Intro to Classical and Physical Cosmology, Feb 2007

Prof. Karl Glazebrook

Homework problem sheet

General Guidelines: If you attempt to do the problems you will gain 4-5× more value than by just listening to the lectures. Classical pedagogy states that 'doing' not 'listening' is the most effective learning technique and this is 100% correct in my experience. The only even more effective way to learn this material would be to teach your own course in cosmology!

Please feel free to consult with me and each other if you get stuck on particular points but you will gain the most if you try to solve them yourself first as much as possible. I will make my solutions available to everyone on the due date and will be available that week to explain!

Due Date: Monday, Feb 19th at 9am.

1. Show that in the radiation dominated early Universe that the scale factor evolves as $a(t) \propto t^{1/2}$. *Estimate* the redshift corresponding to baryogenesis at $t=10^{-6}$ secs.

2. If the Universe is closed and reaches a maximum size 200× it's present day value then what is the value of Ω_m? How long is it to the time of the maximum expansion and the Big Crunch? (Take h=0.7)

3. In a simple Ω_m=1, H_0 = 100 km/s/Mpc cosmology what is the *proper distance* a photon emitted at t=0, a=0 will travel in 1 billion years? How does this distance compare to 1 billion light years? Explain the discrepancy.

4. A galaxy at z=1 has a spectral *luminosity* (L_ν) of 10^{21} Watts/Hz at wavelength 2800Å. What is the observed *flux*, in μJy, at 5400Å? Convert this, roughly, to a V magnitude. [Assume a matter dominated Ω_m=0.3, H_0 = 70 km/s/Mpc cosmology].

5. Taking the speed of sound as $c/\sqrt{3}$, H_0 = 70 km/s/Mpc and $\Omega=\Omega_m$=1 work out the size of the sound horizon at the epoch of recombination (z=1100) in both co-moving Mpc and actual physical Mpc. Explain the meaning of these two scales. If the CMB is imprinted with ripples on the scale of the sound horizon what would these correspond to as an observed angular scale?

6. Taking the CMB temperature today as 3K and the baryon density as Ω_b = 0.06 work out the neutral fraction of the Universe and the mean free path at z=1100 and z=1400. How does the m.f.p. of photons at these redshifts compare to the

"size" of the Universe, explaining what you mean by "size"? [Take $H_0 = 70$ km/s/Mpc and $\Omega=\Omega_m=1$]

7. In a $\Omega=\Omega_m=1$ $h=0.5$ Universe a spherical galaxy with $10^{11}M_\odot$ of stars forms at z=7. It is 20 kpc in radius and forms all it's stars in it's dynamical time (270 Myr). At this epoch the Universe's IGM is neutral HI. If a star-formation rate of 1 $M_\odot$ per yr produces 10^{53} ionizing UV photons per second, calculate the physical size (in Mpc) of the Strömgren sphere surrounding the galaxy (take $\Omega_b=0.05$ *TODAY* to work out the IGM density at z=7). Work out the angular diameter of the Strömgren sphere in arcminutes as it appears to an observer on Earth for this cosmology.

8. Write a computer code to compute the luminosity distance (in Mpc) to z=1 and z=3 in a dark energy dominated universe with $H_0 = 70$ km/s/Mpc $\Omega_m=0.3$, $\Omega_{DE}=0.7$, w=const.= –0.8. How do the values compare with a w = –1 Universe?