MCMC and estimating the properties of binary mergers with LIGO data

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Harley Wood School June 2018
Overview

- Gravitational waves
- Observations by LIGO & Virgo
- What do we get from the detector and what can we learn about the source from the data?
- Bayesian inference
  - Parameter estimation
  - Markov Chain Monte Carlo
  - Example
- Some places to find out more
Gravitational waves

- Predicted by Einstein’s General Relativity
- First observed in 2015

Images: NASA; R. Hurt/Caltech-JPL
Searching for Gravitational Waves with Interferometers

Images: Caltech/MIT/LIGO Lab & The Virgo collaboration/CCO 1.0
What are we searching for?

- **Compact Binary Coalescence**
  - Binary black holes, binary neutron stars, black hole-neutron star binaries

- **Continuous waves**
  - Rotating pulsar / neutron star

- **Bursts**
  - Supernova, other unknown sources

- **Stochastic background**
  - White dwarfs...
What do you get from the interferometer?
What can we learn? → Masses

Image: LIGO/Virgo
What can we learn? → Sky position

LIGO/Virgo/NASA/Leo Singer (Milky Way image: Axel Mellinger)
What can we learn? → Spins

Image: LIGO/Virgo
How do you go from data to information on the sources? → inference
Bayes Theorem

- Start with the product rule for $A$, $B$ and $C$

\[
P(A, B|C) = P(A|C)P(B|A, C)
\]

\[
= P(B|C)P(A|B, C)
\]

- Equate these two expressions $\rightarrow$ Bayes Theorem

\[
P(A|B, C) = \frac{P(A|C)P(B|A, C)}{P(B|C)}
\]
Bayes Theorem

\[ P(A|B, C) = \frac{P(A|C)P(B|A, C)}{P(B|C)} \]

- \( P(A|B, C) \): posterior
- \( P(A|C) \): prior
- \( P(B|A, C) \): likelihood
- \( P(B|C) \): evidence
Bayesian inference

Normally falls into two categories

- Model selection
  - which one of two models is preferred by the data?

- Parameter estimation
  - assuming my model is true, what are the values of its parameters?

\[
P(\theta|d, M, I) = \frac{P(\theta|M, I)P(d|\theta, M, I)}{P(d|M, I)}
\]
Model Selection

- Compare two or more models

- Which model is preferred by the data?

\[
P(M_j|d, I) = \frac{P(M_j|I)P(d|M_j, I)}{P(d|I)}
\]

- Compare models with the odds ratio

\[
O_{A,B} = \frac{Z_A}{Z_B} \frac{P(M_A)}{P(M_B)} = B_{A,B} \frac{P(M_A)}{P(M_B)}
\]
Parameter Estimation

- The model $M$ is assumed

- Given the data, what values do the parameters take?

\[
P(\theta|d, M, I) = \frac{P(\theta|M, I)P(d|\theta, M, I)}{P(d|M, I)}
\]
Model

- LIGO/Virgo – waveform models with parameters
- Many parameters → large dimensional space to search → lots of computation
- Grid up parameter space??
MCMC

- Markov Chain Monte Carlo
- Metropolis Hastings Algorithm (Metropolis+1953, Hastings 1970)
Metropolis Hastings Algorithm

- Choose a starting position in parameter space, calculate the likelihood for that position

- Select a trial position to ‘jump’ to

- Accept or reject?
  - Accept criteria
  - Reject criteria

- Repeat steps 2 and 3 until you have enough samples!

→ Python notebook
Some places for more information

- Some other things with samplers
  - Checking for independent samples → auto-correlation length
  - Checking for convergence
  - How much have you updated your prior? e.g. K-L divergence
  - Other types of samplers e.g. Nested Sampling

- A few sampling codes (there are many more!)
  - Stan [http://mc-stan.org/](http://mc-stan.org/)
  - tupak [https://monash.docs.ligo.org/tupak/index.html](https://monash.docs.ligo.org/tupak/index.html)

- Getting LIGO data

- Some resources
  - Often sampling codes have good tutorials to follow
  - Gregory, *Bayesian Logical Data Analysis for the Physical Sciences*
  - Jaynes, *Probability Theory*
  - Mackay, *Information Theory, Inference, and Learning Algorithms*