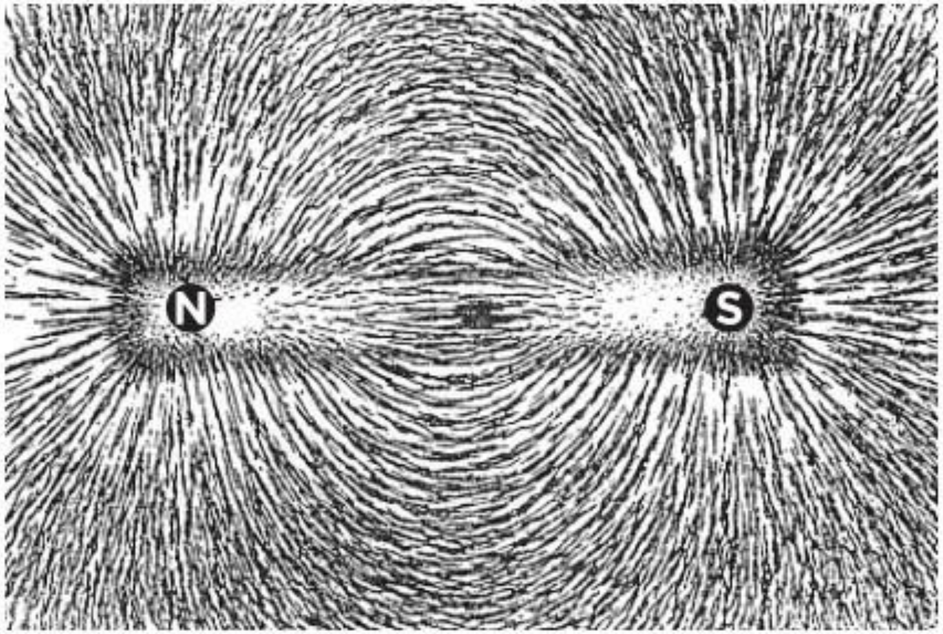
- When a material is **magnetized**, the current loops (or magnetic dipoles) align in a process of **magnetic polarization**



- We define the **magnetization** $\vec{M}$ of the material as the *magnetic dipole moment per unit volume*

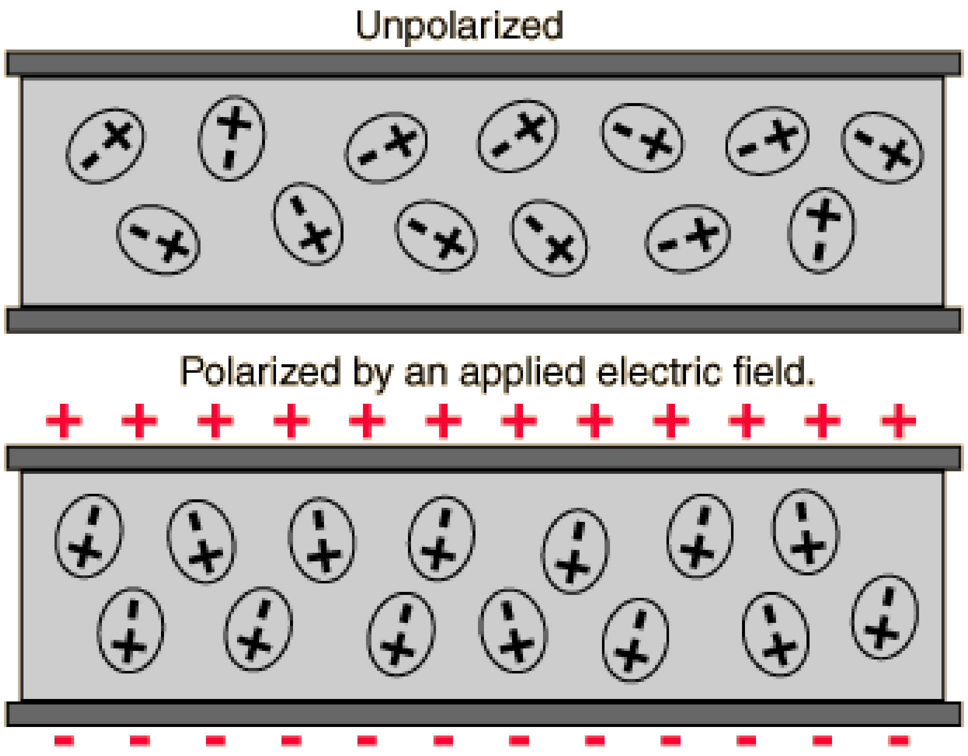
Magnetization

- **Ferromagnets** are materials in which these *magnetic domains* can readily align, such that the material becomes a *permanent magnet*

Magnetization

- This is similar to the *polarization* $\vec{P}$ *of an insulating material* into electric dipoles by an applied $\vec{E}$-field (recall, $\vec{P}$ is defined as the *electric dipole moment* per unit volume)



Magnetization

- This is similar to the *polarization* $\vec{P}$ *of an insulating material* into electric dipoles by an applied $\vec{E}$-field (recall, $\vec{P}$ is defined as the *electric dipole moment* per unit volume)

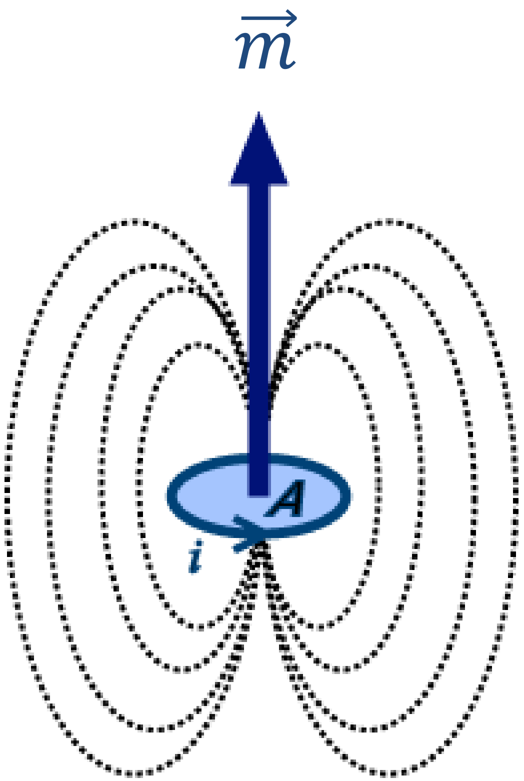
- However, note that magnetizing a medium **increases** the magnetic field ($\vec{B}$ increases), whereas polarizing a dielectric medium **reduces** the electric field ($\vec{E}$ decreases)

Please note in workbook

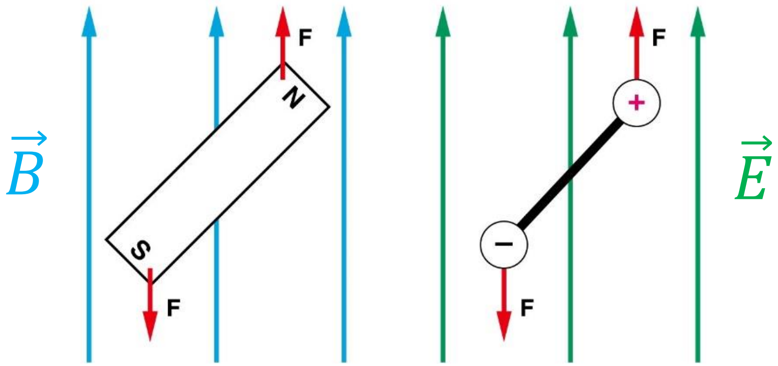
- **Magnetization is equivalent to an effective current density** $\vec{J}_m = \vec{\nabla} \times \overrightarrow{M}$ (the next three slides are optional and explain this for students who would like to follow up)

Magnetization

- The **magnetic dipole moment** $\vec{m}$ of a current loop I enclosing area $\vec{A}$ is $\vec{m} = I\vec{A}$

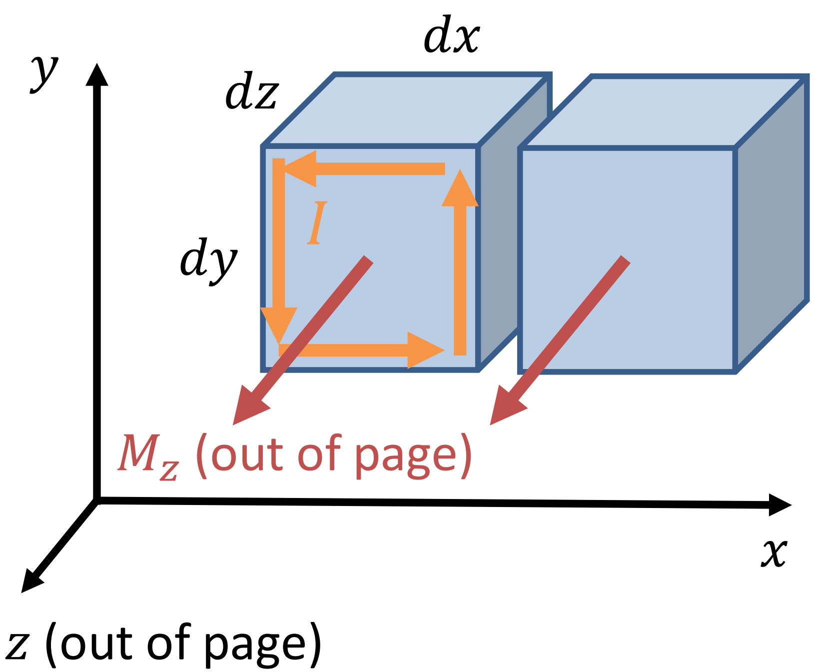


- $\vec{m}$ is analogous to the electric dipole moment $\vec{p}$ in electrostatics
- In an external electric field $\vec{E}$, an electric dipole feels a torque $\vec{p} \times \vec{E}$
- In an external magnetic field $\vec{B}$, a magnetic dipole feels a torque $\vec{m} \times \vec{B}$



Magnetization

- What is the net effect of all these current loops?

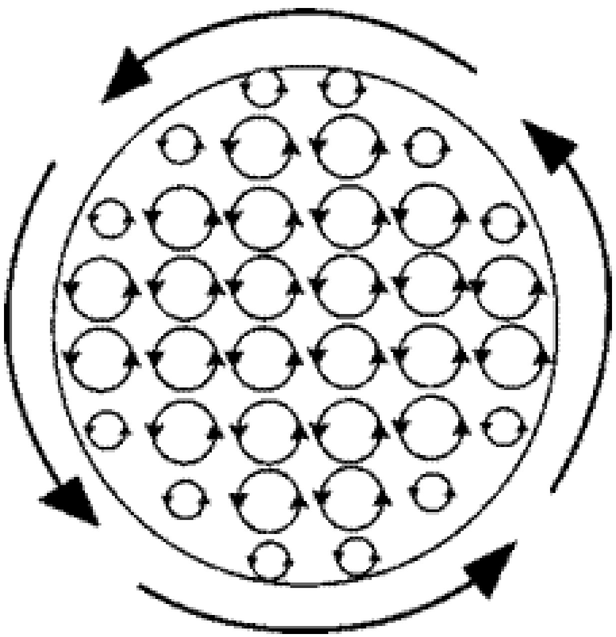


- Consider current loops in xy-plane
- Magnetic dipole moment of 1st element $= I\,dx\,dy = M_z\,dx\,dy\,dz$
- Hence, $I = M_z\,dz$
- For 2nd element, $I' = \left(M_z + \frac{\partial M_z}{\partial x}dx\right)dz$
- Net current $I_y = I - I' = -\frac{\partial M_z}{\partial x}\,dx\,dz$
- Apply the same argument to loops in the yz-plane : $I_y = \frac{\partial M_x}{\partial z}\,dx\,dz$

- Current density $J_y = \frac{I_y}{dx\,dz} = \frac{\partial M_x}{\partial z} - \frac{\partial M_z}{\partial x} = (\vec{\nabla} \times \vec{M})_y$

Magnetization

- The magnetization $\vec{M}$ of a material may hence be modelled via an effective **magnetization current** $\vec{J}_m = \vec{\nabla} \times \vec{M}$



Surface currents

- For constant magnetization, this results in an effective **surface current**
- This is analogous to the electric polarization $\vec{P}$ of an insulator being represented by an effective bound charge density $\rho_b = -\vec{\nabla}.\vec{P}$
- For constant electric polarization, this results in a layer of effective **surface charge**

Magnetic intensity

- We can use magnetization current to **re-write Ampere's Law** $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ in a form that applies in *all magnetic materials*
- Including both free current $\vec{J}_f$ and magnetization current $\vec{J}_m$: $\vec{\nabla} \times \vec{B} = \mu_0(\vec{J}_f + \vec{J}_m) = \mu_0(\vec{J}_f + \vec{\nabla} \times \vec{M})$
- We define the **magnetic intensity** $\boldsymbol{\vec{H}} = \frac{\boldsymbol{\vec{B}}}{\boldsymbol{\mu_0}} - \boldsymbol{\vec{M}}$
- Ampere's Law then takes the general form $\boldsymbol{\vec{\nabla}} \times \boldsymbol{\vec{H}} = \boldsymbol{\vec{J}_f}$ (in differential form) or $\oint \boldsymbol{\vec{H}.d\vec{l}} = \boldsymbol{I_{enc}}$ (in integral form)
- By working in terms of $\vec{H}$, we can ignore the effects of magnetization and only consider free current

Magnetic intensity

- In many materials the magnetization $\vec{M}$ is proportional to the applied field $\vec{B}$
- We can then write $\vec{H} = \vec{B}/\mu_r\mu_0$ where $\mu_r \gg 1$ is the **relative permeability** of the material

Each magnetic material will have a different permeability

Please note in workbook

Magnetic intensity

Magnetic materials

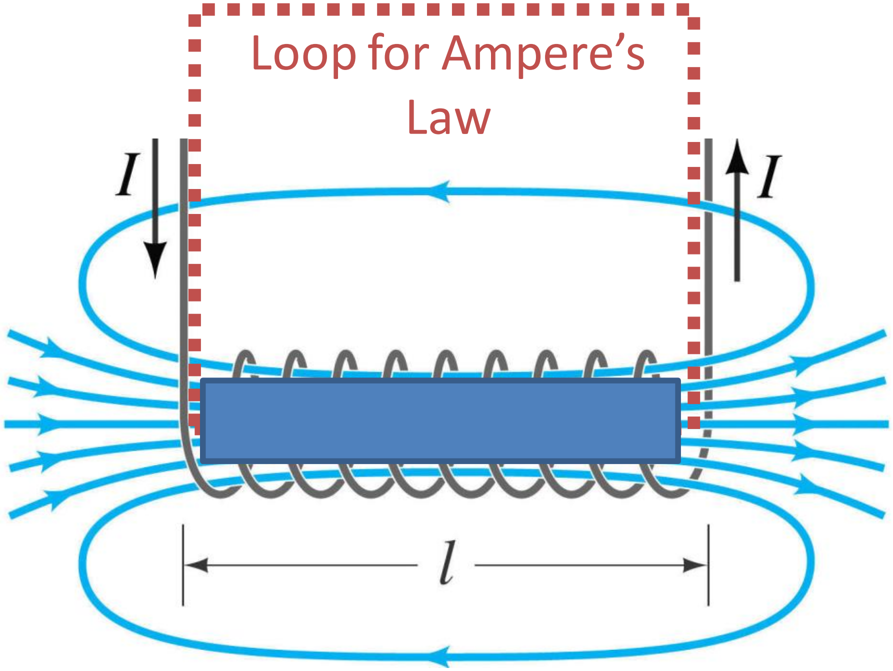
- The magnetic intensity $\vec{H}$ is produced by free currents $\vec{J}_f$
- It is computed from Ampere's Law $\vec{\nabla} \times \vec{H} = \vec{J}_f$ or $\oint \vec{H}.d\vec{l} = I_{enc}$
- The medium determines the magnetic field $\vec{B} = \mu_r \mu_0 \vec{H}$, where $\mu_r \gg 1$
- The $\vec{B}$-field determines the force on a current

Dielectric materials

- The electric displacement $\vec{D}$ is produced by free charges ρ_f
- It is computed from Gauss's Law $\vec{\nabla}.\vec{D} = \rho_f$ or $\int \vec{D}.d\vec{A} = Q_{enc}$
- The medium determines the electric field $\vec{E} = \vec{D}/\varepsilon_r \varepsilon_0$, where $\varepsilon_r \gg 1$
- The $\vec{E}$-field determines the force on a charge

Electromagnet

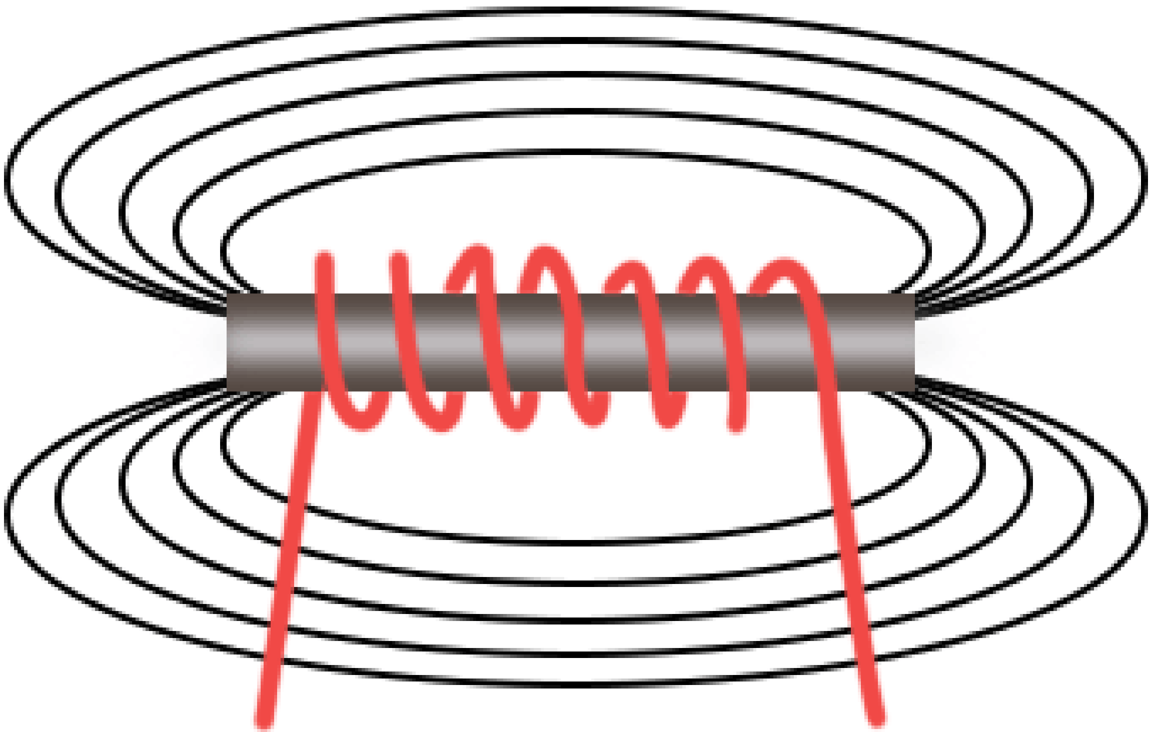
- Now, consider a solenoid with an iron core of relative permeability μ_r. What is the $\vec{B}$-field in the middle?



- Apply Ampere's Law $\oint \vec{H} \, . \, d\vec{l} = I_{enc}$ to the closed loop shown
- $\oint \vec{H} . d\vec{l} = H \, l = \dfrac{B}{\mu_r \mu_0} \, l,$ neglecting $\vec{B}$ outside coil
- $I_{enc} = N \times I$
- Hence $B = \dfrac{\mu_r \, \mu_0 \, N \, I}{l}$

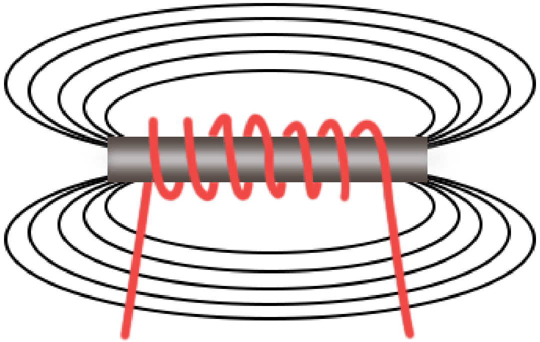
Electromagnet

- In this way, a magnetic field $\vec{B}$ generated by a solenoid can be amplified by using an iron core (since $\vec{B} = \mu_r \mu_0 \vec{H}$ and $\mu_r \gg 1$)



Clicker question

An iron bar is placed inside a solenoid to make an **electromagnet**. What can we say about $\vec{B}$ and $\vec{H}$ inside the solenoid before and after?

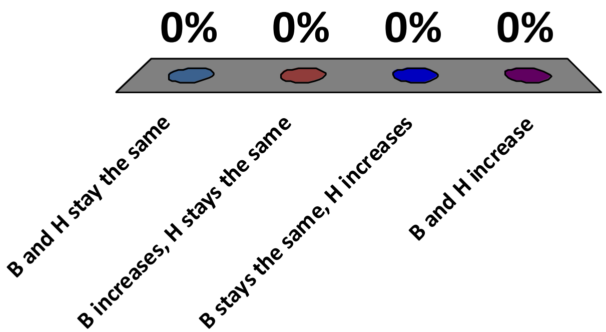


A. B and H stay the same

B. B increases, H stays the same

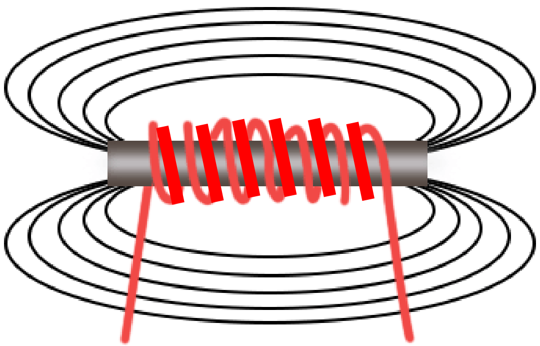
C. B stays the same, H increases

D. B and H increase



Clicker question

The number of coils of wire in the solenoid is now doubled. What can we say about $\vec{B}$ and $\vec{H}$ inside the solenoid before and after?



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B. B increases, H stays the same

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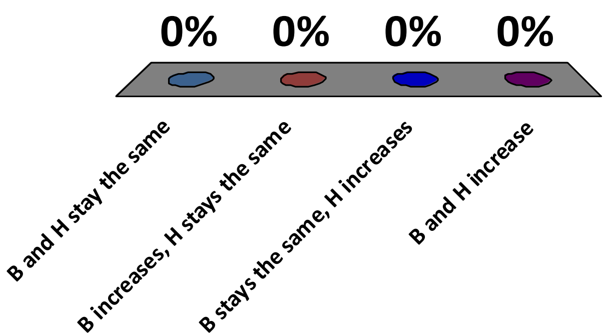
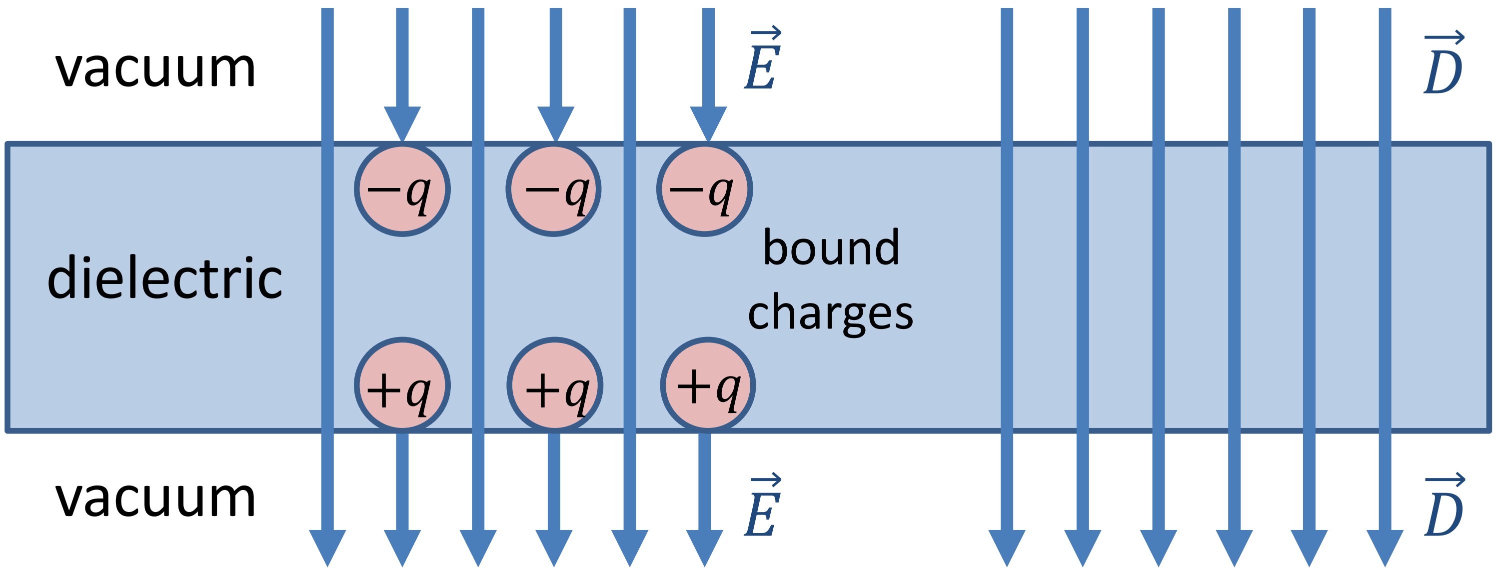


Boundary conditions

- We previously found that on the boundary between two dielectrics, the *normal component of* $\vec{D}$ and the *tangential component of* $\vec{E}$ are **continuous**

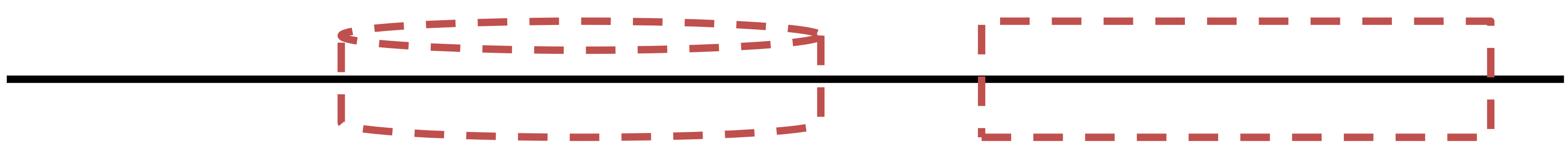


Boundary conditions

- We previously found that on the boundary between two dielectrics, the *normal component of* $\vec{D}$ and the *tangential component of* $\vec{E}$ are **continuous**

- We can similarly derive that the *normal component of* $\vec{B}$ and the *tangential component of* $\vec{H}$ are continuous

Medium 1



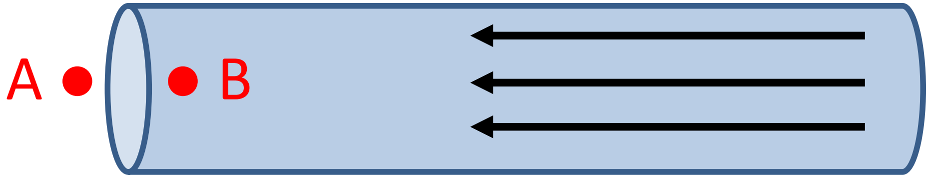
Medium 2

$\vec{\nabla}.\vec{B} = 0$ or $\int \vec{B}.d\vec{A} = 0$ $\quad\quad$ $\oint \vec{H}.d\vec{l} = 0$

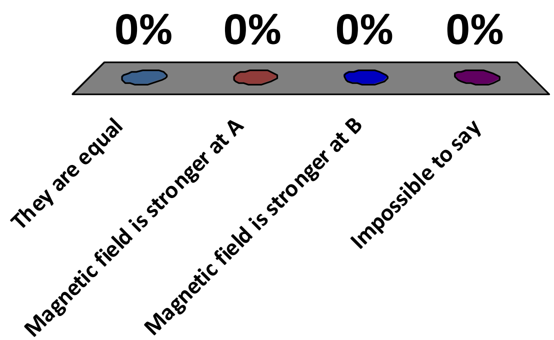
$\rightarrow B_{\perp}$ is continuous $\quad\quad$ $\rightarrow H_{\parallel}$ is continuous

Clicker question

Consider the magnetized iron core of the previous question. What can you say about the magnetic field $\vec{B}$ at A and B?



A. They are equal
B. Magnetic field is stronger at A
C. Magnetic field is stronger at B
D. Impossible to say



Clicker question

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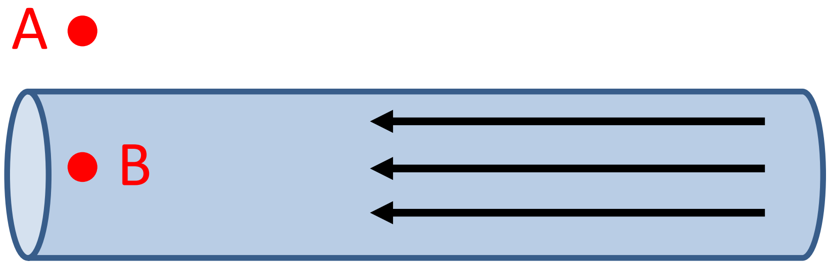

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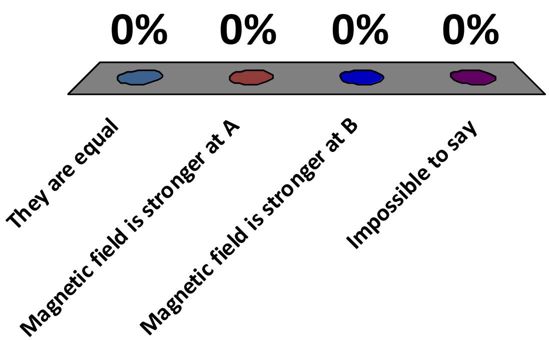

Summary

- Magnetic materials consist of *tiny dipoles* which can be modelled as *current loops*

- When a magnetic field is applied, alignment of these loops causes **magnetization** $\vec{M}$ which is *equivalent to current density* $\vec{J}_m = \vec{\nabla} \times \vec{M}$

- We define the **magnetic intensity** $\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$, in terms of which $\vec{\nabla} \times \vec{H} = \vec{J}_f$